Supplementary Material – Not for Publication

I Estimation Appendix

I.1 Estimation Algorithm

In this section we describe the estimation algorithm in detail, which we break down into several steps for expositional clarity.

Before we proceed, remember that value added for domestic producers in sector k is given by:

$$VA_k(z,\ell) = \Theta_k (P_k^m)^{-(1-\delta_k)\Lambda_k} (\exp(d_{H,k}))^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

where

$$P_k^m \equiv \frac{P_C^{\lambda_k} P_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}}, \tag{S.1}$$

$$\Theta_k \equiv \left(\frac{1}{(1-\delta_k)\Lambda_k}\right)\left(\frac{(1-\delta_k)(\sigma_k-1)}{\sigma_k}\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k},$$

and

$$\Lambda_k \equiv \frac{\sigma_k - 1}{\sigma_k - (1-\delta_k)(\sigma_k-1)}.$$

Rewrite value added for domestic producers as

$$VA_k(z,\ell) = \Theta_k \Psi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k},$$

with

$$\Psi_k \equiv (P_k^m)^{-(1-\delta_k)\Lambda_k} (\exp(d_{H,k}))^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}. \tag{S.2}$$

Note that Θ_k is a solely a function of model's parameters. On the other hand, Ψ_k is a function of model's parameters but also of equilibrium objects such as P_C, P_S and $d_{H,k}$. In turn, value added for exporters is given by:

$$VA_C(z,\ell) = \Theta_C \Psi_C (\exp(d_F))^{\frac{\sigma_C}{\sigma_C-1}\Lambda_C} \left(z\ell^{\delta_C}\right)^{\Lambda_C}.$$

It will be convenient to define and work with

$$\vartheta_{J_u} \equiv b + \frac{1}{1+r} J^u.$$

Ψ_C, Ψ_S, ϑ_{J_u} are treated as parameters to be estimated along with the remaining ones, but these are all endogenous variables. The procedure below makes sure that the values guessed for Ψ_C and Ψ_S are equilibrium outcomes (see Step 9 for details). The number of entrants M_C and M_S will be set to match Ψ_C and Ψ_S. Given knowledge of ϑ_{J_u} and the remaining parameters, we can recover the flow utility of unemployment b and the value of unemployment J^u post-estimation.

Step 1a: λ_C and λ_S are obtained from input-output tables and fixed throughout.

Step 1b: Fix μ^{υ} and obtain ϕ using equation (23):

$$\phi = \left(\frac{\mu^{\upsilon}}{\left(Transition_{Data}^{U \to E}\right)^{\frac{\xi-1}{\xi}}} \right)^{\xi}$$

where $Transition_{Data}^{U \to E}$ is the transition rate from unemployment to employment in the data.

Step 2: Start with a parameter vector guess Ω, including values for Ψ_C, Ψ_S and ϑ_{J_u}.

Step 3: Obtain δ_k using $P_k^m \iota_k(z,\ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k(z,\ell)$:

$$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data} = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k}$$

$$\Rightarrow \delta_k = 1 - \frac{\sigma_k}{\sigma_k - 1} \left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$$

$\left(\frac{Total\ Expenditures\ with\ Intermediates_k}{Total\ Gross\ Revenues_k} \right)_{Data}$ is obtained from input-output tables.

Step 4: Obtain d_F using equation (32):

$$E\left[Export\ Share | Exporter = 1\right]_{Data} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right)$$

$$\Rightarrow d_F = -\frac{1}{\sigma_C} \log\left(1 - E\left[Export\ Share | Exporter = 1\right]_{Data}\right)$$

$E\left[Export\ Share | Exporter = 1\right]_{Data}$ is the average share of exporters' gross revenues in sector C coming from exports, obtained from PIA and SECEX.

Step 5: This step solves for wage schedules $w_{kf}(z,\ell')$, $w_{ki}(z,\ell')$ as well as value functions $V_{kf}(z,\ell)$, $V_{ki}(z,\ell)$, $J_{kf}^e(z,\ell')$, $J_{ki}^e(z,\ell')$, and firms' policy functions.

Step 5a: Compute value added functions $VA_k(z,\ell)$.

Step 5b: Compute wage schedules $w_{kf}(z,\ell')$

- Guess a wage schedule $w_{kf}(z,\ell')$
- Compute the resulting $V_{kf}(z,\ell')$ using (13)
- Compute $J_{kf}^e(z,\ell')$ using (A.46)
- Compute $w_{kf}^u(z,\ell')$ using equation (27)
- Let $\widehat{w}_{kf}^u(z,\ell') = \omega_0 + \omega_1 \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w_{kf}^u(z,\ell')$ on $\left[1, \frac{VA_k(z,\ell')}{\ell'}\right]$

- Update $w_{kf}(z,\ell') = \max\left\{\widehat{w}^{u}_{kf}(z,\ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r}J^{e}_{kf}(z,\ell'), \underline{w}\right\}$
- Restart until convergence

Step 5c: Compute wage schedules $w_{ki}(z,\ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (17)
- Compute $J^{e}_{ki}(z,\ell')$ using (A.47)
- Compute $w^{u}_{ki}(z,\ell')$ using equation (30)
- Let $\widehat{w}^{u}_{ki}(z,\ell') = \omega_0 + \omega_1\left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^{u}_{ki}(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)}p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^{u}_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r}J^{e}_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 6: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_{k}(z) = 1\right) = \int_z I^{formal}_{k}(z)\, g^{e}_{k}(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_{k}(z) = 1\right) = \int_z I^{informal}_{k}(z)\, g^{e}_{k}(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^{e}_{ki}(z) = \frac{g^{e}_{k}(z)\, I^{informal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\, I^{informal}_{k}(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^{e}_{kf}(z) = \frac{g^{e}_{k}(z)\, I^{formal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\, I^{formal}_{k}(\widetilde{z})\, d\widetilde{z}}.$$

Step 7: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k)\int_z\int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right)\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1 - \varrho_{ki}^{exit}\right) N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = {} & \mathcal{I}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}(z) \\ & + \mathcal{I}[\ell \geq 1] \times (1-\alpha_k) \times \psi_{ki}(z,\ell)\, I_{ki}^{stay}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}\left(z,\ell'\right) = \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}\left(z',\ell'\right) = \int_{z} \widehat{\psi}_{ki}\left(z,\ell'\right) g_k\left(z'|z\right) dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\varrho_{kf}^{exit} = \alpha_k + (1-\alpha_k) \int_z \int_{\ell} I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz,$$

$$\varrho_{ki}^{change} = (1-\alpha_k) \int_z \int_{\ell} I_{ki}^{change}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state:

$$\begin{aligned}\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\ &= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}\right)\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \begin{array}{c} \mathcal{I}[\ell = 1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \psi_{kf}^{e}(z) \\ +\mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_k)\,\psi_{kf}(z,\ell)\, I_{kf}^{stay}(z,\ell) \\ +(1-\alpha_k)\underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}}\psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell) \end{array} \right) \end{array}$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\,\mathcal{I}\left(L_{kf}(z,\ell) = \ell'\right) d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.21), (A.25) and (A.27)).

