Supplementary Appendix to "The Foundations of Limited Authoritarian Government: Institutions and Power-sharing in Dictatorships"

This Appendix contains proofs of those technical results that do not follow directly from the discussion in the text as well as additional empirical results and robustness checks.

An alternative derivation of the results in Proposition 1:

As in the paper, suppose that allies use a threshold strategy according to which ally i supports the ruler if her signal k_i is below some threshold k^*, $k_i \leq k^*$, and she rebels if $k_i > k^*$. Since the signal k_i is distributed uniformly on the interval $[\kappa^t - \varepsilon, \kappa^t + \varepsilon]$, a threshold signal k^* implies the existence of a threshold regime strength κ^* such that a rebellion fails if $\kappa^t \leq \kappa^*$ and succeeds if $\kappa^t > \kappa^*$. (Recall that a high κ^t corresponds to a weak regime, thus a rebellion succeeds when the regime is weaker than some threshold κ^*.)

Suppose an ally observes the signal $k_i = k^*$. What is the probability that a rebellion succeeds, $\kappa^t > \kappa^*$? If $k_i = k^*$, then $\kappa^* \sim U[k^* - \varepsilon, k^* + \varepsilon]$. In turn,

$$\Pr(\kappa^t > \kappa^*) = \frac{k^* + \varepsilon - \kappa^*}{2\varepsilon} \quad \text{and} \quad \Pr(\kappa^t \leq \kappa^*) = \frac{\kappa^* - (k^* - \varepsilon)}{2\varepsilon} \,. \tag{1}$$

An ally who observes the signal $k_i = k^*$ must be indifferent between supporting and rebelling against the ruler, and the discussion in the paper (equation (2) in the paper) implies that this will be the case when

$$\Pr(\rho \leq \rho^*) = \frac{b_C}{b_C + b_I + r} \,. \tag{2}$$

Recall that $\Pr(\rho \leq \rho^*)$ is the probability that a rebellion will fail and is thus identical to $Pr(\kappa^t \leq \kappa^*)$. Hence we can substitute $Pr(\kappa^t \leq \kappa^*)$ from (1) for $\Pr(\rho \leq \rho^*)$ in the equilibrium indifference condition (2). Then, in equilibrium,

$$\frac{\kappa^* - (k^* - \varepsilon)}{2\varepsilon} = \frac{b_C}{b_C + b_I + r} \ . \tag{3}$$

Now suppose $\kappa^t = \kappa^*$. What is the proportion of allies that will rebel because they observe a signal k_i greater than the threshold signal k^*, $k_i > k^*$? If $\kappa^t = \kappa^*$, then $k^* \sim U[\kappa^* - \varepsilon, \kappa^* + \varepsilon]$. In turn,

$$\Pr(k_i > k^*) = \frac{\kappa^* + \varepsilon - k^*}{2\varepsilon} \ . \tag{4}$$

When $\kappa^t = \kappa^*$, the probability $\Pr(k_i > k^*)$ is identical to the proportion of allies whose participation in the rebellion is just sufficient for the rebellion to succeed, ρ^*. Thus we can substitute (4) into condition (1) in the paper and obtain

$$\frac{\kappa^* + \varepsilon - k^*}{2\varepsilon} = \frac{\kappa^0 - \kappa^t}{\kappa^0 - \lambda} \ . \tag{5}$$

Equations (3) and (5) constitute the equilibrium conditions that implicitly characterize the threshold signal k^* and the threshold regime strength κ^*. Since (3) and (5) form system of two linear equations in two unknowns, we can solve this system for k^* and κ^*. We obtain

$$k^* = \frac{(\lambda - \varepsilon)b_C + (\kappa^0 + \varepsilon)(b_I + r)}{b_C + b_I + r} \ , \quad \kappa^* = \frac{\lambda b_C + \kappa^0(b_I + r)}{b_C + b_I + r} \ , \tag{6}$$

which is also the result in the paper.

The uniqueness of the equilibrium in Proposition 1:

Carlsson and van Damme (1993, 995-996, 1003-1005) and Morris and Shin (2003, 65-71) summarize and discuss the conditions for a unique equilibrium in a global game. Morris and Shin (2003, 65-67) prove that an equilibrium of a global game is unique (by surviving iterated elimination of strictly dominated strategies) if it satisfies the following five properties: i) action monotonicity, ii) state monotonicity, iii) strict Lapacian state monotonicity, iv) limit dominance, and v) continuity.

