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The financing of innovation: learning and stopping

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We consider the financing of a research project under uncertainty about the time of completion and the probability of eventual success. We distinguish between two financing modes, namely relationship financing, where the allocation decision of the entrepreneur is observable, and arm's-length financing, where it is unobservable. We find that equilibrium funding stops altogether too early relative to the efficient stopping time in both financing modes. The rate at which funding is released becomes tighter over time under relationship financing, and looser under arm's-length financing. The tradeoff in the choice of financing modes is between lack of commitment with relationship financing and information rents with arm's-length financing.

1. Introduction

■ **Motivation.** Typically, when decisions are made to start an R&D project or an innovative venture, much uncertainty exists about the chances of the project and about the time and capital needed to secure success. It has been estimated that it takes about 3,000 raw ideas to eventually achieve a single major commercially successful innovation (Stevens and Burley, 1997). Research and development is tantamount to winnowing down a vast amount of ideas and alternatives through trial and error, and it is therefore subject to considerable variance in terms of the time and money spent: it may be the 3rd or the 2,997th idea that, when tried out, produces a major success.

The research and development process for a new pharmaceutical product may serve as an illustration. The idea for a new drug is most likely based on some initial and very preliminary research, opening a vast field of possible combinations or ideas. The development itself requires substantial amounts of trials and investments before the value of the initial approach can be

719

assessed. More information will be produced over time as to whether the project will be successful or should be abandoned due to poor results.

The uncertainty about the time and capital required is a source of potential conflict between the financiers providing the capital and the researchers or entrepreneurs carrying out the project. The purpose of the present article is to study agency problems that are directly linked to the open-endedness of the funding in R&D projects, in particular conflicts surrounding the timing of the decision to terminate a research project. Venture capitalists often refer to the decision to discontinue a project as the most important source of conflict between them and startup entrepreneurs, since entrepreneurs almost never want to abandon a project that is under way. Entrepreneurs express a strong preference for continuation regardless of present-value considerations under most circumstances, be it because they are (over)confident or because they rationally try to prolong the search, and they tend to use their discretion to (mis)represent the progress that has been made in order to secure further funding (Cornelli and Yosha, 2003).

Agency conflicts of this kind potentially occur in every situation where a researcher or entrepreneur uses external funding for her R&D efforts, as exemplified by the following three areas. First, they will affect venture capital firms financing high-tech startups. Empirical research on the venture capital industry reveals that venture capitalists are well aware of such problems and that they go to great length to build possible safeguards into their contracts.[1] Second, the optimal financing of research is also a concern for the capital budgeting for R&D expenditures process *within* a firm. Third, the problems that we investigate arise also for governments, universities, research foundations, and other organizations that sponsor research. They need to evaluate the progress of research projects and to determine the timing for grant renewal or the decision to abandon.

In many cases, the investors in innovative projects will keep a hands-on approach on their investment. Venture capitalists are known to monitor their portfolio companies intensely, for example by soliciting reports and by visiting the company on a monthly basis, and by being involved in the decision making via board membership and other channels and control rights. Besides monitoring, venture capitalists also play an active role as advisors of startup companies, for example by getting involved in the recruitment of key employees and executives (Gompers and Lerner, 1999; Casamatta, 2003). In other words, the venture capital industry provides predominantly *relationship financing*. But not all investors in innovative startups are in fact relationship investors. Informal business angels have been characterized as being less involved in monitoring and to provide only limited advisory services (Barry, 1994; Fenn, 1997). Angel investors, who until recently channelled more money to startups than formal venture capitalists, can thus be viewed as the prototypical *arm's-length* investors in the financing of innovation.[2] Recent theory articles have cast the choice between venture capitalists and angel investors as a choice between informed and less informed investors (Chemmanur and Chen, 2003; Leshchinskii, 2003).

The distinction between relationship investors and arm's-length investors is not limited to venture capital versus angel financing. A difference between better- or worse-informed investors also arises when comparing large and experienced venture capital funds to small and young ones, close to distant, industry specialist versus generalist, independent versus corporate venture fund.

The terms relationship funding and arm's-length funding were originally introduced to distinguish between informed commercial banks and other, less-informed creditors like bondholders. In his seminal article, Rajan (1992) argues that relationship investors may use their exclusive knowledge to subsequently extract rents from successful projects. The value of relationship bank lending has been empirically confirmed (Petersen and Rajan, 1994; Berger and Udell, 1995; Degryse and Ongena, 2005). Debt in its various forms, including for example trade credit, is an

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[1] For example, the following instruments (documented in Sahlman, 1990; Hellmann, 1998; Kaplan and Stromberg, 2002; and Gompers and Lerner, 1999): Venture capitalists retain extensive control rights, in particular rights to claim control on a contingent basis and the right to fire the founding management team; they keep hard claims in form of convertible debt or preferred stock, underpinning the right to claim control and abandon the project; and staged financing and the inclusion of explicit performance benchmarks make it possible to fine-tune the abandonment decision.

[2] Chemmanur and Chen (2003) discuss the flow of funds estimation.

important source of funding for startup firms (Berger and Udell, 1998), and credit markets offer a natural choice between relationship lending and credit with a greater informational distance.[3]

□   **Analysis.** We examine a stylized model of the funding of a research project where the merit of an idea and the time and money needed for completion are uncertain. We specifically investigate how stopping decisions are taken in the presence of agency conflicts in the form of entrepreneurial opportunism. The project will succeed with a positive probability in every period in proportion to the volume of funds provided, so that uncertainty is represented by a simple stochastic process. As continued research efforts are undertaken and no success is forthcoming, Bayesian learning will lead to a gradual downgrading of the belief in the project's prospects. The project either ends with a success or will eventually be abandoned in the light of persistent negative news. We assume that the time horizon itself is infinite, to address the essence of the uncertainty about the time to completion, but abandonment will occur in finite time.

The entrepreneur controls the allocation of the funds. She can choose to invest the funds efficiently into the project or to divert them to private ends. This agency conflict is rich because of the dynamic nature of the investment problem. When diverting the funds, the entrepreneur not only enjoys the immediate benefit from consuming the money meant for investment. She also secures the option of continued funding in the future, since nothing can be learned about the project when the funds are not invested as supposed. Thus, the entrepreneur's discretion over the funds is intimately linked to the timing of the abandonment decision.

We consider a sequence of short-term contracts that in our setting is equivalent to requiring that the contract, short or long term, can be renegotiated at all times. Thus, any decision to abandon the project after a given horizon of funding, or to reduce the speed at which funding is released, must be time-consistent. A fundamental contribution of our analysis is the fact that we embed the agency conflict about the use of resources in the context of an open-ended funding horizon, coupled with the requirement that any equilibrium be renegotiation-proof.

We model relationship and arm's-length financing by distinguishing whether the investor can or cannot observe the entrepreneur's investment decision.[4] With relationship financing, the action of the entrepreneur is *observable,* or more precisely "observable, but not verifiable" as it is usually described in the incomplete-contracts literature, and the environment is at all times one of symmetric information. With arm's-length financing, actions by the entrepreneur are *unobservable*, and we are investigating a standard moral hazard problem between investor and entrepreneur.

The basic conflict between entrepreneur and investor can be described as follows. For the entrepreneur, the project represents the possibility to win a single large prize, but also a stream of rents that she could possibly divert to her private ends. The tension between investing and diverting the funds is accentuated by the fact that the successful completion of the project automatically stops the flow of funds. The direct incentives for the entrepreneur then have to be adequate to offset the possible loss in future rents, hence they have to be increasing in the volume of future funding that the entrepreneur expects in equilibrium.

The combination of an infinite funding horizon and contract renegotiation becomes truly important in light of this fundamentally intertemporal nature of the incentive problem. The longer the funding horizon, the more valuable the entrepreneur's option to increase the probability of access to future funding by diverting funds. The natural candidate for a contractual remedy would be to declare *ex ante* that no financing will be provided after a certain funding horizon, but such a commitment would necessarily not be time-consistent by the nature of the problem presented here.

In the equilibrium analysis, we examine how entrepreneur and investor share the proceeds of the project as a function of the elapsed time, and whether funding is released at the efficient rate and until the efficient stopping point, by first considering *relationship financing* (observable actions).

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[3] In our model (as in countless others), debt and equity funding are indistinguishable because there is only a single positive cash flow realization, R, that is to be split between entrepreneur and investors.

[4] We would like to thank Patrick Bolton for a suggestion to include this distinction.

The information about the project is then always common for both parties, and funding renewal is negotiated under symmetric information. As funding continues and the outlook becomes less promising, the participation constraint of the investor leaves less for the direct incentive of the entrepreneur. At some point, this residual will fall short of what is needed to provide incentives. The only possible solution is that the investor slows down the release in funds, which happens in the form of a reduced funding rate by the investor. This reduces the entrepreneur's option value of prolonging the project, and the incentive constraint can be met again. As time goes on and the posterior belief decreases, the slowdown in funding becomes more serious, and funding will come down to a trickle as the belief approaches the final abandonment point, which is too early relative to the efficient policy.

As we consider the case of *arm's-length financing* (actions are unobservable), we need to take into account the dynamics of the moral hazard problem. The moral hazard problem about the entrepreneur's decision in the current period translates into an adverse selection problem about beliefs in future periods. For the entrepreneur, control over the investment flow also means control over the information flow, knowing that the private beliefs of entrepreneur and investor about the project can diverge. We find that while the tension between immediate incentives and intertemporal rents remains, there is one subtle, yet important, difference in the value of a deviation for the entrepreneur. With symmetric information, the entrepreneur could renew her proposal after a deviation based on the belief held in the previous period, since nothing in the perception of the project has changed on either side. In contrast, with unobservable actions, the investor will automatically downgrade his belief after a deviation and insist in the continuation game to be compensated on the basis of his belief, which is more pessimistic than warranted. This change in the belief limits the maximal financing horizon, which relaxes the incentive constraint and facilitates funding. On the other hand, the entrepreneur commands an additional information rent since she controls the information flow.

We are then in a position to compare the overall efficiency of arm's-length and relationship financing. We identify the following basic tradeoff: Under relationship financing, there is no informational asymmetry, and the information rent that compensates the entrepreneur for her control of the information flow can be saved; but under arm's-length funding, the investor is committed to stick to a finite stopping time, reducing the option value of the entrepreneur to prolong the project through deviations. We find that the second effect always dominates and arm's-length contracts allow for a higher project value.

The equilibrium is shown to be unique in both cases. We require the equilibrium to be weakly renegotiation-proof, meant to capture the inability of entrepreneur and investor to prevent recontracting or renegotiation. More precisely, we first derive the unique Markov equilibrium and then show that this equilibrium is identical to the equilibrium derived under the renegotiation-proofness assumption. We argue that our results are consistent with the typical financing cycle of startup firms where relationship financiers are gradually replaced by arm's-length sources.

In conclusion, the fundamental contracting difficulty in this article comes about by the combination of an infinite funding horizon and the possibility of renegotiation. As our analysis shows, the only way to resolve these conflicts is via *delays* in the financing of innovation—in the form of a slowdown in the release of funds. The temporal occurrence of these delays depends on the informational relationship between investor and entrepreneur: it will be frontloaded for profitable projects under arm's-length funding, and backloaded in all other cases.

☐ **Related literature.** Besides the articles already mentioned, in particular the ones on relationship financing, our article is related to three distinct strands of the literature. First, it is linked to the literature on the financing of innovation and venture capital, in particular articles focusing on the entrepreneur's discretion to influence the stopping decision in innovative and risky projects. Qian and Xu (1998) observe that soft budget constraint problems of this kind are endemic in bureaucratic systems of R&D funding. Cornelli and Yosha (2003) address the window-dressing of performance signals to have the project continued. Dewatripont and Maskin (1995) note that

having multiple investors may be a device to mitigate this problem. In the venture capital literature, moral hazard–driven stopping problems have served as the background to explain the use of such remedies as stage financing, convertible securities, and the dismissal of incumbent managers (e.g., Repullo and Suarez, 2004; Hellmann, 1998). But all contractual devices of this sort are in principle open to renegotiation. To fully account for the relevant time-consistency issues, it seems desirable to go beyond the static (two- or three-period) models employed in this literature. The question is what happens if the horizon is extended and time-consistent devices to commit to an abandonment decision are not available. This is the starting point of our article.

Second, our problem is related to a strand of the incomplete-contracts literature that investigates what is known as the strategic default problem (e.g., Hart and Moore, 1994, 1998; Bolton and Scharfstein, 1996). In this literature, the agent can threaten to default on obligations despite being solvent, and the principal's power to liquidate assets or dismiss the agent enforces payment. Our model is different in that there is only a single cash flow, with uncertain arrival time, and that the outlook of the project under consideration deteriorates over time. We are closest in spirit to three infinite-horizon models of strategic default. Gromb (1994) investigates repetitions of projects à la Bolton and Scharfstein (1996) and finds that the efficiency of the best feasible contracts deteriorates as the horizon of such repeated investments is extended. DeMarzo and Fishman (2000) address long-term contracts and the agent's ability to save. In Fluck's (1998) repeated game, only infinite-maturity outside equity can solve the agent's as well as the principal's incentive problem. Neher (1999) considers a variant of Hart and Moore (1994) where delay via staged financing enables the build-up of collateral, in contrast to our model where delay reduces the present value of future rents.

Third, our article is clearly related to literature on the advantages of arm's-length relationships in agency models. It is closest to Crémer (1995), who shows that better information about the agent's circumstances makes it more difficult for the principal to commit to sanctions. Marquez (1998) explores this idea in the context of financial contracts and relates it to competition as an alternative commitment device.

In an earlier article, Bergemann and Hege (1998) undertake a preliminary analysis of the same basic model, but the present article goes beyond the earlier one in two important dimensions. First, it thoroughly allows for renegotiation, which is entirely ignored in the earlier article. Renegotiation is at the heart of our dynamic agency problem, since the entrepreneur would like to commit *ex ante* to a finite funding horizon when she encounters financing constraint, but such a commitment would necessarily be time-inconsistent. Second, Bergemann and Hege (1998) examine only the case of unobservable actions, whereas here we account for relationship funding, which is more typical for the funding of innovative projects and puts the comparison between relationship and arm's-length financing at center stage.

The article is organized as follows. The model is formally presented in Section 2. We consider a two-period version of our model in Section 3. The equilibrium analysis begins in Section 4 with observable actions of the entrepreneur. In Section 5 we examine equilibrium financing when the allocation decision of the entrepreneur is unobservable to the investor. The structure and efficiency of the equilibria under symmetric and asymmetric information are compared in Section 6. We present some concluding remarks in Section 7. The proofs of all results are relegated to the Appendix.

2 The model and first-best policy

■ We first describe the project, the investment technology, and the evolution of the posterior beliefs. Next, we introduce the contracting problem. Finally, we derive the efficient stopping posterior.

□ **Project with unknown returns.** The entrepreneur owns a project with unknown return. The project is either good with prior probability α_0 or bad with prior probability $1 - \alpha_0$. If the project is good, then the probability of success in a given period is proportional to the funds

invested into the project in that period. If the project is bad, then the probability of success is zero independent of the investment flow. The project can at most generate a *single success*, which generates a fixed monetary return $R > 0$.