Step 8: Obtain the entry costs $c_{e,k}$ $(k = C, S)$:

$$c_{e,k} = V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z)\right] g_k^e(z)\, dz$$

These costs will be subtracted from aggregate income, and will be added to the expenditure on S-sector goods.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with Ψ_C, Ψ_S, d_F and μ^{υ}.
Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.
Step 9b: Write P_C and P_S as functions of M_C and M_S.
Step 9c: Write X_C^{int} as a function of M_C and M_S.
Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.
Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum\limits_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.
Step 9fe: Express masses of firms N_{kj} as functions of L_u.

Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.
Step 9h: Use equation for μ^v (and the value initially guessed in Step 1 for μ^v) to obtain L_u consistent with Ψ_C, Ψ_S, d_F and μ^v.
Step 9i: Go back and obtain masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s.
Step 9j: Recover price indices P_C and P_S.
Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Step 10: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Step 9.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u} \right)^{\xi}$$

Step 11: Use equations (A.28)-(A.29) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}}$$
$$L_{Si} = \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}}$$
$$L_{Cf} = \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci \to f} L_{Ci}}{\chi^{layoff}_{Cf}}$$
$$L_{Sf} = \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si \to f} L_{Si}}{\chi^{layoff}_{Sf}}$$

Step 12: Compute deviation from the labor market clearing equation:

$$Dev_L = abs \left(\frac{\overline{L} - (L_{Cf} + L_{Ci} + L_{Sf} + L_{Si})}{\overline{L}} \right) ,$$

Step 13: Compute all moments to be matched with those in the data.

Step 14: Compute Loss Function. Add Model/Data deviations to equilibrium penalty $EQ\_Penalty$. The objective function is therefore given by

$$L = L_{mom} + EQ\_Penalty$$

Where L_{mom} penalizes deviations between moments in the data and $EQ\_Penalty$ penalizes deviations from the labor market clearing condition:

$$EQ\_Penalty = W_L Dev_L + W_T abs \left(\min \{ Dev_T, 0 \} \right)$$

With W_L and W_T denoting large weights and Dev_T is the relative deviation between government revenues and spending with unemployment insurance (see section I.2 for details). We highly penalize a negative Dev_T.

Step 15: Optimization routine picks new parameter vector Ω. Go back to Step 1 until convergence.

Step 16 (Post estimation): Obtain J^u using

$$
\begin{aligned}
J^u = & \sum_{k,j} \mu_{kj}^e \int_\ell \int_z \overline{J}_{kj}^e \left(z, L_{kj}\left(z,\ell\right)\right) g_{kj}\left(z,\ell\right) dz d\ell \\
& + \left(1 - \sum_{k,j} \mu_{kj}^e\right) \vartheta_{J_u}.
\end{aligned}
$$

Step 17 (Post estimation): At this point, we know J^u and can compute

$$
b = \vartheta_{J_u} - \frac{1}{1+r} J^u,
$$

Step 18 (Post-estimation): Obtain D_F^* (this is the parameter that we need for the counterfactuals as d_F is endogenous):

$$
D_F^* = \frac{\left(\exp\left(\sigma_C \times d_F\right) - 1\right) \left(P_C^m\right)^{(1-\delta_C)(\sigma_C - 1)} \Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}}{\overline{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}},
$$

where $\overline{\epsilon}$ is the exchange rate value that balances trade:

$$
\overline{\epsilon} = \frac{1}{\tau_a \tau_c} \left(P_C^m\right)^{(1-\delta_C)} \Psi_C^{\frac{1}{\Lambda_C}} \left(\tau_a Exports\right)^{\frac{1}{1-\sigma_C}}.
$$

I.2 Estimation Algorithm – Further Details

This section details the steps within Step 9 of the estimation procedure.

Step 9: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Ψ_C, Ψ_S and d_F.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 8, that is, after solving for the steady state distribution of states.

$$
Avg\_wbill_{ki} = \int_z \int_{\ell'} \left[w_{ki}\left(z,\ell'\right)\ell'\right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C, S
$$

$$
Avg\_wbill_{kf} = \int_z \int_{\ell'} \left[\max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\}\ell'\right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \text{ for } k = C, S
$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_z \int_\ell \left[\left(\ell - L_{kf}\left(z,\ell\right)\right)\left(1 - I^{hire}_{kf}\left(z,\ell\right)\right)\right] \widetilde{\psi}_{kf}\left(z,\ell\right) d\ell dz \text{ for } k = C,S$$

$$Avg\_Hiring\_Costs_{kj} = \int_z \int_\ell \left[H_{kj}\left(\ell, L_{kj}\left(z,\ell\right)\right) I^{hire}_{kj}\left(z,\ell\right)\right] \widetilde{\psi}_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg\_Revenue_{kj} = \int_z \int_{\ell'} R_k\left(z,\ell'\right) \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ \ j = i,f$$

$$Avg\_InfPenalty_{ki} = \int_z \int_{\ell'} \left[p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right)\right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg\_Vacancies_{kj} = \int_z \int_\ell v_{kj}\left(z,\ell\right) \widetilde{\psi}_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg\_Exports_{Cf} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right) \int_z \int_{\ell'} \left[R_C\left(z,\ell'\right) I^x_C\left(z,\ell'\right)\right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz$$

$$Fraction\_Export_{Cf} = \int_z \int_{\ell'} I^x_C\left(z,\ell'\right) \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz$$

$$Avg\_size_{kj} = \int_z \int_\ell \ell \psi_{kj}\left(z,\ell\right) d\ell dz \text{ for } k = C,S;\ j = i,f$$

