

## Supplementary Appendix to “Contracting on Violence: Authoritarian Repression and Military Intervention in Politics”

This Appendix contains technical proofs from Section 3, a numerical example, an extension of the model that examines the model’s implications for our understanding of military interventions in military dictatorships, and details of the multiple imputation procedure in Section 4.

### Proofs

#### Upper bound on $g$

In order to focus on interesting political scenarios, we must exclude large values of  $g$  according to which the government would be so attracted to its preferred policy that it would renege even if it knew that an intervention would certainly follow. The government’s expected payoff from reneging when  $\beta = 1$  is

$$\pi_{aB}(g - r) + \pi_{bB}[\rho(r)(-r) + [1 - \rho(r)](g - r)] = [1 - \pi_{bB}\rho(r)]g - r,$$

whereas the government’s expected payoff from complying is  $1 - \gamma$ . The latter is greater as long as

$$g < \frac{1}{1 - \pi_{bB}\rho(r)} = \bar{g}.$$

### Comparative statics on $\alpha^*$ and $\beta^*$

To see that the government's equilibrium probability of defecting is decreasing in  $\rho(r)$  when  $\rho(r) > c/m$ , differentiate  $\alpha^*$  with respect to  $\rho(r)$ ,

$$\frac{\partial \alpha^*}{\partial \rho(r)} = \frac{-\pi_{bA}(1 + \pi_{aB})}{[(\pi_{aA} - \pi_{aB})(\rho(r)m - c) + \pi_{bA}m]^2} < 0.$$

To see that the military's equilibrium probability of intervening is decreasing in  $\rho(r)$  when  $\rho(r) > c/m$ , differentiate  $\beta^*$  with respect to  $\rho(r)$ ,

$$\frac{\partial \beta^*}{\partial \rho(r)} = -\frac{g - 1}{\pi_{bB}\rho(r)^2 g} < 0.$$

Recall from the paper that the government's expected equilibrium payoff in the mixed strategy equilibrium  $g' - r$  does not depend on  $\rho(r)$ , hence we can treat  $r$  as a parameter when interpreting the above partial derivatives.

Finally, to see that the equilibrium likelihood of military intervention is increasing in the divergence of policy preferences between the government and the military, observe that only  $\beta^*$  depends on  $g$ . Differentiating  $\beta^*$  with respect to  $g$ , we obtain

$$\frac{\partial \beta^*}{\partial g} = \frac{1}{\pi_{bB}\rho(r)g^2} > 0.$$

### The government's equilibrium choice of the military's resources $r$

Recall that the government's expected payoff is

$$\phi(\hat{R}, r)(\hat{g} - r) - [1 - \phi(\hat{R}, r)](r + \hat{R}). \tag{1}$$

By differentiating (1) with respect to  $r$ , we obtain the first-order condition

$$\frac{\partial \phi(\hat{R}, r)}{\partial r} = \frac{1}{\hat{g} + \hat{R}}. \quad (2)$$

Substituting  $\frac{\partial \phi(\hat{R}, r)}{\partial r} = \frac{\hat{R}}{(r + \hat{R})^2}$ , the first-order condition (2) becomes a quadratic equation with a unique positive root

$$r^{max} = -\hat{R} + \sqrt{\hat{R}(\hat{g} + \hat{R})}. \quad (3)$$

Differentiating  $r^{max}$  with respect to  $\hat{R}$ , we obtain

$$\frac{\partial r^{max}}{\partial \hat{R}} = \frac{g + 2\hat{R}}{2\sqrt{\hat{R}(\hat{g} + \hat{R})}} - 1 > 0.$$

Hence  $r^{max}$  is increasing in the expected value of the threat  $\hat{R}$ , regardless of whether  $\hat{g} = g$  or  $\hat{g} = g'$ .

Now consider the optimal choice of  $r \in [0, \bar{r}]$  under alternative values of  $\hat{R}$ . Since  $\lim_{\hat{R} \rightarrow 0^+} r^{max} = 0$ ,  $\rho(r)$  is increasing, and  $\rho(0) = 0$ , there will be sufficiently small values of  $\hat{R}$  such that  $\rho(r^{max}) \leq c/m$ . Thus the optimal choice of  $r$  will be (3) with  $\hat{g} = g$ ,  $r_1^* = -\hat{R} + \sqrt{\hat{R}(g + \hat{R})}$ , as long as  $\hat{R} \leq \hat{R}_1$ , where  $\hat{R}_1$  solves  $\rho(r_1^*) = c/m$ .