More precisely, if the project is good and receives an investment flow of $c\gamma$, with $c > 0$, then the probability of success is given by γ. The parameter c thus represents the constant marginal cost of increasing the success probability. We assume that $\gamma \in [0, \lambda]$ with $\lambda < 1$. In consequence, the project can never succeed with certainty in any given period. If the project succeeds, then a cash flow R is realized and distributed among the parties, and the game ends immediately. We refer to an investment flow $\gamma = \lambda$ as *full* or *maximal* funding, and to an investment $\gamma < \lambda$ as *limited* or *restricted* funding.

The uncertainty about the nature of the project is resolved over time as the flow of funds either produces a success or leads to a stopping of the project. The time horizon is discrete and infinite, time periods are denoted by $t = 0, 1, \ldots, \infty$, and the discount factor is $\delta \in (0, 1)$.

The investment process represents an experiment that produces information about the future likelihood of success. The current information is represented by the posterior belief α_t that the project is good. The evolution of the posterior belief α_t, conditional on no success in period t, is given by Bayes' rule as a function of the prior belief α_t and the investment flow γ_t:

$$\alpha_{t+1} = \frac{\alpha_t(1 - \gamma_t)}{1 - \gamma_t\alpha_t}. \tag{1}$$

The posterior belief α_t decreases over time if success doesn't arise. The decline in the posterior belief is stronger for larger investments flows γ_t as the agents become more pessimistic about the likelihood of future success. The posterior belief changes only slowly for very precise beliefs about the nature of the project, i.e., if α_t is close to either zero or one. Correspondingly, the event of no success is most informative with diffuse beliefs, or when α_t is close to $1/2$.

We refer to the special case of $\alpha_0 = 1$ as the "certain success project" or the shorter "certain project." With $\alpha_0 = 1$, the posterior never changes as the agents do *not* entertain the possibility that the project might be bad and α_t remains at $\alpha_t = 1$ for all $t \geq 0$. For this obvious reason, we refer to the case of $\alpha_0 = 1$ as the "certain success project" or "certain project." As a consequence of the constant posterior beliefs of $\alpha_t = 1$ for all $t \geq 0$, if funding at $\alpha_0 = 1$ can be provided at the maximal level λ, then it will be provided forever until the project succeeds, hence "certain project."

□ **Contracting.** The entrepreneur has initially no wealth and seeks to obtain external funds to realize the project. Financing is available from a competitive market of investors, which is represented in the model by a *single* investor who can only accept or reject contract proposals by the entrepreneur. Entrepreneur and investor share initially the same assessment about the likelihood of success represented by the prior belief α_0. The funds are supplied by the investor, and the entrepreneur controls the allocation of the funds. She can either invest the funds into the project or divert the capital flow to her private ends. We assume that the entrepreneur consumes any diverted funds immediately, i.e., she cannot accumulate funds in order to finance the project on her own in the future.[5]

The time structure in every period t is as follows. At the beginning of period t the entrepreneur can offer the investor a *share contract* s_t and a success probability $\gamma_t \geq 0$. The share s_t represents the share of the entrepreneur in the proceeds if the project succeeds in period t. The investor receives the remaining share $1 - s_t$. The restriction to share contracts is without loss of generality due to the binary nature of the project. After the contract proposal, the investor can decide whether

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[5] Alternatively, the same incentive constraints would arise if the efficient investment of the funds required costly effort by the entrepreneur. In this case the entrepreneur evidently cannot save any funds that have not been invested. We are grateful to an anonymous referee for pointing out this interpretation. In both cases, one could imagine that one unit diverted from the funds increases the entrepreneur's utility by a monetary equivalent of only $\beta < 1$ units. The smaller is β, the less attractive is diversion and hence the less acute is the agency problem that we study.

FIGURE 1

TIMELINE OF EVENTS
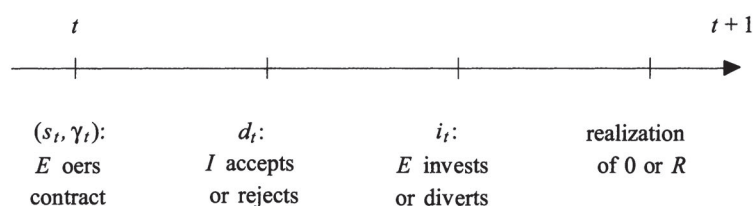
to accept or reject the new contract (decision d_t). If he accepts the contract, then he provides the entrepreneur with the requested funds $c\gamma_t$ in period t to support the development of the project. If he rejects the contract, then a new proposal can be made by the entrepreneur in the subsequent period. Finally, and conditional on funding, the entrepreneur decides whether to invest the funds in the project or divert them to her private ends (decision i_t). The sequence of decisions in every period is illustrated by Figure 1.

□ **First-best policy.** The project should receive funds as long as current expected returns of the investment exceed costs:

$$\alpha_t \gamma_t R - c\gamma_t \geq 0.$$

As both return and cost are linear in γ_t, it follows that if investment is socially efficient, it should occur at the maximal level, or $\gamma_t = \lambda$. The project should receive its final investment at the lowest α_T where the current net return is positive:

$$\alpha_T \lambda R - c\lambda \geq 0,$$

which yields a socially efficient stopping point, described in terms of the posterior belief, at

$$\alpha_T \lambda R - c\lambda = 0$$

or, equivalently,

$$\alpha^* = \frac{c}{R}. \tag{2}$$

The social value of the project, denoted by $V(\alpha_t)$, is given by a familiar dynamic programming equation:

$$V(\alpha_t) = \max_{\gamma \in [0,\lambda]} \{\alpha_t \gamma R - c\gamma + \delta(1 - \gamma\alpha_t)V(\alpha_{t+1}(\gamma))\}.$$

The value of the program can be decomposed into the flow payoffs and continuation payoffs. The flow payoffs are the returns multiplied by the current probability of success minus the investment costs. The continuation payoffs arise conditional on no success, or with probability $(1 - \alpha_t \lambda)$, in which case the future is assessed at a new posterior, namely α_{t+1}. The efficient stopping condition (2) can be recovered from the dynamic programming equation at α_T by setting $V(\alpha_{T+1}) = 0$.[6]

The value of the project under the first-best policy can be determined as

$$V(\alpha_0) = \alpha_0 \lambda (R - c)\frac{1 - \delta^{T^*}(1 - \lambda)^{T^*}}{1 - \delta(1 - \lambda)} - (1 - \alpha_0)c\lambda\frac{1 - \delta^{T^*}}{1 - \delta},$$

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[6] The *intertemporally* optimal stopping point is thus determined by a *static* revenue condition. The stopping point condition does not include any intertemporal element in terms of a value of information, as the posterior belief (conditional on no success) declines deterministically and thus there is no option value in the evolution of the posterior belief.

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where T^* is the maximal number of periods such that the updated belief after T^* periods of full funding is still weakly above α^*, or

$$\frac{\alpha_0(1 - \lambda)^{T^*}}{\alpha_0(1 - \lambda)^{T^*} + 1 - \alpha_0} \geq \alpha^*.$$

3. A simple two-period model

■   This section presents some basic insights and tradeoffs in a simple two-period example with $t = 0, 1$. We compare the funding decision in the symmetric- and asymmetric-information environments.

In this finite-horizon setting, we can analyze the contracting equilibrium by backward induction. We shall start with the symmetric environment. Suppose then that in the final period, investor and entrepreneur share a common posterior α_1 that describes the probability that the project is good. If the entrepreneur makes a funding proposal (s_1, γ_1), then the investor will accept it only if his participation constraint is satisfied, or

$$\gamma_1 \alpha_1 (1 - s_1) R \geq c \gamma_1, \tag{3}$$

and if the incentive constraint for the entrepreneur is satisfied as well:

$$\gamma_1 \alpha_1 s_1 R \geq c \gamma_1. \tag{4}$$

The incentive constraint simply requires that the expected return from allocating the funds properly exceeds the value of a diversion. Due to the linear structure of the model, we can safely neglect partial diversion. In other words, inequality (4) also guarantees that the incentive constraints for partial diversion, or

$$\gamma_1 \alpha_1 s_1 R \geq c \gamma + (\gamma_1 - \gamma) \alpha_1 s_1 R,$$

are satisfied for all $\gamma \in [0, \gamma_t]$. From the participation and the incentive constraints, it follows that financing is provided only if

$$\gamma_1 \alpha_1 R \geq 2 c \gamma_1 \Leftrightarrow \alpha_1 \geq \frac{2c}{R}.$$

We can make a first observation regarding the social efficiency of the funding decision. We just saw that equilibrium funding stops at

$$\alpha_S \triangleq \frac{2c}{R}, \tag{5}$$

whereas the socially efficient stopping point is given by the posterior belief

$$\alpha^* = \frac{c}{R}.$$

Hence equilibrium funding ends too early in comparison with the socially efficient funding policy. The divergence between equilibrium and social stopping arises from the rent the entrepreneur can extract due to the nonverifiability of her allocation decision.

In equilibrium, the entrepreneur offers a break-even contract to the investor that solves his participation constraint (3) at equality, or

$$1 - s_1 = \frac{c}{\alpha_1 R} \Leftrightarrow s_1 = \frac{\alpha_1 R - c}{\alpha_1 R}.$$

The share of the entrepreneur depends on the posterior belief about the quality of the project. Her expected profit in period $t = 1$ is given by

$$\alpha_1 \gamma_1 R - c\gamma_1 \geq 0.$$

It follows that she will always suggest a maximal funding level $\gamma_1 = \lambda$. We can then describe the expected profit of the entrepreneur as a function of the posterior belief α_1 as follows:

$$V_E(\alpha_1) = \begin{cases} \alpha_1 \lambda R - c\lambda & \text{if } \alpha_1 \geq 2c/R, \\ 0 & \text{if } \alpha_1 < 2c/R. \end{cases}$$

Going backward to period $t = 0$, we find that the participation constraint of the investor remains unchanged (except that the posterior α_1 is replaced by the prior α_0):

$$\alpha_0 \gamma_0 (1 - s_0) R \geq c\gamma_0.$$

In contrast, the incentive constraint of the entrepreneur contains an intertemporal element, or

$$\alpha_0 \gamma_0 s_0 R + (1 - \alpha_0 \gamma_0)\delta V_E(\alpha_1) \geq c\gamma_0 + \delta V_E(\alpha_0). \tag{6}$$

The left-hand side of inequality (6) represents the discounted value of the funding proposal (s_0, γ_0) on the equilibrium path. If the entrepreneur allocates the funds properly, then the project is successful with probability $\alpha_0 \gamma_0$, and an agreed share s_0 of the cash flow R is paid to the entrepreneur. With the remaining probability, namely $1 - \alpha_0 \gamma_0$, the project is not successful. In this case, there is still a chance to realize the project tomorrow. The net value of this option for the entrepreneur is given by $V_E(\alpha_1)$, taking into account the update in the belief to α_1 following the the failure to succeed in period 0.

The right-hand side of the inequality (6) represents the value of a diversion, which now arises from two sources. First, there is a direct private benefit of $c\gamma_0$, but second, the failure to pursue the project today leads to the opportunity to realize it tomorrow. It should be noted that in contrast to the left-hand side of the inequality, the opportunity of pursuing the project tomorrow now arises with certainty, as the diversion of the funds guaranteed that the project could not be realized in period 0. Moreover, the prior α_0 is not updated, as no information regarding the project was generated in period 0.

By the same argument developed for $t = 1$, the entrepreneur will offer a break-even contract with maximal funding to the investor. Using the continuation value $V_E(\alpha_1)$, we can write the incentive constraint of the entrepreneur as follows:

$$\alpha_0 \lambda R - c\lambda + (1 - \alpha_0 \lambda)\delta(\alpha_1 \lambda R - \lambda c) \geq c\lambda + \delta(\alpha_0 \lambda R - \lambda c).$$

The posterior α_1 is determined via Bayes' rule (see (1)) as

$$\alpha_1 = \frac{\alpha_0(1 - \gamma_0)}{1 - \alpha_0 \gamma_0} < \alpha_0, \tag{7}$$

and after replacing the posterior belief α_1 in (7), we get the following condition on the prior for funding over two periods to be possible:

$$\alpha_0 \geq \frac{2c}{R(1 - \delta\lambda) + \delta\lambda c}.$$

We make several observations. First, for $\delta = 0$, the funding condition in period 0 is identical to the funding condition in period 1, as the entrepreneur acts purely myopically. But for all $\delta > 0$, the

funding condition becomes more severe in period 0, as the denominator is a convex combination of R and c (with $R > c$). Moreover, as δ increases, more weight should be given to c and the funding condition increases in severity. In consequence, it follows that for all α_0 satisfying

$$\frac{2c}{R(1 - \delta\lambda) + \delta\lambda c} > \alpha_0 \geq \frac{2c}{R}, \tag{8}$$

there will be no equilibrium funding in period 0, though the project will eventually be funded in period 1. The reason for the equilibrium delay in the funding decision emerges from the incentive constraint. If the project will get funded anyhow in period 1, the entrepreneur has a strong incentive to divert the funds in period 0 and thereby guarantee himself further funding and yet maintain the possibility of success in period 1. This equilibrium delay can be prevented only if the discount factor is very low, or if the project has a very high probability of success, so that the immediate rewards outweigh the future benefits.

We consider now the asymmetric-information environment. Along the equilibrium path, entrepreneur and investor maintain symmetric information, as the investor uses his equilibrium belief about the allocation decision of the entrepreneur. The sole difference arises along the possible deviation of the entrepreneur, i.e., the off-the-equilibrium-path decision. Thus, if we consider the incentive constraint of the entrepreneur in period 0, it now reads

$$\alpha_0 \gamma_0 s_0 R + (1 - \alpha_0 \gamma_0) \delta V_E(\alpha_1) \geq c\gamma_0 + \delta \frac{\alpha_0}{\alpha_1} V_E(\alpha_1), \tag{9}$$

where the only, but important, difference compared with the symmetric-information incentive constraint (6) arises on the right-hand side. Following a deviation by the entrepreneur in period 0, the investor observes only that the project did not succeed and continues to update his prior with the information coming from the failure to succeed. The investor therefore continues to hold his equilibrium belief α_1 formed by Bayes' rule as in (7). In consequence, he still wishes to be rewarded in period 1 as if the true posterior were α_1. In contrast, the entrepreneur knows that the belief α_1 is too pessimistic because conditional on the funds being diverted in period 0, no new information about the project arose, and she maintains the correct belief α_0. As the value in period 1 comes from a successful realization, the term α_0/α_1 corrects the misperception of the investor. In sum, the correct expected value of the entrepreneur following the diversion of the funds is given by $(\alpha_0/\alpha_1)V_E(\alpha_1)$. As the true probability of success is α_0, but the share is negotiated on the basis of the less optimistic belief $\alpha_1 < \alpha_0$, it follows that

$$\frac{\alpha_0}{\alpha_1} V_E(\alpha_1) < V_E(\alpha_0).$$

Since the investor's required value is larger in the arm's-length case following the investor's informational handicap, the arm's-length environment makes a deviation of the entrepreneur less attractive. This effect, which we call the *commitment effect*, is at the heart of our analysis, and we will encounter it extensively in our main analysis. Solving (9) in a manner similar to (8) leads to the condition that financing only in the second period is possible if

$$\frac{2c - \dfrac{1}{1 - \lambda}\delta\lambda c}{R(1 - \delta\lambda) - \dfrac{\lambda}{1 - \lambda}\delta\lambda c} \geq \alpha_0 \geq \frac{2c}{R}.$$

In comparison with the symmetric environment, we find that the set of prior beliefs without financing constraints is larger in the asymmetric case, since

$$\frac{2c}{R(1 - \delta\lambda) + \delta\lambda c} > \frac{2c - \dfrac{1}{1 - \lambda}\delta\lambda c}{R(1 - \delta\lambda) - \dfrac{\lambda}{1 - \lambda}\delta\lambda c}.$$

Thus, as a consequence of the commitment effect, more projects will receive immediate funding in the asymmetric-information environment than in the symmetric-information environment.