Now, define

$$\begin{aligned} Avg\_Price_{kj} &= \int_z \int_{\ell'} p_{kj}\left(z,\ell'\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k\left(z,\ell'\right)}{q_k\left(z,\ell',\iota_k\left(z,\ell'\right)\right)}\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ j = i,f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Ψ_C and Ψ_S. However, note that:

$$\begin{aligned} Avg\_Price_{kj} = &\left(\frac{\left(1-\delta_k\right)\left(\sigma_k - 1\right)}{\sigma_k}\right)^{\left(1-\delta_k\right)\Lambda_k} \left(P^m_k\right)^{\left(1-\sigma_k\right)\left(1-\delta_k\right)} \Psi_k^{\left(1-\sigma_k\right)\delta_k} \times \\ &\int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_k}\right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz, \end{aligned}$$

$$\begin{aligned} Avg\_Price_{Cf} = &\left(\frac{\left(1-\delta_C\right)\left(\sigma_C - 1\right)}{\sigma_C}\right)^{\left(1-\delta_C\right)\Lambda_C} \left(P^m_C\right)^{\left(1-\sigma_C\right)\left(1-\delta_C\right)} \Psi_C^{\left(1-\sigma_C\right)\delta_C} \times \\ &\int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_C}\right)^{\Lambda_C} \left(\exp\left(d_F \times I^x_C\left(z,\ell'\right)\right)\right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz. \end{aligned}$$

So, given Ω, Ψ_C and Ψ_S we can compute:

$$\begin{aligned} \widetilde{Avg\_Price}_{kj} &\equiv \left(\frac{\left(1-\delta_k\right)\left(\sigma_k - 1\right)}{\sigma_k}\right)^{\left(1-\delta_k\right)\Lambda_k} \Psi_k^{\left(1-\sigma_k\right)\delta_k} \int_z \int_{\ell'} \left(z\left(\ell'\right)^{\delta_k}\right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \left(P^m_k\right)^{\left(\sigma_k - 1\right)\left(1-\delta_k\right)} Avg\_Price_{kj}, \end{aligned}$$

$$\widetilde{Avg\_Price}_{Cf} \equiv \left(\frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C}\right)^{(1-\delta_C)\Lambda_C} \Psi_C^{(1-\sigma_C)\delta_C} \times$$
$$\int_z \int_{\ell'} \left(z(\ell')^{\delta_C}\right)^{\Lambda_C} \left(\exp\left(d_F \times I_C^x(z,\ell')\right)\right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}(z,\ell')\, d\ell' dz$$
$$= (P_C^m)^{(\sigma_C-1)(1-\delta_C)} Avg\_Price_{Cf}.$$

At this point, we can compute the following variables, as functions of M_C and M_S

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.3}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.4}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{S.5}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{S.6}$$

$$M_{Ci} = \omega_{Ci} M_C$$

$$M_{Si} = \omega_{Si} M_S$$

$$M_{Cf} = \omega_{Cf} M_C$$

$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry costs, fixed export costs)

$$E_S = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg\_Hiring\_Costs_{Cf} + \bar{c}_{Cf}\right)$$
$$+ \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \bar{c}_{Ci}\right)$$
$$+ \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf}\right)$$
$$+ \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si}\right)$$
$$+ \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction\_Export_{Cf} f_x$$
$$+ M_C c_{e,C}$$
$$+ M_S c_{e,S}$$

Define c_C:

$$\begin{aligned}
c_C \equiv \frac{E_{S,C}}{M_C} &= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}\left(Avg_Hiring_Costs_{Cf} + \bar{c}_{Cf}\right) \\
&+ \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\left(Avg_Hiring_Costs_{Ci} + \bar{c}_{Ci}\right) \\
&+ \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction_Export_{Cf} f_x \\
&+ c_{e,C},
\end{aligned} \tag{S.7}$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$\begin{aligned}
c_S \equiv \frac{E_{S,S}}{M_S} &= \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}\left(Avg_Hiring_Costs_{Sf} + \bar{c}_{Sf}\right) \\
&+ \frac{\omega_{Si}}{\varrho_{Si}^{exit}}\left(Avg_Hiring_Costs_{Si} + \bar{c}_{Si}\right) \\
&+ c_{e,S},
\end{aligned} \tag{S.8}$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned}
E_S &= E_{S,C} + E_{S,S} \\
&= c_C M_C + c_S M_S
\end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$

$$(1-\zeta) I + X_S^{int} + E_S = Rev_S$$

$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k (z,\ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k (z,\ell),$$

which leads to:

$$X_C^{int} = \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C + \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S$$

$$X_S^{int} = (1-\lambda_C) \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C + (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) Rev_C \\ & + \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) Rev_S \\ & - E_S \\ & + (\tau_a - 1) Exports \end{aligned}$$

Using

$$\begin{aligned} Rev_C &= Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \\ Rev_S &= Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \\ Exports &= Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \\ E_S &= c_C M_C + c_S M_S \end{aligned}$$

Step 9a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$
\begin{aligned}
I = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \end{array} \right) \\
& + \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \end{array} \right) \\
& - (c_C M_C + c_S M_S) \\
& + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \right)
\end{aligned}
$$

Therefore:

$$
I = a_C M_C + a_S M_S \tag{S.9}
$$

Where

$$
\begin{aligned}
a_C = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{array} \right) \\
& + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \right) \\
& - c_C
\end{aligned}
$$

$$
\begin{aligned}
a_S = & \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{array} \right) \\
& - c_S
\end{aligned}
$$

Step 9b: Write P_C and P_S as functions of M_C and M_S.

Price Index Sector C

$$
P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}
$$

The domestic component is given by:

$$\begin{aligned}
P_{H,C}^{1-\sigma_C} &= N_{Cf} Avg_Price_{Cf} + N_{Ci} Avg_Price_{Ci} \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Price_{Cf} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Price_{Ci} \end{pmatrix} M_C \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci} \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \end{pmatrix} M_C
\end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{H,C}^{1-\sigma_C} = b_C^1 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned}
&\Rightarrow \left(\epsilon\tau_a\tau_c\right)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\
&= \frac{\tau_a \times N_{Cf} Avg_Exports_{Cf}}{D_{H,C}} \\
&= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp\left(\sigma_C \times d_{H,C}\right)} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\
&= \left(P_C^m\right)^{-(\sigma_C-1)(1-\delta_C)} \frac{\tau_a \times Avg_Exports_{Cf}}{\Psi_C^{\frac{\sigma-1}{\Lambda}}} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C.
\end{aligned}$$

Where we have used

$$\exp\left(\sigma_C \times d_{H,C}\right) = \left(\frac{\Psi_C}{\left(P_C^m\right)^{-(1-\delta_C)\Lambda_C}}\right)^{\frac{\sigma_C-1}{\Lambda_C}}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = b_C^2 \left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)} M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg\_Exports_{Cf}}{\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Therefore:

$$\frac{P_C^{1-\sigma_C}}{\left(P_C^m\right)^{(1-\sigma_C)(1-\delta_C)}} = \underbrace{\left(b_C^1 + b_C^2\right)}_{b_C} M_C \tag{S.10}$$