If  $\hat{R} > \hat{R}_1$ , the government's choice of  $r$  will weigh the risk of being overthrown by the threat  $R$  at low values of  $r$  against the risk of the military's intervention at high values of  $r$ . In turn, the government will compare the expected payoff under the largest  $r$  consistent with perfect control over the military,  $r_2^*$  solves  $\rho(r) = c/m$ , against the optimal choice of  $r$  when the military has the capacity to intervene,  $r_3^* = -\hat{R} + \sqrt{\hat{R}(g' + \hat{R})}$ . Since  $g > g'$  and  $r^{max}$  is continuous, (by the intermediate value theorem) there will be a range of values for  $\hat{R}$ , such that for  $\hat{R}_1 < \hat{R} < \hat{R}_2$ , the government's expected payoff when  $r = r_2^*$  is greater

than its expected payoff when  $r = r_3^*$ ; since  $r^{max}$  is increasing in  $\hat{g}$ ,  $r = r_2^*$  will be unique.

Hence

$$r^* = \begin{cases} -\hat{R} + \sqrt{\hat{R}(g + \hat{R})} & \text{if } \hat{R} \leq \hat{R}_1, \\ \text{solves } \rho(r) = c/m & \text{if } \hat{R}_1 < \hat{R} < \hat{R}_2, \\ -\hat{R} + \sqrt{\hat{R}(g' + \hat{R})} & \text{if } \hat{R} \geq \hat{R}_2, \end{cases}$$

where  $R_1$  solves  $\rho(r_1^*) = c/m$  and  $R_2$  solves  $u(r_2^*) = u(r_3^*)$ , where  $u(r)$  is the government's expected payoff when it endows the military with the resources  $r$ . Equilibrium values of  $r_2^*$ ,  $R_1$ , and  $R_2$  are characterized only implicitly because we did not assume a specific functional form for  $\rho(r)$ . In fact, since  $\lim_{\hat{R} \rightarrow \infty} r_1^* = g/2$ , there may be functional forms for  $\rho(r)$  according to which  $u(r_2^*) > u(r_3^*)$  for arbitrarily large values of  $\hat{R}$ .<sup>1</sup> In the latter case,  $\hat{R}_2$  does not exist and the government's equilibrium choice of  $r$  is either  $r_1^*$  or  $r_2^*$ . In the numerical example below, I work with a specific functional form for  $\rho(r)$  and compute examples of  $r_2^*$ ,  $R_1$ , and  $R_2$ .

## A numerical example

Assume

$$\rho(r) = \frac{1}{1 + e^{-kr}} - \frac{e^{-kr}}{2},$$

and suppose  $\pi_{aA} = \pi_{bB} = 4/5$ ,  $\pi_{bA} = \pi_{aB} = 1/5$ ,  $c = 1/5$ ,  $\gamma = 1/2$ ,  $k = 3$ ,  $g = 2$ , and  $m = 2$ . Then  $R_1 = 0.011$ ,  $R_2 = 0.375$ , and  $r_2^* = 0.134$ . Figure 1 plots the equilibrium values

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<sup>1</sup>To see that  $\lim_{\hat{R} \rightarrow \infty} r_1^* = g/2$ , note that  $\lim_{\hat{R} \rightarrow \infty} \left[ \sqrt{\hat{R}(g + \hat{R})} - \hat{R} \right] = \lim_{\hat{R} \rightarrow \infty} \sqrt{\hat{R}} \left[ \sqrt{(g + \hat{R})} - \sqrt{\hat{R}} \right]$ . This is an indeterminate form that can be transformed to  $\lim_{\hat{R} \rightarrow \infty} \frac{\sqrt{(g + \hat{R})} - \sqrt{\hat{R}}}{1/\sqrt{\hat{R}}}$ ; rationalizing the numerator, we obtain  $\lim_{\hat{R} \rightarrow \infty} \frac{\sqrt{(g + \hat{R})} - \sqrt{\hat{R}}}{1/\sqrt{\hat{R}}} \frac{\sqrt{(g + \hat{R})} + \sqrt{\hat{R}}}{\sqrt{(g + \hat{R})} + \sqrt{\hat{R}}} = \lim_{\hat{R} \rightarrow \infty} \frac{g}{\left[ \sqrt{(g + \hat{R})} + \sqrt{\hat{R}} \right] / \sqrt{\hat{R}}} = \frac{g}{1 + \sqrt{\lim_{\hat{R} \rightarrow \infty} \frac{g + \hat{R}}{\hat{R}}}}$ . Applying L'Hospital's Rule, we have  $\lim_{\hat{R} \rightarrow \infty} \frac{g + \hat{R}}{\hat{R}} = 1$ . Hence,  $\lim_{\hat{R} \rightarrow \infty} \left[ \sqrt{\hat{R}(g + \hat{R})} - \hat{R} \right] = g/2$ .