Costly delay is more frequent under symmetric information than under asymmetric information, even though it may arise in both cases.

This simple two-period model generates two important results: (i) it demonstrates the difficulties in creating efficient investment arrangements when the option of continued future funding undermines the incentives for current investment, and (ii) it shows that asymmetric information can improve the efficiency of the equilibrium by acting as a commitment device for the investor.

The two-period model contains an artificially strong discontinuity regarding the provision of incentives. In period 0, incentives to invest the funds into the project are weak, as further funding is forthcoming for sure in the period 1. In contrast, in period 1, incentives to invest the funds are strong, as by assumption no further occasions to realize the project arise. This stark contrast between the periods gave rise to the extreme nature of the equilibrium, with no funding in period 0 and maximal funding in period 1. By itself, the two-period model thus gives little guidance as to how the results carry over to a model with a more general time horizon, finite or infinite. In the remainder of the article, we shall analyze an infinite time horizon model and examine the role of the discount factor δ in the equilibrium provision of funds. The removal of the artificial final period will lead to the elimination of the discontinuity in the funding volume observed in the two-period model. We will obtain an intertemporal characterization of the funding volume that will evolve smoothly over time and display not only minimal and maximal funding levels, but typically intermediate funding at various stages of the project.

4. Relationship financing

■ In this section we analyze contracting with symmetric information, that is, the entrepreneur's actions are observable (but not verifiable) for the investor. We first define the concept of Markov-perfect equilibrium. We next investigate the properties of this equilibrium and show that it coincides with the weakly renegotiation-proof equilibrium. Finally, we discuss how the contracting results would be affected if the agents could commit to long-term contracts yet could recontract in every period.

□ **Equilibrium.** In the environment with observable actions, the information of entrepreneur and investor is symmetric in every period. Formally, we can describe the strategies in the game as follows. Let H_t denote the set of possible public histories up to, but not including, period t. A proposal strategy by the entrepreneur is given by

$$s_t : H_t \to \mathbb{R}, \gamma_t : H_t \to [0, \lambda].$$

A decision rule by the investor is a mapping from the history and the contract proposal into a binary decision to reject ($d_t = 0$) or to accept ($d_t = 1$):

$$d_t : H_t \times \mathbb{R} \times [0, \lambda] \to \{0, 1\}.$$

Finally, an investment policy by the entrepreneur is given by

$$i_t : H_t \times \mathbb{R} \times [0, \lambda] \to \{0, \gamma_t\}.$$

The above policies all describe pure rather than mixed strategies and indeed throughout the article we focus on pure-strategy equilibria. A generic public history of the game is denoted by $h_t \in H_t$ and is simply a realized sequence of offers, funding, and investment decisions:

$$h_t = \{s_0, \ldots, s_{t-1}; \gamma_0, \ldots, \gamma_{t-1}; d_0, \ldots, d_{t-1}; i_0, \ldots, i_{t-1}\}.$$

The evolution of the posterior belief α_t is not included in the history, as it can be inferred from the sequence of public funding and investment decisions by Bayes' rule. By default, updating occurs

only conditional on failure of the project, since the game ends as soon as the project succeeds and realizes the return R. Thus given any prior α_0, an arbitrary history h_t uniquely determines the current posterior belief $\alpha_t = \alpha(h_t)$. For a given quadruple $\{s_t, \gamma_t, d_t, i_t\}_{t=0}^{\infty}$ of strategies, denote the value function of the entrepreneur at the beginning of period t by $V_E(h_t)$ and the value function of the investor by $V_I(h_t)$.

We restrict our attention initially to Markovian equilibria where strategies are allowed to depend only on the payoff-relevant part of the history of the game, which in this model is fully represented by a single state variable, the posterior belief α_t in every period t. The Markov equilibrium outcome is subsequently shown to be identical to a (weakly) renegotiation-proof equilibrium outcome. Formally, a Markov-perfect equilibrium following Maskin and Tirole (2001) is defined as

Definition 1 (Markov-perfect equilibrium). A Markov-perfect equilibrium is a subgame-perfect equilibrium

$$\{s_t^*, \gamma_t^*, d_t^*, i_t^*\}_{t=0}^{\infty},$$

such that the sequence of policies satisfies $\forall \ h_t \in H_t, \forall \ h_{t'}' \in H_{t'}, \forall \ s_t, \ s_{t'}', \forall \ \gamma_t, \ \gamma_{t'}', \forall \ d_t, \ d_{t'}'$:

$$\alpha(h_t) = \alpha\left(h_{t'}'\right) \quad\quad\quad\quad\quad\quad\quad\quad \Rightarrow \quad\quad s_t^*(h_t) = s_{t'}^*(h_{t'}'), \quad \gamma_t^*(h_t) = \gamma_{t'}^*\left(h_{t'}'\right);$$
$$\alpha(h_t) = \alpha\left(h_{t'}'\right), \quad s_t = s_{t'}', \gamma_t = \gamma_{t'}' \quad\quad \Rightarrow \quad\quad d_t^*(h_t, s_t, \gamma_t) = d_{t'}^*(h_{t'}', s_{t'}', \gamma_{t'}');$$
$$\alpha(h_t) = \alpha\left(h_{t'}'\right), \quad s_t = s_{t'}', \gamma_t = \gamma_{t'}', d_t = d_{t'}' \quad \Rightarrow \quad i_t^*\left(h_t, s_t, \gamma_t, d_t\right) = i_{t'}^*\left(h_{t'}', s_{t'}', \gamma_{t'}', d_{t'}'\right).$$

$$(10)$$

Thus, a Markov-perfect equilibrium imposes the requirement that the continuation play be identical after any two histories h_t and $h_{t'}'$ with an identical belief, $\alpha(h_t) = \alpha(h_{t'}')$, noting that the particular history h_t may differ from $h_{t'}'$ in its date, $t \neq t'$, and/or in its past actions. Since the moves in any period are sequential, the relevant state for the investor is not only the belief about α_t but must also include the entrepreneur's contract offer, and similarly for the entrepreneur's final capital allocation decision. In the Markovian setup, we can write the entrepreneur's and the investor's value functions simply as a function of the current belief, $V_E(\alpha_t)$ and $V_I(\alpha_t)$, respectively.

☐ **Analysis.** Consider the situation of the investor at an arbitrary point of time. He receives a proposal by the entrepreneur to fund a project for the current period in exchange for shares in the proceeds of the project should it succeed in the current period. As the current contract commits neither investor nor entrepreneur to any future course of action, the investor is willing to accept the proposal (s_t, γ_t) as long as the expected returns are nonnegative, or

$$\alpha_t(1 - s_t)\gamma_t R \geq c\gamma_t. \quad\quad\quad (11)$$

The inequality then represents the participation constraint of the investor. However, the expected returns can materialize only if the entrepreneur decides to put the funds to work in the project rather than divert them to her private ends. This is the incentive problem of the entrepreneur.

Consider first the final period where the entrepreneur receives funding in equilibrium. This final period will arise when the belief $\alpha = \alpha_t$ has deteriorated so much that it will be impossible to solicit any future funds. In that final period the entrepreneur has to choose between investing and diverting, or

$$\alpha_t s_t \gamma_t R \geq c\gamma_t. \quad\quad\quad (12)$$

Exactly as shown in the two-period model above, jointly the inequalities (11) and (12) imply that for any funding to occur in equilibrium, the expected flow return from the investment must cover both the cost of the funds for the investor and the opportunity costs for the entrepreneur,

$$\alpha_t \gamma_t R \geq 2c\gamma_t.$$

The critical posterior belief at which funding will certainly cease is therefore given by α_S defined by the identity (5) above, where α_S is twice as large as the efficient stopping belief, $\alpha_S = 2\alpha^*$.

In all preceding periods, the incentive constraint for the entrepreneur has to take into account her future opportunities. As in the two-period model (see equation (6)), the constraint can be represented in terms of her value function:

$$\alpha_t \gamma_t s_t R + (1 - \alpha_t \gamma_t)\delta V_E(\alpha_{t+1}) \geq c\gamma_t + \delta V_E(\alpha_t). \tag{13}$$

For any sharing rule s_t, the entrepreneur can either invest the funds (left-hand side) or divert them (right-hand side). She benefits from complying with the contract via two different sources. Either the project succeeds in period t, giving her a share of the return $s_t R$, or it does not succeed, in which case she has access to future rounds of funding. With a funding flow of γ_t, the probabilities of these events are $\alpha_t \gamma_t$ and $1 - \alpha_t \gamma_t$ respectively. If the project fails in the current period, then the posterior belief declines to α_{t+1}. The alternative action for the entrepreneur is to simply divert the funds today and then face a similar problem tomorrow as the state of the project remains unchanged. Therefore, the equilibrium can be characterized by a sequence of participation constraints for the investor (as in (11)) and a sequence of incentive constraints for the entrepreneur (as in (13)).

In equilibrium, the entrepreneur will never leave the investor with more net value than is necessary to obtain the funding. The equilibrium share s_t^* is therefore determined by the exact fulfillment of the participation constraint. The investor receives zero net utility when the participation constraint (11) is binding and we can solve for the equilibrium sharing rule:

$$s_t^* = \frac{\alpha_t R - c}{\alpha_t R}. \tag{14}$$

We refer to contracts that leave the investor with zero net utility as break-even contracts and observe that the break-even share is independent of the funding flow.[7] Using (14), we may rewrite the incentive constraint (13) as

$$\alpha_t \gamma_t R - c\gamma_t + (1 - \alpha_t \gamma_t)\delta V_E(\alpha_{t+1}) \geq c\gamma_t + \delta V_E(\alpha_t). \tag{15}$$

The dynamic incentive constraint shows that the return R has to be sufficiently high to cover the static as well as the dynamic incentive costs. The static part simply says that the gross return $\alpha_t \gamma_t R$ has to be sufficiently large to compensate the investor for his cost $c\gamma_t$ as well as dissuade the entrepreneur from diverting the current flow, an additional $c\gamma_t$. The dynamic part accentuates the incentive problem. By rewriting (15) as

$$\alpha_t \gamma_t (R - \delta V_E(\alpha_{t+1})) \geq 2c\gamma_t + \delta(V_E(\alpha_t) - V_E(\alpha_{t+1})), \tag{16}$$

it says that the net return for the entrepreneur after paying for the static cost is $R - \delta V_E(\alpha_{t+1})$ rather than R itself. This is natural, as the success today leads to an end of the project and preempts future payouts to the entrepreneur. On the other hand, even if the project is funded yet without success, then the future value of the project is determined by α_{t+1} rather than α_t. By diverting the funds, the entrepreneur could escape the downgrade that constitutes the dynamic part of incentive costs.

A lower level of funding γ_t certainly affects both sides of the inequality (16), but at different rates. A lower equilibrium level reduces the gains on the left-hand side but only with the weight $\alpha_t < 1$, whereas on the right-hand side it affects the current costs of funding and, what is more important, the intertemporal incentive costs. A low funding level γ_t reduces the current value function, $V_E(\alpha_t)$, but a lower γ_t also decreases the difference between α_t and α_{t+1} and with it the difference of the value functions $V_E(\alpha_t) - V_E(\alpha_{t+1})$. This argument suggests that a marginal decrease in γ_t always leads to a larger decrease in the right-hand side of the incentive constraint than the left-hand side of the incentive constraint.

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[7] We prove the result that only break-even contracts will be offered in equilibrium formally in Lemma 2 as a property that holds for all subgame-perfect equilibria, and not only for Markovian equilibria.

Given the impact of the funding rate on the intertemporal incentive constraints, we might then ask whether it is conceivable that the project receives full funding with $\gamma_t = \lambda$ until the last period $\alpha_T = \alpha_S$. With period T being the last period of funding, the continuation value at α_{t+1} would be zero, $V_E(\alpha_{T+1}) = 0$, and the value function at α_T would be given by

$$V_E(\alpha_T) = \alpha_T \lambda R - c\lambda.$$

But if we insert these two continuation valuations into the incentive constraint (15), we are led to a contradiction because we obtain, after rearranging,

$$\alpha_T \lambda R \geq 2c\lambda + \delta(\alpha_T \lambda R - c\lambda).$$

The inequality clearly cannot be satisfied at $\alpha_T. = \alpha_S$, as we already have $\alpha_S \lambda R = 2c\lambda$ and $\alpha_S \lambda R - c\lambda = c\lambda$. This argument already indicates that in equilibrium, funding has to (eventually) slow down from the maximal level λ to a lower level $\gamma_t < \lambda$, which can be sustained by the incentive constraint (15).

In light of this finding, we might ask whether full funding with $\gamma_t = \lambda$ will ever occur. To answer this question, it is helpful to consider the limit case of the incentive constraint (16) with $\alpha_0 = 1$, the case of the "certain project." Using the fact that with the certain project, $\alpha_0 = 1$, the posterior beliefs remain constant and equal to the prior beliefs, the intertemporal incentive constraint (15) can then be written as

$$\lambda R - c\lambda + (1-\lambda)\delta V_E(1) \geq c\lambda + \delta V_E(1). \tag{17}$$

In equilibrium, the value function of the entrepreneur is the discounted and risk-adjusted sum of the per-period returns:

$$V_E(1) = \frac{\lambda R - \lambda c}{1 - \delta(1-\lambda)}.$$

We then insert $V_E(1)$ into the incentive constraint (17),

$$\lambda R - c\lambda + (1-\lambda)\delta \frac{\lambda R - \lambda c}{1 - \delta(1-\lambda)} \geq c\lambda + \delta \frac{\lambda R - \lambda c}{1 - \delta(1-\lambda)}, \tag{18}$$

and obtain a necessary and sufficient condition on R for full funding to occur:

$$R \geq 2c + \lambda c \frac{\delta}{1-\delta}. \tag{A}$$

By considering the deviation option of the entrepreneur, the sources of the determination for the critical value R become more transparent. If we consider $\alpha_0 = 1$, then if full funding is to occur in equilibrium, we know that the entrepreneur can always guarantee herself at least $c\lambda$ in every period, since a diversion would simply lead to renewed attempts of funding in next period. With $\alpha_0 = 1$ and $V_E(\alpha_t) = V_E(\alpha_{t+1})$, we can rearrange the incentive constraint (17) to read

$$\lambda R \geq 2\lambda c + \delta \lambda V_E(1). \tag{19}$$

Now the entrepreneur has the option to divert the funds in every period, which secures him at least a perpetuity of $c\lambda$, hence $V_E(1) = \lambda c/(1-\delta)$. The inequality (19) then states that the return from the project, R, has to cover at least $c + c$, which are the current costs for entrepreneur and investor, and the increment in the perpetual rent that is at the discretion of the entrepreneur via his option to deviate and thereby to increase by λ the probability of getting access to the perpetual rent. The rent of the entrepreneur thus has a contemporaneous and an intertemporal component.