Price Index Sector S

$$\begin{aligned}
P_S^{1-\sigma_S} &= N_{Sf} Avg_Price_{Sf} + N_{Si} Avg_Price_{Si} \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Price_{Si} \end{array} \right) M_S \\
&= \left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Sf} \left(P_S^m\right)^{-(\sigma_S - 1)(1-\delta_S)} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg_Price}_{Si} \left(P_S^m\right)^{-(\sigma_S - 1)(1-\delta_S)} \end{array} \right) M_S \\
&\Rightarrow P_S^{1-\sigma_S} = b_S \left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)} M_S
\end{aligned}$$

Where

$$b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si}$$

Therefore:

$$\frac{P_S^{1-\sigma_S}}{\left(P_S^m\right)^{(1-\sigma_S)(1-\delta_S)}} = b_S M_S. \tag{S.11}$$

Step 9c: Write X_C^{int} as a function of M_C and M_S.

$$\begin{aligned}
X_C^{int} &= \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right) M_C \\
&+ \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right) M_S \\
&= d_C M_C + d_S M_S
\end{aligned}$$

Where

$$d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} \left(Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right)$$

$$d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} \left(Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right)$$

Step 9d: Solve for $\frac{M_S}{M_C}$ that matches Ψ_C.
Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}}\right)^{\frac{1}{\sigma_C}}$$

Using (S.9), (S.10), (S.2) and manipulating, we obtain:

$$\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} = \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right) + \frac{\zeta a_S + d_S}{b_C}\frac{M_S}{M_C}$$

$$\frac{M_S}{M_C} = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$

Step 9e: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.

To separately pin down M_C and M_S, use the labor market clearig equation.

$$\begin{aligned}
\overline{L} - L_u &= N_{Cf} Avg_Size_{Cf} + N_{Ci} Avg_Size_{Ci} + N_{Sf} Avg_Size_{Sf} + N_{Si} Avg_Size_{Si} \\
&= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Size_{Cf} + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg_Size_{Ci} + \\
&\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg_Size_{Sf} + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg_Size_{Si} \\
&= \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci}\right) M_C + \\
&\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si}\right) M_S
\end{aligned}$$

At this point, we can only express M_C and M_S as functions of L_u.
From now on write

$$\left(\frac{M_S}{M_C}\right)^* = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)$$

$$\Rightarrow M_S = \underbrace{\frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right)}_{AA} M_C$$

Therefore:

$$M_S = AA \times M_C$$
$$AA = \frac{b_C}{\zeta a_S + d_S}\left(\Psi_C^{\frac{\sigma_C - 1}{\Lambda_C}} - \left(\frac{\zeta a_C}{b_C} + \frac{d_C}{b_C}\right)\right) \tag{S.12}$$

So that:

$$\begin{aligned}\overline{L} - L_u = &\left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Size_{Ci}\right) M_C + \\ &\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Size_{Si}\right) AA \times M_C \\ &= BB \times M_C\end{aligned}$$

$$\begin{aligned}BB = &\left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Size_{Ci}\right) + \\ &\left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Size_{Si}\right) AA\end{aligned} \tag{S.13}$$

Finally:

$$M_C = \frac{\overline{L} - L_u}{BB} \tag{S.14}$$
$$M_S = \frac{AA}{BB}\left(\overline{L} - L_u\right) \tag{S.15}$$

Step 9f: Express masses of firms N_{kj} as functions of L_u.

Substituting (S.14) and (S.15) into (S.3)-(S.6) to obtain the masses of firms:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C = \underbrace{\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}}\frac{1}{BB}}_{EE_C}\left(\overline{L} - L_u\right) = EE_C\left(\overline{L} - L_u\right)$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S = \underbrace{\frac{\omega_{Si}}{\varrho_{Si}^{exit}}\frac{AA}{BB}}_{EE_S}\left(\overline{L} - L_u\right) = EE_S\left(\overline{L} - L_u\right)$$

$$N_{Cf} = \underbrace{\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}\frac{1}{BB}}_{DD_C}\left(\overline{L} - L_u\right)$$

$$N_{Sf} = \underbrace{\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}}\frac{AA}{BB}}_{DD_S}\left(\overline{L} - L_u\right)$$

Step 9g: Express aggregate posted vacancies V_{kj} as functions of L_u.

Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$\begin{aligned} V_{Cf} &= N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^{\upsilon}} \\ &= Avg\_Vacancies_{Cf} \times DD_C \left(\overline{L} - L_u\right) + \frac{\omega_{Cf}}{\mu^{\upsilon}}\frac{1}{BB}\left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg\_Vacancies_{Cf} \times DD_C + \frac{\omega_{Cf}}{\mu^{\upsilon}}\frac{1}{BB}\right)}_{FF_C}\left(\overline{L} - L_u\right) \\ &= FF_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.16}$$

$$\begin{aligned} V_{Ci} &= N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^{\upsilon}} \\ &= Avg\_Vacancies_{Ci} \times EE_C \left(\overline{L} - L_u\right) + \frac{\omega_{Ci}}{\mu^{\upsilon}}\frac{1}{BB}\left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg\_Vacancies_{Ci} \times EE_C + \frac{\omega_{Ci}}{\mu^{\upsilon}}\frac{1}{BB}\right)}_{GG_C}\left(\overline{L} - L_u\right) \\ &= GG_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.17}$$

$$\begin{aligned} V_{Sf} &= N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^{\upsilon}} \\ &= \underbrace{\left(Avg\_Vacancies_{Sf} \times DD_S + \frac{\omega_{Sf}}{\mu^{\upsilon}}\frac{AA}{BB}\right)}_{FF_S}\left(\overline{L} - L_u\right) \\ &= FF_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.18}$$

$$\begin{aligned} V_{Si} &= N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^{\upsilon}} \\ &= \underbrace{\left(Avg\_Vacancies_{Si} \times EE_S + \frac{\omega_{Si}}{\mu^{\upsilon}}\frac{AA}{BB}\right)}_{GG_S}\left(\overline{L} - L_u\right) \\ &= GG_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{S.19}$$

$$
\begin{aligned}
\widetilde{V} &= V_{Cf} + V_{Ci} + V_{Sf} + V_{Si} \\
&= \underbrace{(FF_C + GG_C + FF_S + GG_S)}_{JJ} \times \left(\overline{L} - L_u\right) \\
&= JJ \times \left(\overline{L} - L_u\right)
\end{aligned}
$$

Step 9h: Use equation for μ^{υ} to obtain L_u.