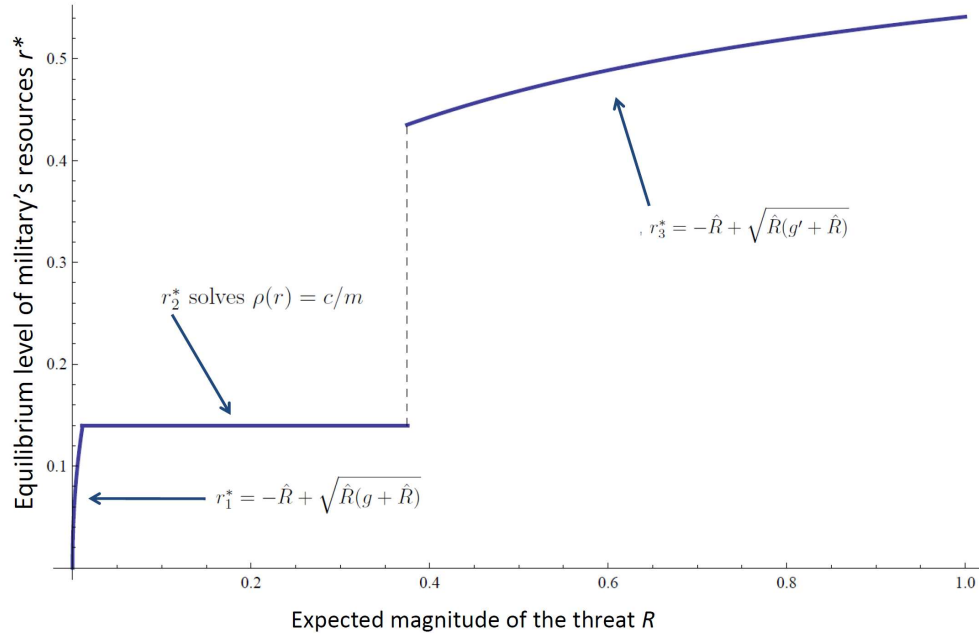


Figure 1: Equilibrium level of military's resources  $r^*$

of  $r$ . For instance, when  $\hat{R} = 1$ ,  $r^* = 0.541$ ,  $\rho(r^*) = 0.737$ ,  $\alpha^* = 0.125$ ,  $\beta^* = 0.848$ , the probability of military intervention is 0.127, and the probability of a successful intervention is 0.094. In expectation, the mass threat succeeds with the probability 0.649.

### An extension: Military intervention in military dictatorships

The model in Section 3 in the paper also helps us understand the vulnerability of military dictatorships to intervention by other factions from within the military. Empirically, military dictators are more likely to be removed from office earlier than are civilian dictators and these removals are typically carried out by other professional soldiers (????). This particular vulnerability may account for the serial correlation in the likelihood of coups in some dictatorships, as estimated by ?. In their words, “once the ice is broken,

more coups follow” (?, 152).

Suppose, therefore, that instead of a civilian government, it is a military government that relies on the rest of the military for the repression of mass opposition. As in the basic model, we assume that the officers in government may differ in their preferences over policies from officers who function strictly within the military. ?, Chapter 2, ?, Chapter 2, and ? describe such differences within the Chilean, Argentine, and Brazilian military dictatorships.

However, when it comes to contracting on violence, military and civilian dictatorships differ from each other in three ways. First, military governments typically lack institutions and norms that facilitate information sharing. We may in turn expect that the policy making process will be less transparent under military than civilian dictatorships. Formally, this corresponds to an increase in  $\pi_{bA}$  and  $\pi_{aB}$ . Second, following its intervention in politics, the military proper is less autonomous from the government and more factionalized. We may therefore expect that an intervention is more likely to fail than when the military acts as an autonomous, unified institution against a civilian government. In terms of our model, the value of  $\rho(r)$  declines. Third, the institutional cost of further intervention  $c$  will be much smaller – essentially zero – after an initial intervention that brings the military to power and thereby diminishes its institutional integrity.