Condition (A) turns out to be a key condition in our analysis. A project where the payoff

R is large enough relative to the marginal cost c of success, the contemporaneous rent c, and the increment in the perpetual rent $\lambda c[\delta/(1-\delta)]$ so as to satisfy (19) is termed a "high-return" project (in incentive-adjusted terms), as opposed to a "low-return" project where the condition is violated.

Definition 2 (low- and high-return projects). The project is a *low-return* project if $R < 2c + \lambda c[\delta/(1-\delta)]$, and it is a *high-return* project if $R \geq 2c + \lambda c[\delta/(1-\delta)]$.

We can now ask what happens to equilibrium funding when the critical inequality (A) is violated. Staying with the special case of the certain project and hence $\alpha_0 = 1$, the solution is almost apparent from the analysis of the incentive constraint (17). For funding to occur, the incentive constraint has to be reestablished. This requires that the rent arising from a diversion is lowered, which can only mean that the funding level is lowered to an appropriate level $\gamma_t^* < \lambda$. In fact, we can obtain the equilibrium funding $\gamma_t = \gamma_t^*$ by solving the incentive constraint (18) as an equality. The solution γ_t^* to the equality (18) is also the unique equilibrium funding level. While it is by now clear that a funding level above γ_t^* could not be sustained in equilibrium, we shall now argue that any funding level strictly lower than γ_t^* could not form an equilibrium either. We observe first that if the funding level is set below γ_t^*, then the incentive constraint (17) would again hold as a strict inequality. Because the entrepreneur then has slack in his incentive constraint, he could ask the investor for a higher funding level, even λ, by "bribing" the investor and offering him a slightly larger share of the surplus than the break-even contract. The investor would agree, since he would be offered a strictly positive net surplus, and given the continuation values, he would be assured that the entrepreneur's incentive constraint still holds. In consequence, an interior level of funding $\gamma \in (0, 1)$ can be sustained in equilibrium only if the incentive constraint of the entrepreneur is met as an equality. The insight that an interior level of funding is always associated with a binding incentive constraint is of course not restricted to $\alpha_t = 1$, but is valid more generally for all $\alpha_t \leq 1$. We now summarize the equilibrium funding decisions for the certain project.

Theorem 1 (certain project).

  (i) If the certain project has high returns, then it receives full funding in all periods.

  (ii) If the certain project has low returns, then it receives restricted funding in all periods:

$$\gamma_t^* = \frac{1-\delta}{\delta c}(R-2c) < \lambda, \quad \forall\, t.$$

The equilibrium funding level of the low-return project is increasing in the final value R and decreasing in cost c. An increase in the discount factor δ increases the value of the option to divert, and hence the investor responds in equilibrium by decelerating the flow of funds as it becomes more difficult to satisfy the incentive constraint.

The equilibrium in the general case of an evolving α_t can now almost be conceived by replacing the constant value R by the dynamically evolving value $\alpha_t R$. As long as $\alpha_t R$ is sufficiently large, unrestricted funding will be possible, yet as $\alpha_t R$ decreases, funding will have to decrease as well so as to maintain the incentive constraint of the entrepreneur.

The critical value of the posterior belief, denoted by $\overline{\alpha}$, at which funding will become restricted can in fact be easily obtained. Under the hypothesis that the value function of the entrepreneur is just equal to, but not larger than, the perpetual rent that the entrepreneur can secure by deviating forever, the incentive constraint allows us to solve for the value function as

$$V_E(\alpha_t) = V_E(\alpha_{t+1}) = \frac{\lambda c}{1-\delta}.$$

The critical posterior belief is then computed by solving (15) with the continuation values given by the perpetual rents, or

$$\overline{\alpha}\lambda R - \lambda c + (1-\overline{\alpha}\lambda)\delta\frac{\lambda c}{1-\delta} = \lambda c + \lambda c\frac{\delta}{1-\delta},$$

from which we can infer the threshold to be

$$\bar{\alpha} = \frac{2c}{R - \lambda c \frac{\delta}{1-\delta}}.$$ (20)

Full funding at α_t is possible if and only if $\alpha_t \geq \bar{\alpha}$. We can summarize our findings as follows.

Theorem 2 (relationship funding). The Markov-perfect equilibrium is unique, and funding is always provided until $\alpha_T = \alpha_S$.

  (i) If the project has low returns, then it receives restricted funding in all periods.

  (ii) If the project has high returns, then it receives full funding for all $\alpha_t \geq \bar{\alpha}$ and restricted funding for all $\alpha_t < \bar{\alpha}$.

  (iii) If funding is restricted, then γ_t is strictly decreasing in t.

The sharing rule associated with the equilibrium is given by (14). For high-return projects, there is a critical value $\bar{\alpha}$ such that the project will receive maximal funding as long as $\alpha_t \geq \bar{\alpha}$. Low-return projects that have insufficient returns to cover current costs and perpetual rents, even at $\alpha_t = 1$, are then always subject to restricted funding. In both cases, the volume of funding will decrease over time with the deterioration in the expected returns $\alpha_t R$.

☐ **Renegotiation-proof equilibrium.** The notion of a Markov equilibrium imposes a stationarity requirement on the offer and acceptance decisions of the agents. In the context of our model, the Markovian assumption has a natural interpretation as a consistency requirement on the process of (re)negotiation between the two parties; namely, the Markovian condition requires that entrepreneur and investor find an arrangement mutually acceptable whenever they have found the same arrangement acceptable in the past and absent any new information about the nature of the project.

We now strengthen this intuition by considering arbitrary history-dependent policies instead. However, we impose a condition that the policies must be *time-consistent* in the sense that if the players can coordinate on a certain policy in a subgame, they are also able to coordinate on the same policy in any other subgame where the circumstances are the same, that is, if they share the same belief about α_t. In other words, we assume that they are able to avoid any Pareto-inferior outcome under exactly the same circumstances. To this end, we invoke the refinement of weakly renegotiation-proof equilibrium first suggested by Farrell and Maskin (1989) for repeated games. The adaptation of the equilibrium notion to dynamic games is straightforward.

Definition 3 (weakly renegotiation-proof). A subgame-perfect equilibrium $\{s_t^*, \gamma_t^*, d_t^*, i_t^*\}_{t=0}^{\infty}$ is weakly renegotiation-proof if there do not exist continuation equilibria at some h_t and $h'_{t'}$ with $\alpha(h) = \alpha(h'_{t'})$ and $h_t \neq h'_{t'}$ such that $V_E(h_t) \geq V_E(h'_{t'})$ and $V_I(h_t) \geq V_I(h'_{t'})$, with at least one strict inequality.

The renegotiation considered here occurs between time periods. It is conceptually different from renegotiation in static principal-agent models as considered by Fudenberg and Tirole (1990) or Hermalin and Katz (1991). The notion of weakly renegotiation-proof is often interpreted as an internal consistency requirement. Indeed, Farrell and Maskin (1989) suggested a strengthening of the notion by defining as strongly renegotiation-proof any weakly renegotiation-proof profile with none of its continuation equilibria being strictly Pareto dominated by another weakly renegotiation-proof profile. This distinction is immaterial to our argument, as they all coincide in this sequential move game with symmetric information.

Theorem 3 (equivalence). The unique Markov-perfect equilibrium is identical to the unique weakly renegotiation-proof equilibrium.

The equivalence can be illustrated by the following simple example of equilibrium strategy profiles that form a subgame-perfect, but not renegotiation-proof, equilibrium. The example also shows that renegotiation-proofness indeed imposes restrictions on the equilibrium set.

(i) The entrepreneur offers in each period break-even contracts and invests funds if her private value to invest exceeds her private value to divert. If the investor has observed no deviations in the past, then the investor provides maximal funding if he breaks at least even and can expect the entrepreneur to invest. He rejects any contract proposal that doesn't meet the above conditions.

(ii) If there were any deviations in the past, then entrepreneur and investor pursue the stationary equilibrium strategies as described earlier.

Consider these strategy profiles for a certain project, $\alpha_0 = 1$, with low returns, $R < 2c + \lambda c[\delta/(1 - \delta)]$. By Theorem 1, the Markov-perfect equilibrium permits only restricted funding with

$$\gamma^* = \frac{1 - \delta}{\delta c}(R - 2c),$$

and the resulting equilibrium value for the entrepreneur is

$$V_E(1) = \frac{1}{\delta}(R - 2c).$$

In contrast, suppose part (i) of the strategy profile indeed forms a subgame-perfect equilibrium. Then the value for the entrepreneur would be

$$\hat{V}_E(1) = \frac{\lambda R - \lambda c}{1 - \delta(1 - \lambda)}.$$

As it is immediately verified that offer and acceptance strategies in (i) have the best-response property if the entrepreneur subsequently invests, it remains to verify her incentive constraint, which can be written as

$$\frac{\lambda R - \lambda c}{1 - \delta(1 - \lambda)} \geq c\lambda + (R - 2c),$$

which leads after the obvious cancellations to

$$R \leq 2c + \lambda c \frac{\delta}{1 - \delta}.$$

This is precisely the condition of low returns that we imposed for this example. Thus, the outlined strategy profile would allow full funding everywhere along the equilibrium path by relying on the stationary equilibrium as an off-the-equilibrium punishment path. The strategy profiles rely in an obvious way on continuation plays that are not renegotiation-proof. As the investor receives zero utility on and off the equilibrium path, it is sufficient to note that the entrepreneur receives different values on and off the equilibrium path to find that the strategy profile is not weakly renegotiation-proof.

☐ **Bargaining and long-term contracts.** We have so far imposed two strong assumptions on the structure of contracts, namely (i) that all the bargaining power rests with the entrepreneur and (ii) that only short-term contracts are possible. Here, we briefly discuss the robustness of the results if we relax either of the assumptions.

Bargaining power. Consider first a change in the bargaining power. Suppose that the investor now makes all contract offers and the entrepreneur accepts or rejects all proposals. Still, the participation constraint of the investor and the incentive constraint of the entrepreneur have to hold in any equilibrium. As long as both constraints are binding, they uniquely determine the equilibrium. In the model, both constraints were binding in the region $\alpha_t < \bar{\alpha}$, where only reduced funding was feasible. Therefore, nothing would change in this region with the redistribution of the bargaining power: The pattern of funding and the distribution of the surplus remain the same.

A change in the allocation can arise only if one of the inequalities is not binding any more, and the change would then pertain to the distribution of the surplus. Now in the benchmark model, the incentive constraint was slack only in the region of optimistic posterior beliefs $\alpha_t \geq \overline{\alpha}$. But there, maximal funding was guaranteed anyhow, and a shift in bargaining power would not alter that. It follows that the funding pattern in equilibrium would remain unaffected by a change in the bargaining power.

Long-term contracts. In this article we analyze short-term contracts in which the participation constraint of the investor has to hold in every period. Consider then an extension of the contracting space to allow for long-term contracts that are valid for any arbitrary horizon of T periods. We maintain our requirement that existing contracts can be renegotiated or new contracts be concluded in every future period. Formally, this allows us to substitute the sequence of participation constraints that had to be met in every period by a single intertemporal participation constraint that has to hold only at the time of entry into the contract. In contrast, the sequence of period-by-period incentive constraints needs to be maintained, as they guarantee the proper allocation of investment funds in every period.

The advantages of a long-term contract reside naturally with a possible intertemporal smoothing of the entrepreneur's expected payoffs. More precisely, it is then possible to reallocate the entrepreneur's payoff stream over time so as to make it coincide with the stream that is necessary to guarantee incentives. Thus, in every moment where the project's current net cash flow $(\alpha_t R - c)\lambda$ exceeds what is needed to maintain the entrepreneur's incentives, or as long as $\alpha_t > \overline{\alpha}$, there is a surplus that can be reallocated. The entrepreneur concedes a larger share to the investor today in exchange for receiving a larger share herself in the future. Conversely, the investor makes profits initially in return for a commitment to *subsidize* the project later on, when α_t falls below $\overline{\alpha}$.

Hence, for high-return projects with an optimistic prior belief, $\alpha_0 > \overline{\alpha}$, a long-term contract can strictly improve upon the allocation of short-term contracts. The region where full funding is provided can then be extended beyond the threshold $\overline{\alpha}$. The funding pattern, however, would remain as before, insofar as the project would receive full funding initially and then switch to lower funding levels. By contrast, we observe that as soon as funding is reduced, our previous argument of the intertemporal smoothing effect of long-term contracting never applies and long-term contracts can do no better than short-term contracts. Therefore, if $\alpha_0 \leq \overline{\alpha}$, which is always the case for low-return projects, there is no role for long-term contracts and the equilibrium is unaffected by the larger set of feasible contracts. The reason is that then the project in no instance has high-enough returns to generate surplus beyond participation and incentive constraints.

5. Arm's-length financing

■   In this section we assume that the investment decision by the entrepreneur is unobservable by the investor. We first consider Markovian equilibria, to maintain consistent equilibrium conditions across different informational structures. Below we define a Markov sequential equilibrium and then present the equilibrium analysis. We then show that the Markovian restriction is immaterial, as the unique Markov sequential equilibrium coincides with the unique sequential equilibrium. Finally, we again discuss robustness when changes in the bargaining power or long-term contracts are introduced.

□   **Equilibrium.** As we consider the contracting problem with unobservable actions by the entrepreneur, the observable history of the game begins to differ for entrepreneur and investor. The entrepreneur still observes all past realizations of the strategic choices, and a private history h_t for her is still given by

$$h_t = \{s_0, \ldots, s_{t-1}; \gamma_0, \ldots, \gamma_{t-1}; d_0, \ldots, d_{t-1}; i_0, \ldots, i_{t-1}\}.$$

The investor, however, is not able to observe the action of the entrepreneur anymore. Along any arbitrary sample path without success, the observable history to him is given by

$$\hat{h}_t = \{s_0, \ldots, s_{t-1}; \gamma_0, \ldots, \gamma_{t-1}; d_0, \ldots, d_{t-1}\}.$$

Denote by $\hat{H}_t$ the set of all possible such histories. In consequence, the evolution of the posterior belief may differ for entrepreneur and investor. We continue to denote by α_t the entrepreneur's posterior belief based on the history h_t, $\alpha_t \triangleq \alpha(h_t)$. We refer to the belief that the investor holds at time t and after observing the restricted public history $\hat{h}_t$ as $\hat{\alpha}_t \triangleq \hat{\alpha}(\hat{h}_t)$, which will depend on the observed history $\hat{h}_t$ as well as on the investor's belief about the entrepreneur's past investment behavior, $\{\hat{i}_0, \ldots, \hat{i}_{t-1}\}$. By Bayes' law there is a one-to-one relationship between the estimate regarding the entrepreneur's past investments $\{\hat{i}_0, \ldots, \hat{i}_{t-1}\}$ and the belief about $\hat{\alpha}(\hat{h}_t)$. The estimate regarding $\{\hat{i}_0, \ldots, \hat{i}_{t-1}\}$ depends on the incentives provided through the past and future share contracts $\{s_0, \ldots, s_t, \ldots\}$. As before, updating occurs only conditional on current failure of the project, since the game ends as soon as the project succeeds.

Because entrepreneur and investor observe different histories, the payoff-relevant part of the history is now represented by two state variables, the two (possibly different) posterior beliefs about the likelihood of success, α_t and $\hat{\alpha}_t$. The suitably adapted Markovian equilibrium concept can then be stated as follows.