In the present context, i) action monotonicity requires that the incentive of an ally to rebel is nondecreasing in the number of other allies who plan to rebel (ρ), ii) state monotonicity requires that the incentive of an ally to rebel is nonincreasing in the strength of the regime (κ^t), iii) strict Lapacian state monotonicity requires that there is a unique regime strength (κ^*) that satisfies the indifference condition (3) for an ally who observes the signal $k_i = k^*$, iv) limit dominance requires that there are high and low levels of κ^t (and by extension of k_i) such that an ally strictly prefers to rebel and support the ruler, respectively, regardless of other allies' actions, and v) continuity requires that the allies' expected payoff from rebelling is continuous in k_i and $\Pr(\rho > \rho^*)$.

Our setting satisfies these conditions and thus the equilibrium characterized by Proposition 1 is unique. Our main technical difference from Morris and Shin (2003) is the bounded support of the probability distribution of k_i, which is unbounded in Morris and Shin (2003). Footnote 3 in Morris and Shin (2003, 65) explains that their proofs extend to the case of bounded support; see also Carlsson and van Damme (1993, 1003-1005).

Comparative static results from Proposition 1:

The relevant partial derivatives of κ^* are

$$\begin{aligned}
\frac{\partial \kappa^*}{\partial \lambda} &= \frac{b_C}{b_C + b_I + r} > 0, \\
\frac{\partial \kappa^*}{\partial b_I} &= \frac{\partial \kappa^*}{\partial r} = \frac{b_C(\kappa^0 - \lambda)}{(b_C + b_I + r)^2} > 0, \quad \text{and} \\
\frac{\partial \kappa^*}{\partial b_C} &= -\frac{(b_I + r)(\kappa^0 - \lambda)}{(b_C + b_I + r)^2} < 0.
\end{aligned}$$

The relevant partial derivatives of k^* are

$$\begin{aligned}
\frac{\partial k^*}{\partial \lambda} &= \frac{b_C}{b_C + b_I + r} > 0, \\
\frac{\partial k^*}{\partial b_I} &= \frac{\partial k^*}{\partial r} = \frac{b_C(2\varepsilon + \kappa^0 - \lambda)}{(b_C + b_I + r)^2} > 0, \quad \text{and} \\
\frac{\partial k^*}{\partial b_C} &= -\frac{(b_I + r)(2\varepsilon + \kappa^0 - \lambda)}{(b_C + b_I + r)^2} < 0.
\end{aligned}$$

Forming a larger than a minimum ruling coalition κ^0:

Suppose that the incumbent and the challenger can choose a larger than a minimum ruling coalition $\kappa' \geq \kappa^0$ (holding κ' the same for the incumbent and the challenger.) This has two implications. First, as κ' increases, a larger fraction of allies needs to rebel in order to depose the incumbent,

$$\rho^* = \frac{\kappa' - \kappa^t}{\kappa' - \lambda} \quad \text{and} \quad \frac{\partial \rho^*}{\partial \kappa'} = \frac{\kappa^t - \lambda}{(\kappa' - \lambda)^2} > 0.$$

But second, holding the fraction of total benefits from joint rule β that the incumbent and the challenger share with the allies equal, each ally obtains a smaller benefit $b_C = \frac{\beta}{\mu} = \frac{\beta}{\kappa' - \lambda}$ as κ' increases.

The former effect dominates the latter. After substituting the updated ρ^* and b_C into the expression for ϕ, we obtain

$$\phi = 1 - \frac{\beta\lambda + \kappa'(\kappa' - \lambda)r}{\beta + (\kappa' - \lambda)r} = \frac{\beta(1-\lambda) + (1-\kappa')(\kappa' - \lambda)r}{\beta + (\kappa' - \lambda)r}\,.$$

Differentiating ϕ with respect to κ', we get

$$\frac{\partial\phi}{\partial\kappa'} = -\frac{(\kappa' - \lambda)r[2\beta + (\kappa' - \lambda)r]}{[\beta + (\kappa' - \lambda)r]^2} < 0\,.$$

Thus forming a larger than a minimum ruling coalition reduces the probability that a rebellion succeeds and cannot help an incumbent to strengthen the credibility of his promise to share benefits as agreed. (Constraint (7) is harder to satisfy as the probability that a rebellion succeeds declines.)

This result also implies that forming a minimum-size ruling coalition κ^0 is challenge-proof at the formation stage. If we think of the formation of the initial ruling coalition as a bidding process where two candidates for the dictator propose the size of the ruling coalition to the notables (again holding β that the incumbent and the challenger share with the allies equal), then the candidate that offers to form a minimum winning coalition κ^0 will not be beatable. A minimum winning coalition gives the allies the greatest influence over the leader because it maximizes the likelihood that a rebellion succeeds if staged, and it gives each ally the largest payoff because he only shares β with the smallest necessary number of allies.