We have written each V_{kj} in terms of L_u. Now, note that

$$
\mu^{\upsilon} = \phi \left(\frac{L_u}{\widetilde{V}} \right)^{1-\xi}
$$

We can invert this equation to obtain L_u.

$$
\begin{aligned}
\mu^{\upsilon} &= \phi \left(\frac{L_u}{JJ \times \left(\overline{L} - L_u\right)} \right)^{1-\xi} \\
\Rightarrow L_u^* &= \frac{(\mu^{\upsilon})^{\frac{1}{1-\xi}} \times JJ \times \overline{L}}{\phi^{\frac{1}{1-\xi}} + (\mu^{\upsilon})^{\frac{1}{1-\xi}} \times JJ}
\end{aligned}
$$

Step 9i: Go back and obtain masses of entrants M_k's (equations (S.14) and (S.15)), masses of firms N_{kj}'s (equations (S.3)-(S.6)), and aggregate vacancies V_{kj}'s (equations (S.16)-(S.19)). We are now able to compute transitions out of unemployment μ^e_{kj} (Step 8).

Step 9j: Recover price indices P_C and P_S.

Equations (S.1) and (S.10) lead to:

$$
P_C = \left((b_C M_C)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C} (1-\lambda_C)^{1-\lambda_C}} \right)^{(1-\delta_C)} \right)^{\frac{1}{1-(1-\delta_C)\lambda_C}} P_S^{\frac{(1-\delta_C)(1-\lambda_C)}{1-(1-\delta_C)\lambda_C}}
$$

Defining

$$
\varpi_C = \left((b_C M_C)^{\frac{1}{1-\sigma_C}} \left(\frac{1}{\lambda_C^{\lambda_C} (1-\lambda_C)^{1-\lambda_C}} \right)^{(1-\delta_C)} \right)^{\frac{1}{1-(1-\delta_C)\lambda_C}}
$$

and

$$
\varkappa_C = \frac{(1-\delta_C)(1-\lambda_C)}{1-(1-\delta_C)\lambda_C}
$$

Allows us to write

$$
P_C = \varpi_C P_S^{\varkappa_C}
$$

Equations (S.1) and (S.11) lead to:

$$P_S = \left((b_S M_S)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} (1-\lambda_S)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}} P_C^{\frac{(1-\delta_S)\lambda_S}{1-(1-\delta_S)(1-\lambda_S)}}$$

Writing

$$\varpi_S = \left((b_S M_S)^{\frac{1}{1-\sigma_S}} \left(\frac{1}{\lambda_S^{\lambda_S} (1-\lambda_S)^{1-\lambda_S}} \right)^{(1-\delta_S)} \right)^{\frac{1}{1-(1-\delta_S)(1-\lambda_S)}}$$

and

$$\varkappa_S = \frac{(1-\delta_S)\lambda_S}{1-(1-\delta_S)(1-\lambda_S)}$$

Allows us to write

$$P_S = \varpi_S P_C^{\varkappa_S}$$

Solving the system leads to:

$$P_C = \left(\varpi_C (\varpi_S)^{\varkappa_C}\right)^{\frac{1}{1-\varkappa_S\varkappa_C}}$$

Step 9k: Compute deviation between government revenues and spending with unemployment insurance Dev_T.

Government Revenue

$$\begin{aligned}
G_{Rev} = & \frac{\sigma_C - (1-\delta_C)(\sigma_C - 1)}{\sigma_C} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_y Avg_Revenue_{Cf} \\
& + \frac{\sigma_S - (1-\delta_S)(\sigma_S - 1)}{\sigma_S} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_y Avg_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_w Avg_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_w Avg_wbill_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Firing_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg_Firing_Costs_{Sf} \\
& + (\tau_a - 1) \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Exports_{Cf}
\end{aligned}$$

Government Spending with Unemployment Insurance

$$G_{UI} = \underbrace{\left(b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D} \right)}_{\text{mass of } \textbf{formal} \text{ workers who transition to unemployment}} \right)}_{\text{Total Expenditure with Unemployment Benefits}}$$

Government Transfers

$$T = G_{Rev} - G_{UI}$$

We impose in the objective function that $Dev_T \geq 0$—in other words, we highly penalize $Dev_T < 0$

$$Dev_T = \frac{G_{Rev} - G_{UI}}{G_{Rev}}$$

When we compute aggregate income, we implicitly assumed that $G_{Rev} - G_{UI} \geq 0$.

II Simulation Appendix

II.1 Simulation Algorithm

Fix P_S at $\overline{P}_S$. Write the value added function as:

$$VA_k(z,\ell) = \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k} \left(z\ell^{\delta_k}\right)^{\Lambda_k}$$

Define

$$\Xi_k \equiv \Theta_k \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k}(1-\lambda_k)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k},$$

and

$$\Phi_k \equiv P_C^{-\lambda_k(1-\delta_k)\Lambda_k} \left(\exp\left(d_{H,k}\right)\right)^{\frac{\sigma_k}{\sigma_k-1}\Lambda_k}.$$

Rewrite the value added function as:

$$VA_k(z,\ell) = \Xi_k \Phi_k \left(z\ell^{\delta_k}\right)^{\Lambda_k}.$$

μ^{υ}, ϑ_{J_u}, d_F, Φ_C, Φ_S are the endogenous variables to be determined in equilibrium. For a given value of these variables, Steps 1 through 11 below compute the deviations from equilibrium conditions given by $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ for $i = 1, ..., 5$. We then need to find values $\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S$ solving $L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta^*_{J_u}, d^*_F, \Phi^*_C, \Phi^*_S\right) = 0$ for all $i = 1, ..., 5$. We discuss potential solutions to this problem in Step 12.

We proceed by first imposing values for ϑ_{J_u}, μ^{υ}, d_F, Φ_C, Φ_S.

Step 1: This step solves for wage schedules $w_{kf}(z,\ell')$, $w_{ki}(z,\ell')$ as well as value functions $V_{kf}(z,\ell)$, $V_{ki}(z,\ell)$, $J^e_{kf}(z,\ell')$, $J^e_{ki}(z,\ell')$, and firms' policy functions.

Step 1a: Compute value added functions $VA_k(z,\ell)$.