Definition 4 (Markov sequential equilibrium). A Markov sequential equilibrium is a sequential equilibrium

$$\{s_t^*, \gamma_t^*, d_t^*, i_t^*\}_{t=0}^{\infty}$$

if $\forall\, h_t \in H_t, \forall\, h_{t'}' \in H_{t'}, \forall\, \hat{h}_t \in \hat{H}_t, \forall\, \hat{h}_{t'}' \in \hat{H}_{t'}$ and $\forall\, s_t, s_{t'}', \forall\, \gamma_t, \gamma_{t'}', \forall\, d_t, d_{t'}'$:

$$
\begin{aligned}
\alpha(h_t) = \alpha\left(h_{t'}'\right) &\Rightarrow & s_t^*(h_t) = s_{t'}^*\left(h_{t'}'\right); \quad \gamma_t^*(h_t) = \gamma_{t'}^*\left(h_{t'}'\right); \\
\hat{\alpha}(h_t) = \hat{\alpha}\left(h_{t'}'\right), \quad s_t = s_{t'}', \gamma_t = \gamma_{t'}'; &\Rightarrow & d_t^*\left(\hat{h}_t, s_t, \gamma_t\right) = d_{t'}^*\left(\hat{h}_{t'}', s_{t'}', \gamma_{t'}'\right); \\
\alpha(h_t) = \alpha\left(h_{t'}'\right), \quad s_t = s_{t'}', \gamma_t = \gamma_{t'}', d_t = d_{t'}' &\Rightarrow & i_t^*\left(h_t, \gamma_t, s_t, d_t\right) = i_{t'}^*\left(h_{t'}', \gamma_{t'}', s_{t'}', d_{t'}'\right).
\end{aligned}
$$

(21)

The Markovian sequential equilibrium ensures that the continuation strategies are time-consistent and identical after any history with an identical pair of rationally updated beliefs, α_t and $\hat{\alpha}_t$. The Markovian restrictions contained in (21) are equivalent to the ones formulated earlier in (10), with the exception being that the underlying histories and beliefs differ for entrepreneur and investor.

☐ **Analysis.** Before we go to the details of the analysis, it might be useful to describe intuitively where the differences in the equilibrium incentives arise and how they matter for the equilibrium funding. Conditional on receiving the funds, the entrepreneur still has the option to either invest or divert the funds. The differences arise in how entrepreneur and investor evaluate these different options. Clearly, the investor is willing to provide the funds only if he is convinced that the funds will be directed to the project. Consider then the counterfactual of a diversion of the funds by the entrepreneur. Following a deviation, the entrepreneur would know that the funds didn't benefit the project and hence a failure of the project to succeed in this period will not surprise her at all. In contrast, for the investor, a deviation remains a counterfactual and thus he is downgrading his beliefs about the future value of the project, as the current failure induces a downward change in his beliefs. Thus a deviation, as an off-the-equilibrium behavior by the entrepreneur, leads to a divergence in the posterior about the future likelihood of success. More precisely, the entrepreneur maintains her estimate $\alpha_{t+1} = \alpha_t$, whereas the investor continues to update his belief to a lower value $\hat{\alpha}_{t+1} < \hat{\alpha}_t$. Such a divergence of beliefs per se could not arise in the environment with observable actions.

How does the possibility of divergent beliefs influence the equilibrium incentives? Ultimately the divergence imposes more discipline on the funding decisions of the investor and therefore tends to ease the funding problem. Because a deviation will still lead to a lowering in the posterior belief of the investor, he will ask for a larger share of the return R. This leads directly to a higher cost of obtaining funds from the point of view of the entrepreneur. The option of delaying the investment decision until the next period thus becomes less attractive.

We examine next how these changes will be reflected in the participation and incentive constraints. The participation constraint of the investor remains unchanged at

$$\hat{\alpha}_t s_t \gamma_t R \geq \gamma_t c,$$

with the exception that it is evaluated at $\hat{\alpha}_t$ rather than α_t. The modification is immaterial along the equilibrium path, as $\alpha_t = \hat{\alpha}_t$. However, the incentive constraint of the entrepreneur changes to reflect the divergence of the beliefs off the equilibrium path. Formally, the incentive constraint is given by

$$\alpha_t \gamma_t s_t R + (1 - \alpha_t \gamma_t) \delta V_E(\alpha_{t+1}) \geq c\gamma_t + \delta V_E\left(\alpha'_{t+1}, \hat{\alpha}_{t+1}\right),$$

where, momentarily, we express the value function as determined by both beliefs. We observe that off the equilibrium path, the posterior belief of entrepreneur and investor diverge, and hence the value function off the equilibrium path depends on the specific beliefs of the entrepreneur, α'_{t+1}, and the investor, $\hat{\alpha}_{t+1}$. Along the equilibrium path, the entrepreneur invests the funds into the project, and a diversion occurs off the equilibrium path. The continuation value conditional on a diversion is therefore described by two different beliefs, the correct belief α'_{t+1} of the entrepreneur and the incorrect belief $\hat{\alpha}_{t+1}$ of the investor. After a one-period deviation, the off-the-equilibrium-path belief of the entrepreneur, α'_{t+1}, is given simply by $\alpha'_{t+1} = \alpha_t$, whereas the investor holds the "equilibrium" belief $\hat{\alpha}_{t+1} = \alpha_{t+1}$. Hence, off the equilibrium path, the investor will accept only contracts that will break even under his posterior belief $\hat{\alpha}_{t+1}$ and all subsequent updates of his posterior. How then does this affect the continuation value of the entrepreneur off the equilibrium path? The answer is rather straightforward. It will be as if her continuation value would indeed be determined by the belief of the investor $\hat{\alpha}_{t+1} = \alpha_{t+1}$, but because the entrepreneur privately knows that the true posterior, conditional on diversion, is still given by α_t, she simply exchanges the posterior belief $\hat{\alpha}_{t+1} = \alpha_{t+1}$ of the investor for her own, $\alpha'_{t+1} = \alpha_t$. This allows us to relate the off-the-equilibrium-path value function to the on-the-equilibrium-path value function as follows:

$$V_E\left(\alpha'_{t+1} = \alpha_t, \hat{\alpha}_{t+1} = \alpha_{t+1}\right) = \frac{\alpha_t}{\alpha_{t+1}} V_E\left(\alpha_{t+1}\right). \tag{22}$$

After all, the value function of the entrepreneur, on and off the equilibrium path, is simply the discounted sum of success probabilities, or

$$V_E(\alpha_{t+1}) = R\left(\alpha_{t+1} s_{t+1}\gamma_{t+1} + \delta(1 - \gamma_{t+1}\alpha_{t+1})\alpha_{t+2}s_{t+2}\gamma_{t+2} + \delta^2(1 - \gamma_{t+2}\alpha_{t+2})\alpha_{t+3}s_{t+3}\gamma_{t+3} + \cdots\right). \tag{23}$$

After replacing the conditional success probabilities $\alpha_{t+2}\gamma_{t+2}$, $\alpha_{t+3}\gamma_{t+3}$, ... with the unconditional success probabilities viewed from α_{t+1}, and making repeated use of Bayes' formula,

$$\alpha_{t+2} = \frac{\alpha_{t+1}(1 - \gamma_{t+1})}{1 - \gamma_{t+1}\alpha_{t+2}},$$

we can rewrite the value function (23) as a sequence of unconditional probabilities,

$$V_E(\alpha_{t+1}) = \alpha_{t+1}R\left(\gamma_{t+1}s_{t+1} + \delta(1 - \gamma_{t+1})\gamma_{t+2}s_{t+2} + \delta^2(1 - \gamma_{t+1})(1 - \gamma_{t+2})\gamma_{t+3}s_{t+3} + \cdots\right), \tag{24}$$

that only invoke the current belief α_{t+1} and the current and future flow probabilities $\gamma_{t+1}, \gamma_{t+2}, \gamma_{t+3}, \ldots$. This shows how the identity (22) arises. The terms of the contract, namely $s_{t+1}, s_{t+2}, \ldots,$ are conditional on a deviation of the entrepreneur and they are determined by the belief of the investor. In consequence, the accepted shares conditional on the deviation will be identical to the shares the parties would agree upon on the equilibrium path as $\hat{\alpha}_{t+1} = \alpha_{t+1}$, hence the term $V_E(\alpha_{t+1})$. But privately, the entrepreneur knows that the true probability of future success is α_t rather than α_{t+1}. Hence the ratio term α_t/α_{t+1} corrects for the fact that the value function $V_E(\alpha_{t+1})$ underestimates the true probability of success when $\alpha'_{t+1} = \alpha_t$.

We can therefore rewrite the incentive constraint in the asymmetric information environment as follows:

$$\alpha_t \gamma_t R - \gamma_t c + (1 - \alpha_t \gamma_t) \delta V_E(\alpha_{t+1}) \geq c \gamma_t + \frac{\alpha_t}{\alpha_{t+1}} \delta V_E(\alpha_{t+1}). \tag{25}$$

The reader may realize that the left-hand side of the inequality, which represents the on-the-equilibrium-path behavior, remains identical to the one in the observable environment (see expression (13)). The change occurs on the right-hand side of the inequality, or the off-the-equilibrium-path behavior. The flow value of a diversion still contains the immediate benefit of $c\gamma_t$. But as the investor continues to believe that an investment occurred, he will only accept future proposals as if an investment today had indeed occurred. In consequence, the value function of the entrepreneur will have to evolve (almost) as if the current failure had to be attributed to the project rather than the diversion of the entrepreneur. There is one benefit, however, for the entrepreneur from the continued updating. She will know that the true probability is still α_t rather than α_{t+1}. Thus instead of multiplying the future probability of success with α_{t+1}, she is certain that it is indeed α_t.

The intertemporal incentive constraint (25) may be rewritten, after cancelling the obvious terms, as

$$\alpha_t \gamma_t R - c \gamma_t \geq c \gamma_t + \gamma_t \frac{\alpha_t}{\alpha_{t+1}} \delta V_E(\alpha_{t+1}). \tag{26}$$

In general, then, funding toward the end of the lifetime of the project will become easier with an arm's-length relationship. But now a complementary problem may arise at the beginning of the project. If indeed funding will be generous close to the end of the project, then the entrepreneur may have less incentive at the beginning of the project to invest funds, as the future will offer plenty of opportunities to generate success. Thus an easing of the incentive constraint near the end of the project may tighten the incentive constraint at the beginning of the project, when the assessment in terms of the beliefs α_t is still very positive. This indicates that the monotonicity in the funding volume may indeed be reversed with unobservable actions. We first state the results and then comment on some of the equilibrium properties. The threshold posterior belief at which the funding policy will switch is denoted by $\overline{\overline{\alpha}}$ and given by

$$\overline{\overline{\alpha}} = \frac{2c - \dfrac{2c\lambda}{1-\delta} + \dfrac{\lambda\delta c}{1-\delta}}{R - \dfrac{2c\lambda}{1-\delta}}.$$

Theorem 4 (arm's-length funding). The Markov sequential equilibrium is unique, and funding stops at $\alpha_T = \alpha_S$.

(i) If the discount factor is low, $\delta < (2 - 2\lambda)/(2 - \lambda)$, then

    (a) a low-return project receives restricted funding at all times, and

    (b) a high-return project receives full funding for $\alpha_t \geq \overline{\overline{\alpha}}$ and restricted funding for $\alpha_t < \overline{\overline{\alpha}}$.

(ii) If the discount factor is high, $\delta \geq (2 - 2\lambda)/(2 - \lambda)$, then

    (a) a low-return project receives restricted funding for $\alpha_t > \overline{\overline{\alpha}}$ and full funding for $\alpha_t < \overline{\overline{\alpha}}$, and

    (b) a high-return project receives full funding at all times.

The difference in the equilibrium funding policies between arm's-length and relationship financing are now easily discussed. For low discount factors, or $\delta < (2 - 2\lambda)/(2 - \lambda)$, the equilibrium funding over time displays exactly the same dynamics under arm's-length and relationship funding. Yet a difference emerges for high discount factors, or $\delta \geq (2 - 2\lambda)/(2 - \lambda)$. The absence of a commitment problem with arm's-length funding completely restores the efficiency of

the funding decision until $\alpha_T = \alpha_S$ for a high-return project. For a project with low returns, it at least allows the reestablishment of funding efficiency close to α_S. The above condition on the discount factor can be restated symmetrically as a condition on the winning probability as

$$\delta \geq \frac{2 - 2\lambda}{2 - \lambda} \Leftrightarrow \lambda \geq \frac{2 - 2\delta}{2 - \delta}. \tag{B}$$

For convenience, we shall henceforth refer to large and small discount factors depending on whether δ does or does not satisfy condition (B).

Definition 5 (small and large discount factors). The discount factor is said to be *small* if $\delta < (2 - 2\lambda)/(2 - \lambda)$, and it is said to be *large* if $\delta \geq (2 - 2\lambda)/(2 - \lambda)$.

We observe that an increase in the winning probability λ leads to a lower bound on the discount factor and vice versa. The role of the discount factor, or for that matter of the success probability, should not come entirely as surprise given our earlier discussion on the inefficiencies in relationship funding. The lack of commitment became especially damaging to the social efficiency of the equilibrium when the discount factor was high; the intertemporal rent of the entrepreneur was then high as well, and the only possible equilibrium resolution of this conflict required the investor to slow down the release of the funds. As the informational asymmetry allows the investor to overcome this lack of commitment, the discrepancy between arm's-length and relationship funding arises precisely where arm's-length funding was most affected by the lack of commitment. The efficiency is thus reestablished for high-return projects and improves the funding volume for low-return projects with high discount factors.

Corollary 1 (funding evolution).

    (i) If the discount factor is small, then the funding volume is decreasing over time.
    (ii) If the discount factor is large, then the funding volume is increasing over time.

Whether funding will eventually become unrestricted as α_t is sufficiently close to one is again determined by the high-return condition, $R \geq 2c + [\delta\lambda/(1 - \delta)]c$, that we encountered earlier in the symmetric environment. The reappearance of the condition is plausible because for α_t sufficiently close to one, the differences in the beliefs of entrepreneur and investor after a deviation become arbitrarily small, as a current failure barely changes the very optimistic view of the investor. More precisely, for any fixed funding flow $\gamma_t > 0$, the difference in the belief before and after a single unsuccessful investment γ_t,

$$\lim_{\alpha_t \to 1} (\alpha_t - \alpha_{t+1}) = \lim_{\alpha_t \to 1} \left(\alpha_t - \frac{\alpha_t(1 - \gamma_t)}{1 - \alpha_t\gamma_t} \right) = 0,$$

converges to zero when the initial belief at α_t is arbitrarily optimistic about the likelihood of eventual success. The asymmetry in the information between entrepreneur and investor is thus arbitrarily small when the incoming belief α_t is close to one, and in consequence the asymmetric contracting problem becomes arbitrarily close to the symmetric contracting problem. These arguments can be retraced formally by comparing the two incentive constraints, the symmetric incentive constraint (see (15)),

$$\alpha_t\gamma_t R - c\gamma_t + (1 - \alpha_t\gamma_t)\delta V_E(\alpha_{t+1}) \geq c\gamma_t + \delta V_E(\alpha_t),$$

and the asymmetric incentive constraint (see (25)),

$$\alpha_t\gamma_t R - c\gamma_t + (1 - \alpha_t\gamma_t)\delta V_E(\alpha_{t+1}) \geq c\gamma_t + \frac{\alpha_t}{\alpha_{t+1}}\delta V_E(\alpha_{t+1}).$$

We now observe that as $\alpha_t \to 1$, we have

$$\lim_{\alpha_t \to 1} (\alpha_t - \alpha_{t+1}) = 0 \quad \text{and} \quad \lim_{\alpha_t \to 1} \frac{\alpha_t}{\alpha_{t+1}} = 1.$$

By continuity of the value function, it then follows from the above that

$$\lim_{\alpha_t \to 1} (V_E(\alpha_t) - V_E(\alpha_{t+1})) = 0,$$

and hence symmetric and asymmetric incentive constraints become identical as $\alpha_t \to 1$. Moreover, as $\alpha_t \to 1$, the posterior beliefs will change for a long period of time only very slowly as the funds fail to generate a success. This means that for a long period of time, symmetric and asymmetric incentive conditions will almost be identical and hence the values generated through them will be very close as the more distant events matter less due to discounting.