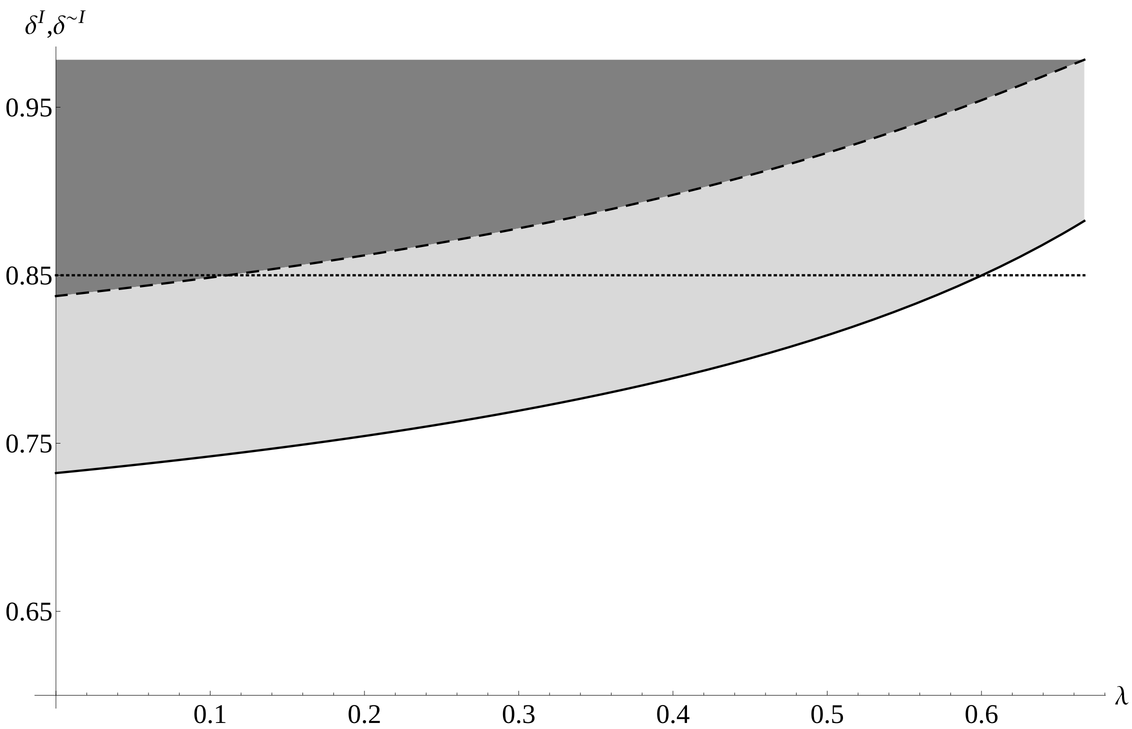

Figure 1: The effect the ruler's power λ vis-à-vis the allies on threshold discount factors δ^I (solid line) and $\delta^{\sim I}$ (dashed line) for $\lambda < \kappa^0$. The dotted line plots a hypothetical discount factor $\delta = 0.85$.

A numerical example:

Suppose $\kappa^0 = 2/3$, $\lambda = 1/3$, $\beta = 0.75$, $b_C = 1.5$ $r = 2$, $\pi = 0.6$ and $\varepsilon = 0.1$. Then $\mu = 1/3$ and, in equilibrium, ally i rebels if his signal k_i is greater than the threshold signal $k^* = 0.52$, and the rebellion succeeds if the regime is weaker that then threshold $\kappa^* = 0.54$.

Figure 1 plots the effect the ruler's power λ vis-à-vis the allies on threshold discount factors δ^I and $\delta^{\sim I}$ for $\lambda < \kappa^0$. For a discount factor $\delta = 0.85$, $\lambda^{\sim I} = 0.11$ and $\lambda^I = 0.6$.

References

Carlsson, Hans and Eric van Damme. 1993. "Global Games and Equilibrium Selection." *Econometrica* 61(5):989–1018.

Morris, Stephen and Hyun Song Shin. 2003. "Global Games: Theory and Applications." In Dewatripont, Mathias, Lars Peter Hansen, and Stephen J. Turnovsky (Eds.), "Advances in Economics and Econometrics," New York: Cambridge University Press, pp. 56–114.