Step 1b: Compute wage schedules $w_{kf}(z,\ell')$

- Guess a wage schedule $w_{kf}(z,\ell')$
- Compute the resulting $V_{kf}(z,\ell')$ using (13)
- Compute $J^e_{kf}(z,\ell')$ using (A.46)
- Compute $w^u_{kf}(z,\ell')$ using equation (27)
- Let $\widehat{w}^u_{kf}(z,\ell') = \omega_0 + \omega_1 \frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^u_{kf}(z,\ell')$ on $\left[1, \frac{VA_k(z,\ell')}{\ell'}\right]$

- Update $w_{kf}(z,\ell') = \max\left\{\widehat{w}^{u}_{kf}(z,\ell'), b_u + \vartheta_{J_u} - \frac{1}{1+r} J^{e}_{kf}(z,\ell'), \underline{w}\right\}$
- Restart until convergence

Step 1c: Compute wage schedules $w_{ki}(z,\ell')$

- Guess a wage schedule $w_{ki}(z,\ell')$
- Compute the resulting $V_{ki}(z,\ell')$ using (17)
- Compute $J^{e}_{ki}(z,\ell')$ using (A.47)
- Compute $w^{u}_{ki}(z,\ell')$ using equation (30)
- Let $\widehat{w}^{u}_{ki}(z,\ell') = \omega_0 + \omega_1\left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)} p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}$ be the linear projection of $w^{u}_{ki}(z,\ell')$ on $\left[1, \left(1 - \frac{\sigma_k}{\sigma_k-(1-\delta_k)(\sigma_k-1)} p_{ki}(\ell')\right)\frac{VA_k(z,\ell')}{\ell'}\right]$
- Update $w_{ki}(z,\ell') = \max\left\{\widehat{w}^{u}_{ki}(z,\ell'), \vartheta_{J_u} - \frac{1}{1+r} J^{e}_{ki}(z,\ell')\right\}$
- Restart until convergence

Step 2: Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_{k}(z) = 1\right) = \int_z I^{formal}_{k}(z)\, g^{e}_{k}(z)\, dz$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_{k}(z) = 1\right) = \int_z I^{informal}_{k}(z)\, g^{e}_{k}(z)\, dz$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

Finally, compute the distribution of z productivities among entrants, conditional on entry into sector kj.

$$\psi^{e}_{ki}(z) = \frac{g^{e}_{k}(z)\, I^{informal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\, I^{informal}_{k}(\widetilde{z})\, d\widetilde{z}},$$

$$\psi^{e}_{kf}(z) = \frac{g^{e}_{k}(z)\, I^{formal}_{k}(z)}{\int_{\widetilde{z}} g^{e}_{k}(\widetilde{z})\, I^{formal}_{k}(\widetilde{z})\, d\widetilde{z}}.$$

Step 3: Compute the steady-state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\varrho^{exit}_{ki} = \alpha_k + (1-\alpha_k)\int_z \int_\ell \left(I^{exit}_{ki}(z,\ell) + I^{change}_{ki}(z,\ell)\right)\psi_{ki}(z,\ell)\, d\ell dz.$$

In steady state $N_{ki} = \left(1 - \varrho_{ki}^{exit}\right) N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned}\widetilde{\psi}_{ki}(z,\ell) = {} & \mathcal{I}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}(z) \\ & + \mathcal{I}[\ell \geq 1] \times (1 - \alpha_k) \times \psi_{ki}(z,\ell)\, I_{ki}^{stay}(z,\ell),\end{aligned}$$

and $\widehat{\psi}_{ki}$:

$$\widehat{\psi}_{ki}(z,\ell') = \int_{\ell} \widetilde{\psi}_{ki}(z,\ell)\, \mathcal{I}\left(L_{ki}(z,\ell) = \ell'\right) d\ell$$

Update ψ_{ki} with:

$$\psi_{ki}(z',\ell') = \int_{z} \widehat{\psi}_{ki}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute:

$$\begin{aligned}\varrho_{kf}^{exit} &= \alpha_k + (1 - \alpha_k) \int_{z} \int_{\ell} I_{kf}^{exit}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz, \\ \varrho_{ki}^{change} &= (1 - \alpha_k) \int_{z} \int_{\ell} I_{ki}^{change}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz.\end{aligned}$$

In steady state:

$$\begin{aligned}\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\ &= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\widetilde{\psi}_{kf}(z,\ell) = \begin{array}{c} \mathcal{I}[\ell=1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \psi_{kf}^{e}(z) \\ +\mathcal{I}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_k)\,\psi_{kf}(z,\ell)\, I_{kf}^{stay}(z,\ell) \\ +(1-\alpha_k)\underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \psi_{ki}(z,\ell)\, I_{ki}^{change}(z,\ell) \end{array} \right) \end{array}$$

and $\widehat{\psi}_{kf}$ as:

$$\widehat{\psi}_{kf}(z,\ell') = \int_{\ell} \widetilde{\psi}_{kf}(z,\ell)\,\mathcal{I}\left(L_{kf}(z,\ell) = \ell'\right) d\ell.$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \int_{z} \widehat{\psi}_{kf}(z,\ell')\, g_k(z'|z)\, dz,$$

and repeat until convergence of ψ_{kf}.

At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, $\widehat{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.21), (A.25) and (A.27)).

Step 4: Compute the values of entry V_k^e $(k = C, S)$:

$$V_k^e = \int_z \left[V_{ki}^e(z)\, I_k^{informal}(z) + V_{kf}^e(z)\, I_k^{formal}(z)\right] g_k^e(z)\, dz$$

and compute the deviations

$$L_5(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_5(\vartheta_{J_u}, \mu^{\upsilon}, \Phi_S) = Dev_{entry,S} = \frac{V_S^e - c_{e,S}}{c_{e,S}}$$

$$L_4(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S) = L_4(\vartheta_{J_u}, \mu^{\upsilon}, d_F, \Phi_C) = Dev_{entry,C} = \frac{V_C^e - c_{e,C}}{c_{e,C}}$$

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F and μ^{υ}.
Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.
Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.
Step 5c: Write X_C^{int} as a function of M_C and M_S

Step 5d: Pin down M_C using the equation defining Φ_C, then obtain M_S.
Step 5e: Obtain masses of firms N_{kj}.
Step 5f: Obtain aggregate posted vacancies V_{kj} and $\widetilde{V}$.
Step 5g: Save the values for P_C and $P_{F,C}$ to be used in Step 9.