☐ **Sequential equilibrium.** The characterization of the equilibrium seemed to rely strongly on the Markovian assumption. In particular, we represented the incentive problem of the entrepreneur through a Bellman equation. But there is one crucial difference to relationship financing: As the investor continues to lower his belief every time he provided funds yet did not observe success, he reaches the posterior belief α_S after finitely many positive funding decisions. This is true on the equilibrium path as well as off. Thus, in contrast to the symmetric environment, the horizon of the game effectively becomes finite. This allows us to analyze the game by backward induction over a finite horizon. As the (static) equilibrium in any final period where $\alpha_T \geq \alpha_S$, yet $\alpha_{T+1} < \alpha_S$, is unique, we can then construct the equilibrium recursively. Moreover, the stage game has a unique equilibrium for any given continuation payoff. In a sequential equilibrium, the investor's beliefs $\alpha(\hat{h}_t)$ are tied down according to Bayes' rule after all possible histories, including off-the-equilibrium-path histories, which is sufficient to guarantee the uniqueness of the continuation equilibrium everywhere. It follows that backward induction leads to a unique sequential equilibrium independent of the Markov assumption.[8]

The construction of the equilibrium in Theorem 4 is thus in fact constructing the unique sequential equilibrium, where the posterior belief α_t merely serves to summarize the beliefs of the players for a given history, but not as a restriction on the conditioning of the strategies.

Corollary 2. The unique Markov sequential equilibrium is the unique sequential equilibrium.

Thus, since the equilibrium play in a sequential equilibrium always follows a finite-horizon logic in the environment with unobservable actions, there is no need to refer to any formal concept of renegotiation-proofness in order to make sure that the outcome is the one that we have in mind, where it is impossible throughout to find a Pareto-improving continuation play by rescinding the equilibrium contracts.

☐ **Bargaining and long-term contracts.** As before, we may ask how sensitive the equilibrium results are to the specifics of the contracting model, in particular the distribution of bargaining power and the restriction to short-term contracts.

Bargaining power. Suppose now that the investor makes all the offers and the entrepreneur can respond only with acceptance or rejection. With a small discount factor, the equilibrium funding pattern is comparable to the one under symmetric information, and changes in bargaining structure do not at all affect the funding volume. With a large discount factor, the funding pattern remains in its qualitative properties but the equilibrium displays fewer inefficiencies. The reason is that whenever funding is unrestricted, the project's expected cash flow leaves some free surplus after participation and incentive constraints are satisfied. The question is then whether a better overall allocation is achieved if this surplus is distributed to the investor rather than the entrepreneur. Giving the surplus to the investor means a lower expected equilibrium payoff for the entrepreneur. Recall that the minimum value of the entrepreneur that guarantees incentive compatibility is recursively constructed. Thus, a lower expected compensation in the future (since the free surplus

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[8] Perfect Bayesian equilibrium cannot be used here, since adverse selection is a consequence of the entrepreneur's unobservable actions, not of chance moves of nature.

is given to the investor) translates into a lower option value of diverting and hence into a lower minimum compensation today. The incentive problem of the entrepreneur in the current period is eased. In consequence, a change in the bargaining power would allow an increase of the area where maximal funding is provided and would increase the volume of funding over the entire horizon.

Long-term contracts. The reasons why there can be benefits from adopting (renegotiation-proof) long-term contracts are closely related. As long-term contracts replace the flow participation constraint of the investor with a single initial constraint, intertemporal smoothing is possible. With a large discount factor, the project is initially constrained, and a free surplus arises toward the end of the relationship. As discussed for changes in the bargaining power, allocating this surplus to the investor lowers the entrepreneur's expected future value, and hence eases the current incentive problem. Moreover, in return for making expected profits toward the end, the investor can agree to *subsidize* the project elsewhere, i.e., to provide full funding while accepting a current share $(1 - s_t)\alpha_t \lambda R$ that falls short of the investment flow $c\lambda$. The question is then when to schedule this subsidy phase. The answer is that this subsidy phase should be scheduled as soon as possible, but the requirement that the equilibrium be immune to renegotiation is an effective constraint on this. As a consequence, if the project has low returns, the intertemporal smoothing arrangement will allow an early start and an extension of the final phase where full funding can be provided, but only limited funding is possible initially. If the project has high returns, then full funding is possible from the start and can be continued even beyond α_S.

By contrast, with a small discount factor, the dynamics of the funding pattern are reversed and resemble roughly the picture with observable actions. The project is constrained toward the end, necessitating a slowdown in the release of funds. The intertemporal smoothing option of long-term contracts allows to prolong the initial full-funding phase. But as soon as the surplus is exhausted, the optimal contract reverts back to the sequence of contracts described above, with the same funding volume. For low-return projects or if α_0 is so small that short-term contracts never allow for full funding, then there is never a surplus to redistribute intertemporally, and long-term contracting cannot improve upon short-term contracts.

6. Observability and the commitment to stop

■ In the previous two sections, we gave separate accounts of the environment with observable actions and with unobservable actions. We provide a comparison of the two cases in this section that we interpret to reflect the initial choice between relationship financing and arm's-length financing when the project is set up.

We will conduct this comparison by maintaining the assumption that once the financing mode is chosen, the investor is committed to the informational environment throughout. The source of this commitment is not explained in the model, and we will informally discuss possible transition from one funding mode to the other at the end of this section.

The immediate benefit of relationship funding is the absence of private information during the development of the relationship. It means in particular that the design of the contract does not have to account for the extraction of private information. It thus circumvents the learning rent that is associated with the private information. We have shown above that under relationship financing, three different components of rents must be awarded to the entrepreneur to make her willing to invest and risk early success, namely the contemporaneous rent equal to the immediate gain in consumption that a deviation affords, the intertemporal rent to compensate for the option to receive sure continued financing when deviating, and finally the learning rent driven by the fact that only the entrepreneur knows whether or not something has actually been learned about α_t. By contrast, only two of these components were present in the case of relationship financing, since there was no need for the learning rent.

The (implicit) cost of the relationship funding resides with the ability of the entrepreneur to restart the relationship after she diverted funds in previous periods. As the investor can't commit himself to refuse a contract with positive net payoffs, the entrepreneur was essentially able to

extract a rent equivalent to an infinite stream of funds λc, worth $\lambda c[\delta/(1-\delta)]$. In contrast, the asymmetry in the arm's-length relationship reduces the ability of the entrepreneur to renegotiate at favorable terms and hence weakens the incentives for the entrepreneur to delay investment into the project.

With this basic tradeoff between the two funding modes, we find that the possible cost of an arm's-length relationship, namely the learning rent, is small in comparison to the benefit from commitment. Therefore, we arrive at the following result.

Theorem 5 (comparison). For all posterior beliefs α_t the funding volume is larger under arm's-length than under relationship financing.

To gain more insight into this result, it is helpful to discuss separately the case of a low and high discount factor. With a small discount factor, we showed earlier that the funding pattern decreases in both informational environments. We then show that for all posterior beliefs α_t, the funding volume is higher with arm's-length funding. With a large discount factor, we have shown that the evolution of the funding levels displays opposite signs: γ_t is (weakly) decreasing in t with observable actions, but it is increasing in t with unobservable actions. Here, in order to show that arm's-length funding always occurs at a higher volume than with relationship funding, it is sufficient to compare the initial equilibrium funding volume, which again can be shown to be higher with arm's-length funding.

The clear Pareto ranking between the two financing modes is a rather striking result. From a naive point of view it may appear counterintuitive, since it says that the financing mode with an informational asymmetry separating financier and entrepreneur is more efficient. While it is well known that asymmetric information may offer advantages in a principal-agent model (see Crémer, 1995), our analysis shows that this argument is particularly prevalent when the agency relationship is open-ended and the agent has the option to extend it over a very long horizon.

The dynamic model shows that the relative advantage of arm's-length funding increases over time. In the beginning, there may be no difference between the speed with which funds can be released, especially for high-return projects. But as projects become protracted and prospects become relatively poor, arm's-length funding eventually offers an increasing advantage compared with relationship funding, which does not offer a commitment to stop at a given time and therefore allows the entrepreneur to threaten an infinite series of deviations even when only few profitable rounds of experimentation are left. As bargaining power shifts to the investor, the advantage of arm's-length contracts toward the end of the projects increases even further.

The typical financing cycle of business startups involves close relationships with financiers early on, and this is exemplified by the activity of venture capitalists who not only provide capital, but also monitor the projects very closely and get involved as advisors (see Gompers and Lerner, 1999; Casamatta, 2003). There is evidence that the value of relationship funding decreases over the typical financial cycle of an innovative firm: for example, in venture funded projects, syndicates tend to grow and to include more uninformed investors later on (Hege, Palomino, and Schwienbacher, 2003). There are also empirical findings that in banking relationships, financiers assume a more passive role over time (e.g., Ongena and Smith, 2001). As projects mature and require more capital, while having more tangible assets and research results to offer, their funding sources tend to become more diverse: funding typically starts with equity-dominated venture financing and adds more and more debt-like instruments over time as the firm grows and its funding needs expand (Berger and Udell, 1998). Thus, even before a successful technology startup reaches financial maturity and is funded by genuine outside investors, such as dispersed shareholders or bondholders, many ventures already go through a process of gradually decreasing the reliance on relationship financiers. This pattern is consistent with the comparison of the two funding modes in our model, which lends support to the notion that the financial cycle of innovative projects evolves from more-informed to less-informed investors.

7. Conclusion

■ In this article we present a dynamic agency model in which time and outcome of the project were uncertain. The model prominently features three aspects that together are defining elements for a wide class of agency problems of research and development activities: (i) the eventual returns from the project are uncertain, (ii) more information about the likelihood of success arrives with investment into the project, and (iii) investor and entrepreneur (innovator) cannot commit to future actions. The analysis focuses on Markovian equilibria, but we showed that this is a rather mild or even immaterial restriction in the context of the model. The equilibrium analysis proceeds sequentially, starting with symmetric information and ending with asymmetric information. The funding level was determined endogenously and depends on the returns of the project, the discount factor, and the informational asymmetry between entrepreneur and investor.

The impatience of the entrepreneur is an important determinant in the volume of funding, as the severity of the incentive constraint increased with the discount factor. This is in contrast to the results in the theory of repeated moral hazard games, where discount factors close enough to one often allow the equilibrium set to reach the efficiency frontier. In addition, we showed that the recursive structure of the incentive constraint leads to distinct funding dynamics under asymmetric information, where with large discount factors, the incentive constraint tends to actually relax over time and allow a larger funding rate as the project approaches its terminal period.

The basic tradeoff between arm's-length and relationship financing revealed in this article is that arm's-length financing offers the advantage that the investor is implicitly committed to a finite stopping horizon, while relationship financing saves up on the learning rent because investor and entrepreneur update beliefs symmetrically.

Finally, some possible extensions of our model should be mentioned. First, a worthwhile extension is to consider the equilibrium behavior when there are *competing projects*, formed by different entrepreneurs. As competition may limit the rent of each entrepreneur, parallel research for an identical objective might be an arrangement that improves efficiency despite the inevitable duplication of R&D efforts. In a *winner-takes-all* competition, the threat of preemption by a competitor will limit the intertemporal rent of each entrepreneur. Similarly, launching competing research teams may increase the *ex ante* value for an organization despite the multiplication of research efforts.

Second, it is conceivable that the entrepreneur may initially own some, perhaps small, investment funds. We then might ask how inside and outside funds are optimally mixed over time. We are confident that a delayed use of the entrepreneur's equity can be shown to be optimal in some cases. This should notably be the case if the entrepreneur's funds help alleviate financing constraints when the promise of the project deteriorates, as is typically the case under relationship financing. The open-horizon principal-agent model developed here should allow us to analyze the relative merits of these different incentive tools and their role in mitigating the contracting problems of compounded information rents.

Appendix

■ Proofs of Theorems 2–4 and Corollary 2 follow.

Lemma A1. In every subgame-perfect equilibrium no funding occurs for $\alpha_t < \alpha_S$.

Proof. The proof is by contradiction. Suppose there exists a subgame-perfect equilibrium with funding in period t, or

$$\alpha_t \gamma_t s_t R + (1 - \alpha_t \gamma_t) \delta V_E(h_{t+1}) \geq c \gamma_t + \delta V_E(h'_{t+1}),$$

or

$$\alpha_t \gamma_t s_t R - \gamma_t c + (1 - \alpha_t \gamma_t) \delta V_E(h_{t+1}) \geq \delta V_E(h'_{t+1}), \tag{A1}$$

and

$$\alpha_t \gamma_t (1 - s_t) R - \gamma_t c + (1 - \alpha_t \gamma_t) \delta V_I(h_{t+1}) \geq \delta V_I(h''_{t+1}), \tag{A2}$$

yet

$$\alpha_t \gamma_t R - 2c\gamma_t < 0. \tag{A3}$$

As entrepreneur and investor can always guarantee themselves at least a zero lifetime utility by offering contracts without funding (i.e., $\gamma_t = 0$) and by refusing all other contracts, respectively, it follows that $V_E(h_t) \geq 0$, $V_I(h_t) \geq 0$ for all histories h_t and all periods t. A necessary condition for the validity of (A1) and (A2) is therefore

$$\alpha_t \gamma_t s_t R - \gamma_t c + (1 - \alpha_t \gamma_t)\delta V_E(h_{t+1}) \geq 0, \tag{A4}$$

and

$$\alpha_t \gamma_t (1 - s_t)R - \gamma_t c + (1 - \alpha_t \gamma_t)\delta V_I(h_{t+1}) \geq 0. \tag{A5}$$

Under the hypothesis of (A3), it follows that at least one of the agents, entrepreneur or investor, must incur a loss in period t in exchange to a strictly positive continuation utility in period $t + 1$, and from (A4) and (A5) we can infer that

$$V_E(h_{t+1}) + V_I(h_{t+1}) \geq \frac{2c\gamma_t - \alpha_t \gamma_t R}{\delta(1 - \alpha_t \gamma_t)} > 0. \tag{A6}$$

It follows that entrepreneur and investor jointly expect to be compensated for the current loss in the future. Yet due to discounting and the possibility of a positive realization, the current loss translates to higher present value gains starting from tomorrow. But future gains can only be generated from the value of the project, yet since α_t is decreasing over time, (A3) implies that $\alpha_{t+1}\gamma_{t+1} R - 2c\gamma_{t+1} < 0$, and a repetition of the same argument allows us to infer that by forwarding (A6) by one period, we obtain the condition

$$V_E(h_{t+2}) + V_I(h_{t+2}) \geq \frac{1}{\delta(1 - \alpha_{t+1}\gamma_{t+1})}\left(2c\gamma_{t+1} - \alpha_{t+1}\gamma_{t+1}R + \frac{2c\gamma_t - \alpha_t \gamma_t R}{\delta(1 - \alpha_t \gamma_t)}\right) > 0,$$

and by induction on t, we then come to the conclusion that the value functions of the entrepreneur and investor are growing without bounds as $t \to \infty$, which delivers the desired contradiction because the sum of the value functions has to be finite because the value of the project is finite.   *Q.E.D.*

Lemma A2. In every subgame-perfect equilibrium only break-even contracts have a positive probability of being accepted.