The earlier equilibrium analysis implies that two of these three factors – reduced transparency of the policy making process and greater factionalization – unambiguously exacerbate the moral hazard problem in contracting on violence and thereby raise the

likelihood of subsequent military intervention. For  $\rho(r) \geq c/m$ ,

$$\begin{aligned}\frac{\partial \alpha^*}{\partial \pi_{aB}} &= \frac{\pi_{bA}([1 - \rho(r)]m + c)(\rho(r)m - c)}{[(\pi_{aA} - \pi_{aB})(\rho(r)m - c) + \pi_{bA}m]^2} > 0, \\ \frac{\partial \alpha^*}{\partial \pi_{aA}} &= -\frac{\pi_{bB}([1 - \rho(r)]m + c)(\rho(r)m - c)}{[(\pi_{aA} - \pi_{aB})(\rho(r)m - c) + \pi_{bA}m]^2} < 0, \\ \frac{\partial \beta^*}{\partial \pi_{aB}} &= \frac{g - 1}{\pi_{bB}^2 \rho(r) g} > 0.\end{aligned}$$

We have seen earlier that both  $\alpha^*$  and  $\beta^*$  decrease in  $\rho(r)$  for  $\rho(r) \geq c/m$ . In turn, the likelihood military intervention increases after a decline in  $\rho(r)$ . Meanwhile, the effect of the decrease in the institutional cost of intervention is more nuanced: On the one hand, it bolsters the credibility of the threat of intervention and thus reduces the temptation of the military government to renege on any policy compromise with their counterparts within the military proper,

$$\frac{\partial \alpha^*}{\partial c} = \frac{m\pi_{bA}\pi_{bB}}{[(\pi_{aA} - \pi_{aB})(\rho(r)m - c) + \pi_{bA}m]^2} > 0.$$

On the other hand, intervention now occurs with a positive probability for any value of  $\rho(r)$ . The present model therefore identifies three, key differences between civilian and military dictatorships in contracting on violence, two of which unambiguously imply that military dictatorships should be more vulnerable to military interventions than civilian ones.

## Multiple Imputation

I follow ? and use the Amelia II package for R (?). This package allows for an imputation procedure that is appropriate for cross-sectional time-series data and leads to very sensible

imputed values.

I properly imputed the data as follows: I used countries and years to index the cross-section and time units. I furthermore used second-order polynomials as well as lags and leads on the inequality measures; these were interacted with the the cross-section units. I transformed the inequality measures as well as ethnic fractionalization so that imputations do not fall outside the sample extremes. The remaining covariates were transformed whenever appropriate. In order to minimize the extent to which missingness in the data depends on unobservables, I included in the imputation additional variables that are potential predictors of inequality but are not relevant for the estimation in the paper. Finally, I used a 5% ridge prior in order to improve the numerical stability of the imputations; instability may be caused by the high missingness in the inequality measures as well as the high correlation between the two measures.

The Theil statistic is available for at most 34% of the 5,393 country-years in the data. I used ?'s (?) concept of relative efficiency in order to determine the number of datasets to impute. ?, 114 shows that the efficiency of an estimate based on  $m$  imputations relative to a fully efficient estimate based on an infinite number of imputations is approximately

$$\frac{1}{\sqrt{1 + \frac{\gamma}{m}}},$$

where  $\gamma$  is the rate of missingness.<sup>2</sup> Given the current rate of missingness of 66%, the relative efficiency of at least 99% is obtained with 33 or more imputations. That is, estimates based on 33 or more imputed datasets will on average have standard errors that are at most 1% larger than those based on an infinite number of imputations.

The R code for the multiple imputation procedure is

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<sup>2</sup>This expression measures relative efficiency on the scale of standard errors of the estimated quantity.



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require(Amelia)

intervention <- read.dta("to_fill_milintervention_short.dta")

a.out <- amelia(intervention, m = 40, ts = "year", cs = "ccode",
+ polytime = 2, intercs = TRUE, empri = 0.05*nrow(intervention),
+ lgstc=c("ethfrac_tr", "theil_lag_tr", "SIDD2_lag_tr", "fuel_tr",
+ "ore_tr", "prop_dem_tr", "SIDD2_sq_lag_tr", "theil_sq_lag_tr"),
+ ords=c("polity","oil","war3","civilwar3","mil_entry", "mil_exit",
+ "mil_leader","pleader_mil"),
+ noms = c("hinst","region","leg_origin","cw_1"),
+ lags = c("SIDD2_lag_tr","SIDD2_sq_lag_tr","theil_lag_tr","theil_sq_lag_tr"),
+ leads = c("SIDD2_lag_tr","SIDD2_sq_lag_tr","theil_lag_tr","theil_sq_lag_tr"),
+ p2s = 2)

```

Finally, in order to avoid working with implausible imputations in the analysis, I only use data for countries which have at least one observation on inequality and do not use imputations that are further than ten years from the closest observed value. In turn, the analysis in the paper is based on a data with a rate of missingness of 44% rather than 66%. The 40 imputations that is use in the analysis is thus very conservative.

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