Online Appendix to "Power-sharing and Leadership Dynamics in Authoritarian Regimes"

This Appendix fills in the details of some of the formal results in Svolik (2009).

Proposition 1: As I argued in the paper, there is no equilibrium in which the dictator uses a pure strategy *and* the ruling coalition conditions its decision to stage a coup on the observed signal.

In any equilibrium in mixed strategies, (i) the ruling coalition stages a coup with probability β_θ such that, given the correlation between his actions and the signal θ, the dictator is indifferent between diverting and complying, and (ii) the dictator diverts with probability α such that the ruling coalition is indifferent between staging and not staging a coup after observing a high signal or after observing a low signal, but not both.

Note that the ruling coalition cannot be indifferent between staging and not staging a coup after both a high and low signal: If the dictator chooses such α as to make the ruling coalition indifferent between staging and not staging a coup after observing a high signal, than the ruling coalition will prefer not to stage a coup after observing a low signal. Alternatively, if the dictator chooses such α as to make the ruling coalition indifferent between staging and not staging a coup after observing a low signal, than the ruling coalition will prefer to stage a coup after observing a high signal.

Thus for the ruling coalition, only the actions $(\beta_L = 0, \beta_H > 0)$ and $(\beta_L > 0, \beta_H = 1)$ can be parts of an equilibrium. To obtain the equilibrium action profile, we solve for the indifference conditions.

In the case when $(\beta_L = 0, \beta_H > 0)$, we have

$$\beta_H^* = \frac{\mu}{\rho\left[\pi_{Hd}(1+\mu) - \pi_{Hc}\right]} \quad \text{and} \quad \alpha^* = \frac{\pi_{Hc}}{\pi_{Hc} + \pi_{Hd}\left(\frac{\epsilon}{1-\rho} - 1\right)} \quad . \tag{1}$$

To verify that $\beta_L^* = 0$, it must be true that the ruling coalition prefers not to stage a coup after it observed a low signal,

$$\rho \leq \Pr(d|L)(1-\epsilon) + 1 - \Pr(d|L). \tag{2}$$

After substituting α^* into

$$\Pr(d|L) = \frac{\pi_{Ld}\alpha^*}{\pi_{Ld}\alpha^* + \pi_{Lc}(1-\alpha^*)},$$

inequality (2) can be reduced to

$$-\frac{(\pi_{Hd} - \pi_{Hc})(1-\rho)[\rho - (1-\epsilon)]}{\epsilon(\pi_{Hd} - \pi_{Hc}\pi_{Hd}) - (1-\rho)(\pi_{Hd} - \pi_{Hc})} \leq 0,$$

which holds as long as $\rho \geq 1 - \epsilon$.

In the case when $(\beta_L > 0, \beta_H = 1)$, the indifference condition implies

$$\beta_L^{**} = \frac{\pi_{Hd} - \pi_{Hc} - (\frac{1}{\rho} - \pi_{Hd})\mu}{\pi_{Hd} - \pi_{Hc} - (1 - \pi_{Hd})\mu} \quad \text{and} \quad \alpha^{**} = \frac{1 - \pi_{Hc}}{\pi_{Hd} - \pi_{Hc} + (1 - \pi_{Hc})\left(\frac{\epsilon}{1-\rho}\right)}.$$

To verify that $\beta_H^{**} = 1$, it must be true that the ruling coalition prefers to stage a coup after it observed a high signal,

$$\rho \geq \Pr(d|H)(1-\epsilon) + 1 - \Pr(d|H). \tag{3}$$

After substituting α^{**} into

$$\Pr(d|H) = \frac{\pi_{Hd}\alpha^{**}}{\pi_{Hd}\alpha^{**} + \pi_{Hc}(1-\alpha^{**})},$$

inequality (3) can be reduced to

$$\frac{(\pi_{Hd} - \pi_{Hc})(1-\rho)[\rho - (1-\epsilon)]}{(\pi_{Hd} - \pi_{Hc})(1-\rho) + \pi_{Hc}(1-\pi_{Hd})\epsilon} \geq 0,$$

which holds as long as $\rho \geq 1 - \epsilon$.