Proof. Suppose in equilibrium a contract (s_t, γ_t) is offered and accepted. Then it has to satisfy

$$\alpha_t \gamma_t s_t R + (1 - \alpha_t \gamma_t)\delta V_E(h_{t+1}) \geq c\gamma_t + \delta V_E(h'_{t+1}) \qquad .$$

and

$$\alpha_t \gamma_t (1 - s_t)R - \gamma_t c + (1 - \alpha_t \gamma_t)\delta V_I(h_{t+1}) \geq \delta V_I(h''_{t+1}). \tag{A7}$$

Denote for the purpose of this proof a break-even contract by $\bar{s}_t$, where $\bar{s}_t$ is given by $\bar{s}_t = (\alpha_t R - c)/\alpha_t R$. We first show that in every equilibrium and at every t, $s_t \geq \bar{s}_t$. The proof is by contradiction. Suppose that the equilibrium contract is given by $s_t < \bar{s}_t$. It then follows that every other contract s'_t with $s_t < s'_t$ that is more advantageous for the entrepreneur must be rejected by the investor. It follows that his outside option, which is given by $V_I(h''_{t+1})$, must satisfy

$$0 < \frac{\alpha_t \gamma_t (1 - s_t)R - \gamma_t c}{\delta} \leq V_I(h''_{t+1}),$$

as the value function along the equilibrium path satisfies $V_I(h_{t+1}) \geq 0$. Consider then the continuation equilibrium starting at h''_{t+1}. It follows that starting at $t + 1$, the investor must be offered some contracts with strictly positive net value to him to generate the strictly positive continuation payoff. Yet, again for him to reject all lower offers by the entrepreneur, it must be that his outside option, represented by a decision to reject a lower offer, must be sufficiently high, and in fact since

$$V_I(h''_{t+1}) = \alpha_{t+1}\gamma_{t+1}(1 - s_{t+1})R - \gamma_{t+1}c + (1 - \alpha_{t+1}\gamma_{t+1})\delta V_I(h_{t+2}) = \delta V_I(h''_{t+2}),$$

it follows that

$$0 < \frac{\alpha_t \gamma_t (1 - s_t)R - \gamma_t c}{\delta^2} \leq V_I(h''_{t+1}),$$

and thus by induction we find a sequence of continuation games in which the equilibrium value of the investor grows without bound, which leads to the desired contradiction because the value of the game is finite and the value of the entrepreneur is guaranteed to be nonnegative.

It remains to discuss the case of $s_t \geq \bar{s}_t$. As $s_t \geq \bar{s}_t$ for all t, it follows that $V_I(h_t) = 0$ for all h_t. In this case, the intertemporal participation constraint (A7) of the investor becomes $\alpha_t \gamma_t (1 - s_t) R - \gamma_t c \geq 0$, which can only be satisfied with $s_t = \bar{s}_t$ for all t, which is the desired conclusion.   *Q.E.D.*

Proof of Theorem 2, parts (i) and (ii). The incentive constraint for the entrepreneur in period t is given by $\alpha_t s_t \gamma_t R + \delta(1 - \alpha_t \gamma_t)V(\alpha_{t+1}) \geq c\gamma_t + \delta V(\alpha_t)$. The participation constraint for the investor is given by

$$\alpha_t (1 - s_t) \gamma_t R \geq c\gamma_t. \tag{A8}$$

The participation constraint is always binding, and the sharing rule is given by $s_t = 1 - c/(\alpha_t R)$. The incentive constraint for the entrepreneur becomes $\alpha_t \gamma_t R - c\gamma_t + \delta(1 - \alpha_t \gamma_t)V(\alpha_{t+1}) \geq c\gamma_t + \delta V(\alpha_t)$, and if funding is constrained in α_t, the incentive constraint is binding with

$$\alpha_t \gamma_t R - c\gamma_t + \delta(1 - \alpha_t \gamma_t)V(\alpha_{t+1}) = c\gamma_t + \delta V(\alpha_t). \tag{A9}$$

The equilibrium value for the entrepreneur can then be expressed by $V(\alpha_t) = c\gamma_t/(1 - \delta)$, and the indifference condition leads to a difference equation determining the equilibrium funding γ_t:

$$\alpha_t \gamma_t R - c\gamma_t + \delta(1 - \alpha_t \gamma_t)\frac{c\gamma_{t+1}}{1 - \delta} = c\gamma_t + \delta\frac{c\gamma_t}{1 - \delta}. \tag{A10}$$

Suppose initially that $\gamma_t = \lambda$ and $\gamma_{t+1} < \lambda$. Then it must be that $\alpha_t \lambda R - \lambda c + (1 - \alpha_t \lambda)\delta V(\alpha_{t+1}) \geq \lambda c + \delta V(\alpha_t)$ and in consequence $V(\alpha_t) \geq c\lambda/(1 - \delta)$, whereas $V(\alpha_{t+1}) = c\gamma_{t+1}/(1 - \delta) < c\lambda/(1 - \delta)$. It follows that a necessary and sufficient condition for maximal funding is given by $\alpha_t \lambda R - \lambda c + (1 - \alpha_t \lambda)\delta[c\lambda/(1 - \delta)] \geq \lambda c + \delta[c\lambda/(1 - \delta)]$, or $\alpha_t \geq \bar{\alpha} = 2c/[R - [\delta c\lambda/(1 - \delta)]]$. As the critical posterior belief $\bar{\alpha}$ is a probability, $\bar{\alpha} = 2c/[R - [c\lambda\delta/(1 - \delta)]] \leq 1 \Leftrightarrow 2c + [c\lambda\delta/(1 - \delta)] \leq R$, we obtain the distinction between low-return and high-return projects. It follows that for all $\alpha_t < \bar{\alpha}, \lambda_t < \lambda$, as the funding volume has to be lowered. By the same argument, there is maximal funding for $\alpha_t \geq \bar{\alpha}$.

Part (iii). We proceed by contradiction in two steps. We first show that if $\gamma(\alpha)$ were to be decreasing in α on some segment, then it can only occur for

$$R < 2c + \frac{c\lambda\delta}{1 - \delta}. \tag{A11}$$

We then argue by contradiction that even if R satisfies inequality (A11), $\gamma(\alpha)$ has to be increasing. To this end, we rewrite the difference equation (A10), using the fact that $\alpha_t = \alpha_{t+1}/(1 - \gamma_t + \alpha_{t+1}\gamma_t)$ to get

$$(\gamma_{t+1} - \gamma_t)\frac{1 - \gamma_t}{\gamma_t} = \alpha_{t+1}\left(\gamma_t\left(\frac{2 - \delta}{\delta}\right) - \frac{(1 - \delta)R}{\delta c}\right) + 2\frac{1 - \delta}{\delta}(1 - \gamma_t). \tag{A12}$$

For an arbitrary and fixed γ_{t+1} and α_{t+1}, we then investigate the nature of the solution for γ_t. The right-hand side of the equality (A12) is linear in γ_t. The left-hand side is a convex function of γ_t, initially decreasing, zero at $\gamma_t = \gamma_{t+1}$, displaying a minimum at $\gamma_t = \sqrt{\gamma_{t+1}}$, and remaining negative thereafter. It follows that a necessary condition for $\gamma_{t+1} \geq \gamma_t$ is that

$$\alpha_{t+1}\left(\lambda\left(\frac{2 - \delta}{\delta}\right) - \frac{(1 - \delta)R}{\delta c}\right) + 2\frac{1 - \delta}{\delta}(1 - \lambda) \geq 0. \tag{A13}$$

By a previous part of the theorem, $\alpha_{t+1} \leq 2c/[R - [c\lambda\delta/(1 - \delta)]]$, the inequality (A13) can be written as

$$\frac{2c}{R - \dfrac{c\lambda\delta}{1 - \delta}}\left(\lambda\left(\frac{2 - \delta}{\delta}\right) - \frac{(1 - \delta)R}{\delta c}\right) + 2\frac{1 - \delta}{\delta}(1 - \lambda) \geq 0,$$

and after cancelling the obvious terms, we have $2c[(1 - \delta)/\delta] - R[(1 - \delta)/\delta] + c\lambda \geq 0$. From this we can infer that $\gamma_{t+1} > \gamma_t$ requires

$$R < 2c + \frac{c\lambda\delta}{1 - \delta}. \tag{A14}$$

We proceed by contradiction using again the difference equation (A10), which is now valid everywhere (for all α_t) by the earlier argument. Consider first the right-hand side of (A10). We first show that for $\alpha_t > \alpha_{t+1}$ and for all equilibrium values $\gamma_t \in [0, \lambda]$,

$$\alpha_t\left(\gamma_t - \frac{1 - \delta}{\delta}\frac{R}{c}\right) + 2\frac{1 - \delta}{\delta} < \alpha_{t+1}\left(\gamma_t - \frac{1 - \delta}{\delta}\frac{R}{c}\right) + 2\frac{1 - \delta}{\delta},$$

or

$$\gamma_t < \frac{1-\delta}{\delta} \frac{R}{c},$$

which has to hold as $c\gamma_t\delta/(1-\delta) < R$, as the agent cannot receive more than the gross value of the project. Next suppose there is an increasing segment, or $\gamma_t \leq \gamma_{t+1}$. Then it follows from the above property and the left-hand side of the difference equation that $\gamma_{t+1} < \gamma_{t+2}$. It follows that if funding is weakly increasing at some time segment t and $t+1$, it will be strictly increasing thereafter. Further, if we view the difference equation as a function expressing γ_{t+1} in dependence of γ_t, then for a fixed α_t it is a convex function and hence $\gamma_{t+2} - \gamma_{t+1} > \gamma_{t+1} - \gamma_t$. As α_t is decreasing over time, this makes the difference equation even more increasing. It follows that if the function is not monotonic decreasing, then it must be increasing at an increasing rate, but this leads to the desired contradiction because funding has to be lower than $\lambda < 1$ everywhere by virtue of (A14).   Q.E.D.

Proof of Theorem 3. ($\Rightarrow$) It is a direct implication of the definition of the Markov-perfect equilibrium that the equilibrium value functions of the players depend only on the payoff-relevant state of the game. The set of equilibrium values at any α is therefore a singleton for every player, and it follows that any Markov-perfect equilibrium is also a weakly renegotiation-proof equilibrium.

($\Leftarrow$) We first show that every weakly renegotiation-proof equilibrium has to be a Markov-perfect equilibrium. The uniqueness of the weakly renegotiation-proof equilibrium then follows from the uniqueness of the Markov-perfect equilibrium. We first observe that every weakly renegotiation-proof equilibrium is a subgame-perfect equilibrium. By Lemma A2, it is then sufficient to consider break-even contracts. This implies that the value function of the investor is equal to zero along every continuation path, or $V_I(h_t) = 0$ for all h_t. By Definition 3 of the weakly renegotiation-proof equilibrium, it then follows that the equilibrium value function of the entrepreneur has to take on the same value for any two histories, h_t and h_s, which generate the same posterior belief. In other words, for all h_t and h_s, we have

$$\alpha(h_t) = \alpha(h_s) \Rightarrow V_E(h_t) = V_E(h_s). \tag{A15}$$

Consider then the incentive constraint of the entrepreneur in any subgame-perfect equilibrium in period t: $\alpha_t\gamma_t s_t R + (1 - \alpha_t\gamma_t)\delta V_E(h_{t+1}) \geq c\gamma_t + \delta V_E(h'_{t+1})$. By Lemma A2, we can restrict our attention to break-even contracts, which leads to $\alpha_t\gamma_t R - \gamma_t c + (1 - \alpha_t\gamma_t)\delta V_E(h_{t+1}) \geq c\gamma_t + \delta V_E(h'_{t+1})$. By the earlier argument, represented by the implication (A15), the posterior belief α has to be a sufficient statistic for the history h_t with respect to the value function of the entrepreneur, and hence

$$\alpha_t\gamma_t R - \gamma_t c + (1 - \alpha_t\gamma_t)\delta V_E(\alpha_{t+1}) \geq c\gamma_t + \delta V_E(\alpha_t). \tag{A16}$$

It is immediate from here that every weakly renegotiation-proof equilibrium must also be a Markov-perfect equilibrium, as it satisfies the equilibrium conditions of the Markov-perfect equilibrium. But since by Theorem 2 the Markov-perfect equilibrium is unique, it follows that the weakly renegotiation-proof equilibrium is unique as well.   Q.E.D.

Proof of Theorem 4. We characterize the equilibrium funding through a sequence of lemmas. Lemma A3 establishes necessary and sufficient conditions for restricted and unrestricted funding at $\alpha_T = \alpha_S$. Lemma A4 establishes the switching point from restricted to unrestricted funding and establishes the difference equation that governs the equilibrium funding as it is restricted. Lemma A5 establishes properties of the fixed point and thereby the areas where funding is restricted and unrestricted. Lemma A6 establishes the monotonicity of the funding volume as a function of time.

Lemma A3. The equilibrium funding volume at $\alpha_T = \alpha_S$ is given by

$$\gamma_T = \lambda \Leftrightarrow \delta \geq \frac{2 - \lambda}{2 - 2\lambda} \tag{A17}$$

and

$$\gamma_T < \lambda \Leftrightarrow \delta < \frac{2 - \lambda}{2 - 2\lambda}. \tag{A18}$$

Proof. We consider the ultimate α_T and penultimate prior beliefs α_{T-1} with $\alpha_T = \alpha_S$. We begin with (A17). Consider the incentive constraint (26) evaluated at the penultimate period, $T-1$:

$$\alpha_{T-1}\lambda R - c\lambda \geq c\lambda + \lambda\frac{\alpha_{T-1}}{\alpha_T}\delta V_E(\alpha_T). \tag{A19}$$

We can express α_{T-1} in terms of λ and α_T, $\alpha_{T-1} = \alpha_T/[1 - \lambda(1 - \alpha_T)]$, and rewrite (A19) as

$$\frac{\alpha_T}{1 - \lambda(1 - \alpha_T)}\lambda R \geq 2c\lambda + \lambda\frac{1}{1 - \lambda(1 - \alpha_T)}\delta V_E(\alpha_T). \tag{A20}$$

By hypothesis $\alpha_T = \alpha_S$, and hence

$$V_E(\alpha_T) = \alpha_T \gamma_T R - c\gamma_T = c\gamma_T. \tag{A21}$$

Inserting the value (A21) into the incentive constraint (A20), we obtain, after rewriting,

$$\lambda(1 - \alpha_T)2c \geq \delta c\gamma_T. \tag{A22}$$

The incentive constraint (A22) is the most difficult to satisfy if γ_T is chosen maximally, i.e., $\gamma_T = \lambda$. Using the fact that $\alpha_T = \alpha_S = 2c/R$, we get $\lambda(R - 2c)2 \geq \delta R\lambda$, which can be written as

$$R(2 - \delta) \geq 4c. \tag{A23}$$

Using the fact that we distinguish between low- and high-return projects, we can express the inequality (A23) in terms of δ and λ exclusively. For suppose that $R < 2c + c\lambda\delta/(1 - \delta)$. Then for $R(2 - \delta) \geq 4c$, it follows that $2c + c\lambda\delta/(1 - \delta) > 4c/(2 - \delta)$ or $\delta > (2 - \lambda)/(2 - 2\lambda)$. On the other hand, for $R \geq 2c + c\lambda\delta/(1 - \delta)$ and $R(2 - \delta) < 4c$, we necessarily have $4c/(2 - \delta) > 2c + c\lambda\delta/(1 - \delta)$ or $\delta < (2 - \lambda)/(2 - 2\lambda)$. Thus, it follows for $R \geq 2c + [c\lambda\delta/(1 - \delta)]$ as well as for $R < 2c + [c\lambda\delta/(1 - \delta)]$ that $\delta \geq (2 - \lambda)/(2 - 2\lambda)$ is a necessary and sufficient condition for full funding in the terminal period. It further follows that if

$$\delta < \frac{2 - \lambda}{2 - 2\lambda},$$

then for (A22) to hold, $\gamma_T < \lambda$. Q.E.D.