Step 6: Compute L_u

$$L_u = \left(\frac{\mu^v}{\phi}\right)^{\frac{1}{1-\xi}} \widetilde{V}$$

Step 7: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Steps 5 and 6.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \phi \frac{V_{kj}}{\widetilde{V}} \left(\frac{\widetilde{V}}{L_u}\right)^{\xi}$$

Step 8: Use equations (A.28)-(A.29) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}}$$
$$L_{Si} = \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}}$$
$$L_{Cf} = \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci\to f} L_{Ci}}{\chi^{layoff}_{Cf}}$$
$$L_{Sf} = \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si\to f} L_{Si}}{\chi^{layoff}_{Sf}}$$

Step 9: Compute

$$\bar{\epsilon} = \frac{P_{F,C}}{\tau_a \tau_c},$$

where $P_{F,C}$ was determined in Step 5.
Compute:

$$d'_F = \log\left(\left(1 + \frac{D^*_F}{\exp\left(\sigma_C \times d_{H,C}\right)} \bar{\epsilon}^{\sigma_C} \tau_c^{1-\sigma_C}\right)^{\frac{1}{\sigma_C}}\right),$$

where

$$\exp\left(\sigma_C \times d_{H,C}\right) = \Phi_C^{\frac{\sigma_C - 1}{\lambda_C}} \left(P_C\right)^{\lambda_C(1-\delta_C)(\sigma_C-1)},$$

and P_C was determined in Step 5. Compute the deviation

$$L_3\left(\mu^v, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_{d_F} = \frac{d_F - d'_F}{d_F}$$

Step 10: Compute deviation from the labor market clearing equation:

$$L_1\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) = Dev_L = \frac{\overline{L} - \left(L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u\right)}{\overline{L}}$$

Step 11: Compute the deviation

$$\begin{aligned} L_2\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right) &= Dev_{J_u} \\ &= 1 - \frac{\left(\sum_{k,j} \mu_{kj}^e \int_\ell \int_z \overline{J}_{kj}^e\left(z, L_{kj}\left(z,\ell\right)\right) g_{kj}\left(z,\ell\right) dz d\ell + \left(1 - \sum_{k,j} \mu_{kj}^e\right) \vartheta_{J_u}\right)}{\left(1+r\right)\left(\vartheta_{J_u} - b\right)} \end{aligned}$$

Therefore, given $\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S$, we can compute deviations L_1, L_2, L_3, L_4, L_5.

Step 12: The equilibrium is given by $\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*$ solving

$$L_i\left(\left(\mu^{\upsilon}\right)^*, \vartheta_{J_u}^*, d_F^*, \Phi_C^*, \Phi_S^*\right) = 0 \text{ for all } i = 1, ..., 5$$

Step 13: Compute the price index for exports

$$P_X^* \equiv \left(\int_{N_{F,C}}^{N_C} \mathcal{I}_C^x\left(n\right) p_x^*\left(n\right)^{1-\sigma_C} dn\right)^{\frac{1}{1-\sigma_C}}$$

Note that

$$Exports = \epsilon D_F^* \left(P_X^*\right)^{1-\sigma_C}$$

So that:

$$P_X^* = \left(\frac{Exports}{\epsilon D_F^*}\right)^{\frac{1}{1-\sigma_C}}$$

A key difficulty is that, given the discrete approximations for the state space, the system above has discontinuities. We list a few solutions we implemented.

- Solve for the system using a sequential bisection method. This procedure has the drawback of being very slow.
- Solve for the system using an optimization routine minimizing the norm of the system. This procedure has the drawback of also being slow and to potentially be stuck in local minima.
- Our preferred solution is to approximate each function $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ with a third degree polynomial on the arguments. To do so, we draw a large number of values for $\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ and follow Steps 1 through 11 above to compute $L_i\left(\mu^{\upsilon}, \vartheta_{J_u}, d_F, \Phi_C, \Phi_S\right)$ at each of these points. We then fit third degree polynomials for each L_i function $i = 1, ..., 5$. Finally, we can use an out-of-the shelf solver to find the root of this approximated system.

II.2 Simulation Algorithm – Details

This section details the steps within Step 5 of the estimation procedure.

Step 5: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with Φ_C, Φ_S, ϑ_{J_u}, d_F, and μ^{υ}.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 4, that is, after solving for the steady state distribution of states.

$$Avg\_wbill_{ki} = \int_z \int_{\ell'} \left[w_{ki}\left(z,\ell'\right)\ell'\right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg\_wbill_{kf} = \int_z \int_{\ell'} \left[\max\left\{w_{kf}\left(z,\ell'\right), \underline{w}\right\}\ell'\right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_z \int_{\ell} \left[\left(\ell - L_{kf}(z,\ell)\right)\left(1 - I_{kf}^{hire}(z,\ell)\right)\right] \widetilde{\psi}_{kf}(z,\ell)\, d\ell dz \text{ for } k = C,S$$

$$Avg\_Hiring\_Costs_{kj} = \int_z \int_{\ell} \left[H_{kj}\left(\ell, L_{kj}(z,\ell)\right) I_{kj}^{hire}(z,\ell)\right] \widetilde{\psi}_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg\_Revenue_{kj} = \int_z \int_{\ell'} R_k\left(z,\ell'\right) \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ \ j = i,f$$

$$Avg\_InfPenalty_{ki} = \int_z \int_{\ell'} \left[p_{ki}\left(\ell'\right) R_k\left(z,\ell'\right)\right] \widehat{\psi}_{ki}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S$$

$$Avg\_Vacancies_{kj} = \int_z \int_{\ell} v_{kj}(z,\ell)\, \widetilde{\psi}_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

$$Avg\_Exports_{Cf} = \left(1 - \exp\left(-\sigma_C \times d_F\right)\right) \int_z \int_{\ell'} \left[R_C\left(z,\ell'\right) I_C^x\left(z,\ell'\right)\right] \widehat{\psi}_{kf}\left(z,\ell'\right) d\ell' dz$$

$$Fraction\_Export_{Cf} = \int_z \int_{\ell'} I_C^x\left(z,\ell'\right) \widehat{\psi}_{Cf}\left(z,\ell'\right) d\ell' dz$$

$$Avg\_size_{kj} = \int_z \int_{\ell} \ell \psi_{kj}(z,\ell)\, d\ell dz \text{ for } k = C,S;\ j = i,f$$

Now, define

$$\begin{aligned} Avg\_Price_{kj} &= \int_z \int_{\ell'} p_{kj}\left(z,\ell'\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \\ &= \int_z \int_{\ell'} \left(\frac{R_k\left(z,\ell'\right)}{q_k\left(z,\ell',\iota_k\left(z,\ell'\right)\right)}\right)^{1-\sigma_k} \widehat{\psi}_{kj}\left(z,\ell'\right) d\ell' dz \text{ for } k = C,S;\ j = i,f. \end{aligned}$$

