Supplemental Appendix to "Formal Models of Nondemocratic Politics"

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The probability that a member of the winning coalition will be among the W members of the selectorate with the highest realization of the affinity parameters A_C^i: The probability that an observation x from a sample of N draws from the standard uniform density is the kth largest is

$$\binom{N-1}{k-1} x^{(N-1)-(k-1)}(1-x)^{k-1}.$$

If all we know about x is that it is drawn from the standard uniform density, then the probability that x will rank as the kth largest observation is

$$\int_0^1 \binom{N-1}{k-1} x^{(N-1)-(k-1)}(1-x)^{k-1}\,\mathrm{d}x\,.$$

In turn, the probability that this observation will be at least the kth largest observation is

$$\sum_{i=1}^{k} \int_0^1 \binom{N-1}{i-1} x^{(N-1)-(i-1)}(1-x)^{i-1}\,\mathrm{d}x\,.$$

After multiplying and dividing the expression through by N, the integrand in each element of this sum can be expressed as the density function of the Beta distribution with the parameters $N-i+1$ and i

$$\sum_{i=1}^{k} \frac{1}{N} \int_0^1 \frac{\Gamma(N+1)}{\Gamma(i)\Gamma(N-i+1)} x^{(N-i)}(1-x)^{i-1}\,\mathrm{d}x\,.$$

Since this density function (by assumption) integrates to 1, the probability that an observation from a sample of N draws from the standard uniform density is at least the kth largest is

$$\sum_{i=1}^{k} \frac{1}{N} = \frac{k}{N}.$$

Thus we see that the probability that a member of the winning coalition of size W who considers defecting to the challenger expects to be among the W members of the selectorate of size S with the highest realization of the affinity parameters A_C^i with the probability $\frac{W}{S}$.