Lemma A4. The equilibrium switching point is given by

$$\bar{\bar{\alpha}} = \frac{2c - 2\dfrac{c\lambda}{1 - \delta} + c\dfrac{\lambda\delta}{1 - \delta}}{R - 2\dfrac{c\lambda}{1 - \delta}}.$$

Proof. We first derive the difference equation for the investor's funding decision, provided that the incentive constraint of the entrepreneur is binding. The beliefs of entrepreneur and investor are symmetric along the equilibrium path. In consequence, the contracts on the equilibrium path are the break-even contracts and satisfy

$$\alpha_t \gamma_t s_t R = \alpha_t \gamma_t R - c\gamma_t. \tag{A24}$$

The value of the entrepreneur along the equilibrium path can then be represented as

$$V_E(\alpha_t) = \alpha_t \gamma_t R - c\gamma_t + \delta(1 - \alpha_t \gamma_t)V_E(\alpha_{t+1}). \tag{A25}$$

We can now directly consider the recursive incentive problem of the entrepreneur. The incentive constraint of the entrepreneur changes to reflect the divergence of the beliefs off the equilibrium path. It is given by

$$\alpha_t \gamma_t R - c\gamma_t + \delta(1 - \alpha_t \gamma_t)V_E(\alpha_{t+1}) \geq c\gamma_t + \delta\frac{\alpha_t}{\alpha_{t+1}}V_E(\alpha_{t+1}).$$

Since $1 - \alpha_t \gamma_t = (\alpha_t/\alpha_{t+1})(1 - \gamma_t)$, the incentive constraint may be rewritten as

$$\alpha_t \gamma_t R - 2c\gamma_t \geq \delta\gamma_t \frac{\alpha_t}{\alpha_{t+1}}V_E(\alpha_{t+1}). \tag{A26}$$

When funding is restricted, (A26) must hold with equality, and hence it can be written as

$$V_E(\alpha_{t+1}) = \frac{1}{\delta}\left(\alpha_{t+1}R - \frac{\alpha_{t+1}}{\alpha_t}2c\right). \tag{A27}$$

Substituting (A27) back into (A25) and using $\alpha_{t+1}/\alpha_t = (1 - \gamma_t)/(1 - \alpha_t \gamma_t)$, we get

$$V_E(\alpha_t) = (\alpha_t R - 2c) + c\gamma_t. \tag{A28}$$

Forwarding (A28) for one period and equating to (A27) yields

$$(\alpha_{t+1}R - 2c) + c\gamma_{t+1} = \frac{1}{\delta}\left(\alpha_{t+1}R - \frac{\alpha_{t+1}}{\alpha_t}2c\right).$$

Solving, substituting for α_{t+1}/α_t leads to the difference equation

$$\gamma_{t+1} = 2 + \left(\frac{\alpha_t(1-\gamma_t)}{(1-\alpha_t\gamma_t)} \frac{R}{c} \frac{1-\delta}{\delta} - \frac{1}{\delta} \left(\frac{(1-\gamma_t)}{1-\alpha_t\gamma_t} \right) 2 \right).$$

Making use of the backward expression of the belief ratios, $\alpha_t/\alpha_{t+1} = 1/(\alpha_{t+1}\gamma_t + (1-\gamma_t))$, and solving, we get the backward difference equation of the form $\gamma_t = f(\gamma_{t+1}, \alpha_{t+1})$,

$$\gamma_t = \frac{1}{(1-\alpha_{t+1})} \left(\frac{1}{2}\gamma_{t+1}\delta - (1-\delta)\left(\frac{\alpha_{t+1}R}{2c} - 1 \right) \right), \tag{A29}$$

where we observe that γ_t is a linear increasing function of γ_{t+1}.

The equilibrium switching point $\overline{\overline{\alpha}}$ is given by the unique α that results in a fixed point of the difference equation $\lambda = f(\lambda, \alpha)$ at full funding level,

$$\overline{\overline{\alpha}} = \frac{2c - 2\dfrac{c\lambda}{1-\delta} + \dfrac{c\lambda\delta}{1-\delta}}{R - 2\dfrac{c\lambda}{1-\delta}},$$

which completes this lemma.   *Q.E.D.*

Lemma A5.

   (i) The switching point $\overline{\overline{\alpha}}$ satisfies $\overline{\overline{\alpha}} \in (\alpha_S, 1)$ if either

     (a) $R \geq 2c + [c\lambda\delta/(1-\delta)]$ and $\delta < (2-2\lambda)/(2-\lambda)$, or

     (b) $R < 2c + [c\lambda\delta/(1-\delta)]$ and $\delta \geq (2-2\lambda)/(2-\lambda)$.

   (ii) The switching point satisfies $\overline{\overline{\alpha}} \notin (\alpha_S, 1)$ otherwise.

Proof. The proof is omitted, as it simply requires the algebraic verification of the conditions stated in Lemma A5 applied to the switching point $\overline{\overline{\alpha}}$.   *Q.E.D.*

Lemma A6.

   (i) If $\delta < (2-\lambda)/(2-2\lambda)$, the funding volume γ_t is decreasing over time.

   (ii) If $\delta \geq (2-\lambda)/(2-2\lambda)$, the funding volume γ_t is increasing in time.

   (iii) The monotonicity is strict in either case provided that $\gamma_t < \lambda$.

Proof. To describe the monotonicity properties of the difference equation, it is useful to analyze the fixed point of the mapping, $\gamma = f(\gamma, \alpha)$ for all $\gamma < \lambda$. It is given by

$$\gamma(\alpha) = \frac{(\alpha R - 2c)\dfrac{1-\delta}{\delta}}{\left(2\alpha - 2\dfrac{1-\delta}{\delta}(1-\alpha) - 1 \right)c}.$$

The fixed point $\gamma(\alpha)$ has a derivative that leads to

$$\gamma'(\alpha) > 0 \Leftrightarrow \delta < \frac{2-\lambda}{2-2\lambda} \tag{A30}$$

and

$$\gamma'(\alpha) < 0 \Leftrightarrow \delta > \frac{2-\lambda}{2-2\lambda}. \tag{A31}$$

We start with $\delta < (2-\lambda)/(2-2\lambda)$ and show that the difference equation γ_t must be strictly decreasing in time t. The argument is by contradiction. Suppose then there exists t and $t+1$ such that $\gamma_t \leq \gamma_{t+1}$. Then it follows by the property of the fixed point as a function of α, as displayed in (A30), and the fact that γ_t is linear increasing in γ_{t+1}, as displayed in (A29), that $\gamma_{s-1} < \gamma_s$ for all $s < t$. The funding flow in the period $t = 0$ is then determined, again using (A29), by

$$\gamma_0 = \frac{1}{1-\alpha_0} \left(\frac{1}{2}\delta\gamma_1 - (1-\delta)\left(\frac{\alpha_0 R}{2c} - 1 \right) \right). \tag{A32}$$

Consider then the limit $\alpha_0 \to 1$. Since

$$\lim_{\alpha_0 \to 1} \frac{1}{1 - \alpha_0} = \infty,$$

and γ_0 is bounded by $\gamma_0 \in [0, \lambda]$, it follows that the expression in brackets in (A32) has to go to zero as $\alpha_0 \to 1$, or

$$\gamma_1 = 2\frac{1-\delta}{\delta}\left(\frac{R}{2c} - 1\right). \tag{A33}$$

Because γ_t is supposed to be a local maximum, it follows from (A29) that an upper bound for γ_t is obtained by setting $\gamma_{t+1} = \gamma_t$, or

$$\gamma_t \leq \frac{(R\alpha_{t+1} - 2c)\dfrac{1-\delta}{\delta}}{\left(2\alpha_{t+1} - 2\dfrac{1-\delta}{\delta}(1 - \alpha_{t+1}) - 1\right)c}.$$

Yet we find that

$$\frac{(R\alpha_{t+1} - 2c)\dfrac{1-\delta}{\delta}}{\left(2\alpha_{t+1} - 2\dfrac{1-\delta}{\delta}(1 - \alpha_{t+1}) - 1\right)c} < 2\frac{1-\delta}{\delta}\left(\frac{R}{2c} - 1\right)$$

provided that $\delta \leq (2 - \lambda)/(2 - 2\lambda)$, which leads to the desired contradiction. Thus there is no local maximum either, and hence, $\gamma_t \geq \gamma_{t+1}$ for all t.

Consider next the case of $\delta \geq (2 - \lambda)/(2 - 2\lambda)$. We argue as above by contradiction. Suppose there is a segment t and $t + 1$ such that $\gamma_t > \gamma_{t+1}$. Then it follows again from the fixed-point property, (A31), that for all $s \leq t$, $\gamma_{s-1} \geq \gamma_s$. As $\bar{\bar{\alpha}} \in (\alpha_S, 1)$, it further follows that there must be at least one local minimum, say at γ_t. For the local minimum at γ_t, we obtain a lower bound by setting as above $\gamma_{t+1} = \gamma_t$,

$$\gamma_t > \frac{(R\alpha_{t+1} - 2c)\dfrac{1-\delta}{\delta}}{\left(2\alpha_{t+1} - 2\dfrac{1-\delta}{\delta}(1 - \alpha_{t+1}) - 1\right)c}. \tag{A34}$$

The right-hand side presents the value of the fixed point $\gamma(\alpha)$, which is decreasing by (A30). The lower bound is therefore lowest for $\alpha_{t+1} = 1$, from which it follows that the local minimum γ_t must satisfy

$$\gamma_t > \frac{(R - 2c)\dfrac{1-\delta}{\delta}}{(2 - 1)c}, \tag{A35}$$

yet because it is a local minimum, it has to satisfy

$$\gamma_t \leq \gamma_1 = 2\frac{1-\delta}{\delta}\left(\frac{R}{2c} - 1\right), \tag{A36}$$

where γ_1 was computed earlier at (A33). But for $\delta \geq (2 - \lambda)/(2 - 2\lambda)$, the conditions (A35) and (A36) lead to a contradiction. $Q.E.D.$

Proof of Corollary 2. It is immediately verified that the derivation of the Markov sequential equilibrium above relied only on a backward-induction argument. In the construction of the equilibrium the belief α_t merely served to summarize the information of the players but never to restrict the history contingency of the strategies employed by the agents. $Q.E.D.$

Proof of Theorem 5. Denote by γ_t^s and γ_t^a the funding volume in the symmetric and asymmetric case, respectively. Consider first the case of $\delta \geq (2 - \lambda)/(2 - 2\lambda)$. If, in addition, $R \geq 2c + [\lambda c\delta/(1 - \delta)]$, then $\gamma_t^a = \lambda$ everywhere, and thus the condition $\gamma_t^a \geq \gamma_t^s$ for all α_t, with strict inequality in the final periods, is satisfied. For $R < 2c + [\lambda c\delta/(1 - \delta)]$, we know by Theorem 2 that the funding volume γ_t^s is increasing in α and by Theorem 4 that the funding volume γ_t^a is (weakly) decreasing in α. A necessary and sufficient condition for $\gamma_t^a \geq \gamma_t^s$ is therefore given by

$$\lim_{\alpha_0 \to 1} (\gamma_t^a - \gamma_t^s) \geq 0. \tag{A37}$$

For the symmetric-information case, condition (A10) in the proof of Theorem 2 can be solved as the following difference

equation:

$$\frac{\gamma_{t+1} - \gamma_t}{\gamma_t} = \alpha_t \left(\gamma_{t+1} - \frac{1-\delta}{\delta} \frac{R}{c} \right) + 2 \frac{1-\delta}{\delta}. \tag{A38}$$

As we solve the respective difference equations, (A38) and (A29) from the proof of Theorem 4, for γ_0 as $\alpha_0 \to 1$, we find

$$\lim_{\alpha_0 \to 1} \gamma_0^s = \lim_{\alpha_0 \to 1} \gamma_0^a = \frac{1-\delta}{\delta} \left(\frac{R}{c} - 2 \right),$$

which establishes the validity of (A37).

Consider next the case of

$$\delta < \frac{2-\lambda}{2-2\lambda},$$

where by Theorems 2 and 4 γ_t^a and γ_t^s are both increasing in α. We proceed by establishing a lower bound on γ_t^a and an upper bound on γ_t^s, denoted by $\underline{\gamma}_t^a$ and $\overline{\gamma}_t^s$, respectively. We then show that $\underline{\gamma}_t^a \geq \overline{\gamma}_t^s$, completing the result. As $\gamma_t^a \leq \gamma_{t-1}^a$ in this case, a lower bound $\underline{\gamma}_t^a$ is established by looking for the fixed point $\gamma_{t-1}^a = \gamma_t^a$ in the difference equation (A29),

$$\underline{\gamma}_t^a = \frac{\frac{1-\delta}{\delta} \left(\frac{\alpha_t R}{2c} - 1 \right)}{\frac{1}{2} - (1 - \alpha_t) \frac{1}{\delta}}. \tag{A39}$$

Considering then the difference equation of the symmetric environment (A38) and using the fact that by hypothesis $\gamma_{t-1}^s \geq \gamma_t^s$ in (A38), we obtain an upper bound:

$$\overline{\gamma}_t^s = \frac{1-\delta}{\delta} \left(\frac{R}{c} - \frac{2}{\alpha_t} \right). \tag{A40}$$

Comparing (A39) and (A40) and requiring that $\underline{\gamma}_t^a \geq \overline{\gamma}_t^s$ leads to, after multiplying and cancelling the obvious terms,

$$\left(\frac{R}{2c} - \frac{1}{\alpha_t} \right) + \left(\frac{R}{c} - \frac{2}{\alpha_t} \right) \frac{1-\delta}{\delta} (1 - \alpha_t) \geq \alpha_t \left(\frac{R}{2c} - \frac{1}{\alpha_t} \right),$$

which holds for all $\alpha_t \geq \alpha_S$. *Q.E.D.*

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