Moreover, the expected payoff to *both* the dictator and the ruling coalition is greater in the equilibrium with $(\beta_L = 0, \beta_H > 0)$ than it is in the equilibrium with $(\beta_L > 0, \beta_H = 1)$. In the equilibrium with $(\beta_L = 0, \beta_H > 0)$, the expected payoff to the dictator is

$$\frac{b(\pi_{Hd} - \pi_{Hc})(1+\mu)}{\pi_{Hd} - \pi_{Hc} + \pi_{Hd}\mu},$$

and it is

$$\frac{b(\pi_{Hd} - \pi_{Hc})(1-\rho)(1+\mu)}{\pi_{Hd} - \pi_{Hc} - \mu(1-\pi_{Hd})}$$

in the equilibrium with $(\beta_L > 0, \beta_H = 1)$. The difference between the former and the latter is

$$(\pi_{Hd} - \pi_{Hc})(1+\mu)(1-\rho)\frac{\rho(\pi_{Hd} - \pi_{Hc}) - \mu(1-\rho\pi_{Hd})}{[\pi_{Hd}(1+\mu) - \pi_{Hc}][\pi_{Hd} - \pi_{Hc} - \mu(1-\pi_{Hd})]},$$

which is positive as long as Assumption 1 in the paper is satisfied.

In the equilibrium with $(\beta_L = 0, \beta_H > 0)$, the expected payoff to the ruling coalition is

$$\frac{(\pi_{Hd} - \pi_{Hc})[\rho - (1-\epsilon)] + \pi_{Hc}\epsilon\rho}{(\pi_{Hd} - \pi_{Hc})[\rho - (1-\epsilon)] + \pi_{Hc}\epsilon},$$

and it is ρ in the equilibrium with $(\beta_L > 0, \beta_H = 1)$. The difference between the former and the latter is

$$\frac{(\pi_{Hd} - \pi_{Hc})(1-\rho)[\rho - (1-\epsilon)]}{(\pi_{Hd} - \pi_{Hc})(1-\rho) + \pi_{Hd}\epsilon},$$

which is positive as long as $\rho \geq 1 - \epsilon$. Thus *both* the dictator and the ruling coalition prefer the equilibrium in which $(\beta_L = 0, \beta_H > 0)$ to the equilibrium in which $(\beta_L > 0, \beta_H = 1)$. This concludes all proofs associated with Proposition 1.

Proposition 2: Recall that the probability of successful power-sharing is

$$\Pr(\text{Successful Power-Sharing}) = (1-\alpha^*)\left[\pi_{Hc}(1-\beta_H^*) + (1-\pi_{Hc})\right] = (1-\alpha^*)(1-\pi_{Hc}\beta_H^*). \quad (4)$$

As I showed in the paper, in a contested dictatorship with $(\beta_L = 0, \beta_H > 0)$, both the probability that the dictator diverts (α^*) and the probability that the ruling coalition stages a coup after observing a high signal (β_H^*) increase as the balance of power (b) shifts in the dictator's favor. In turn, the probability of successful power-sharing is decreasing in the dictator's power.

The probability of a successful diversion is

$$\Pr(\text{Successful Diversion}) = \alpha^* \left[\pi_{Ld} + \pi_{Hd}(1 - \beta_H^*) + \pi_{Hd}\beta_H^*(1 - \rho)\right] = \alpha^*(1 - \pi_{Hd}\rho\beta_H^*).$$

Substituting α^* and β_H^* from (1), we obtain

$$\Pr(\text{Successful Diversion}) = \frac{b\pi_{Hc}(\pi_{Hd} - \pi_{Hc})}{[\pi_{Hd}\epsilon - b(\pi_{Hd} - \pi_{Hc})][\pi_{Hd}(1 + \mu) - \pi_{Hc}]}\,.$$

Finally, differentiating with respect to b, we obtain

$$\frac{\partial \Pr(\text{Successful Diversion})}{\partial b} = \frac{\pi_{Hc}\pi_{Hd}(\pi_{Hd} - \pi_{Hc})\epsilon}{[\pi_{Hd}\epsilon - b(\pi_{Hd} - \pi_{Hc})]^2[\pi_{Hd}(1 + \mu) - \pi_{Hc}]} > 0\,.$$

Thus the probability of a successful diversion is increasing in the dictator's power.

Proposition 3: By inspection of (1), we see that both α^* and β_H^* are decreasing in π_{Hd} and increasing in π_{Hc}. To see that α^* is increasing in π_{Hc}, differentiate α^* with respect to π_{Hc}, to obtain

$$\frac{\partial \alpha^*}{\partial \pi_{Hc}} = \frac{\pi_{Hd} b(\epsilon - b)}{[\pi_{Hd}(\epsilon - b) + \pi_{Hc} b]^2} > 0\,.$$

In turn, the probability of successful power-sharing in (4) is increasing in π_{Hd} and decreasing in π_{Hc}.

References

Svolik, Milan. 2009. "Power-sharing and Leadership Dynamics in Authoritarian Regimes." *American Journal of Political Science* 53(2).