We cannot compute Avg_Price_{kj}—given Ω, Φ_C and Φ_S. However, note that:

$$Avg\_Price_{kj} = \widetilde{\Xi}_k P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k} \Phi_k^{\delta_k(1-\sigma_k)} \int \int \left(z\left(\ell'\right)^{\delta_k}\right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) dz d\ell'$$

$$Avg\_Price_{Cf} = \widetilde{\Xi}_C P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \Phi_C^{\delta_C(1-\sigma_C)} \int \int \left(z\left(\ell'\right)^{\delta_C}\right)^{\Lambda_C} \left(\exp\left(d_F \times I_C^x\left(z,\ell'\right)\right)\right)^{-\delta_C \sigma_C \Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) dz d\ell'$$

$$\widetilde{\Xi}_k = \left(\frac{\overline{P}_S^{1-\lambda_k}}{\lambda_k^{\lambda_k} \left(1-\lambda_k\right)^{1-\lambda_k}} \right)^{-(1-\delta_k)\Lambda_k} \left(\frac{\left(1-\delta_k\right)\left(\sigma_k - 1\right)}{\sigma_k} \right)^{(1-\delta_k)\Lambda_k}$$

So, given Ω, Φ_C and Φ_S we can compute:

$$\begin{aligned} \widetilde{Avg\_Price}_{kj} &\equiv \widetilde{\Xi}_k \Phi_k^{\delta_k(1-\sigma_k)} \int\int \left(z \left(\ell'\right)^{\delta_k} \right)^{\Lambda_k} \widehat{\psi}_{kj}\left(z,\ell'\right) dz d\ell' \\ &= \frac{Avg\_Price_{kj}}{P_C^{(1-\sigma_k)(1-\delta_k)\lambda_k}} \end{aligned}$$

$$\begin{aligned} \widetilde{Avg\_Price}_{Cf} &\equiv \widetilde{\Xi}_C \Phi_C^{\delta_C(1-\sigma_C)} \int\int \left(z \left(\ell'\right)^{\delta_C} \right)^{\Lambda_C} \left(\exp\left(d_F \times I_C^x\left(z,\ell'\right)\right)\right)^{-\delta_C\sigma_C\Lambda_C} \widehat{\psi}_{Cf}\left(z,\ell'\right) dz d\ell' \\ &= \frac{Avg\_Price_{Cf}}{P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C}} \end{aligned}$$

At this point, we can compute the following variables, as functions of M_C and M_S:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{S.20}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{S.21}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{S.22}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{S.23}$$

$$M_{Ci} = \omega_{Ci} M_C$$

$$M_{Si} = \omega_{Si} M_S$$

$$M_{Cf} = \omega_{Cf} M_C$$

$$M_{Sf} = \omega_{Sf} M_S$$

Firm-level expenditures with sector S goods (fixed operating costs, hiring costs, entry

costs, fixed export costs)

$$\begin{aligned}
E_S = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg_Hiring_Costs_{Cf} + \bar{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}M_C}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \bar{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg_Hiring_Costs_{Sf} + \bar{c}_{Sf}\right) \\
& + \frac{\omega_{Si}M_S}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \bar{c}_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}$$

Define c_C:

$$\begin{aligned}
c_C \equiv \frac{E_{S,C}}{M_C} = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Cf} + \bar{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg_Hiring_Costs_{Ci} + \bar{c}_{Ci}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction_Export_{Cf} f_x \\
& + c_{e,C},
\end{aligned}$$

Where $E_{S,C}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from C-sector activity.

Define c_S:

$$\begin{aligned}
c_S \equiv \frac{E_{S,S}}{M_S} = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Sf} + \bar{c}_{Sf}\right) \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg_Hiring_Costs_{Si} + \bar{c}_{Si}\right) \\
& + c_{e,S},
\end{aligned}$$

Where $E_{S,S}$ is firm-level expenditures with sector S goods (fixed costs, etc) coming from S-sector activity.

We can therefore write:

$$\begin{aligned} E_S &= E_{S,C} + E_{S,S} \\ &= c_C M_C + c_S M_S \end{aligned}$$

Market Clearing (C and S sectors)

Let I denote aggregate income. Then, market clearing in the C and S sectors must lead to:

$$\zeta I + X_C^{int} = Rev_C - Exports + \tau_a Imports$$

$$(1-\zeta) I + X_S^{int} + E_S = Rev_S$$

$$Imports = Exports$$

Note that expenditures on intermediates are proportional to gross revenues:

$$P_k^m \iota_k (z,\ell) = \frac{(1-\delta_k)(\sigma_k - 1)}{\sigma_k} R_k (z,\ell),$$

which leads to:

$$\begin{aligned} X_C^{int} = &\lambda_C \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C \\ &+ \lambda_S \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S \end{aligned}$$

$$\begin{aligned} X_S^{int} = &(1-\lambda_C) \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C} Rev_C \\ &+ (1-\lambda_S) \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S} Rev_S \end{aligned}$$

Where Rev_C and Rev_S are total gross revenues in sectors C and S respectively. Therefore:

$$\begin{aligned} I = &\left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) Rev_C \\ &+ \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) Rev_S \\ &- E_S \\ &+ (\tau_a - 1) Exports \end{aligned}$$

Using

$$Rev_C = Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$Rev_S = Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$Exports = Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C$$

$$E_S = c_C M_C + c_S M_S$$

Step 5a: Write aggregate income I as a function of masses of entrants M_C and M_S.

$$\begin{aligned} I = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \end{array} \right) \\ & + \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \end{array} \right) \\ & - (c_C M_C + c_S M_S) \\ & + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \right) \end{aligned}$$

Therefore:

$$I = a_C M_C + a_S M_S \tag{S.24}$$

Where

$$\begin{aligned} a_C = & \left(1 - \frac{(1-\delta_C)(\sigma_C - 1)}{\sigma_C}\right) \left(\begin{array}{c} Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} \\ + Avg\_Revenue_{Ci} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \end{array} \right) \\ & + (\tau_a - 1) \left(Avg\_Exports_{Cf} \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} \right) \\ & - c_C \end{aligned}$$

$$\begin{aligned} a_S = & \left(1 - \frac{(1-\delta_S)(\sigma_S - 1)}{\sigma_S}\right) \left(\begin{array}{c} Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} \\ + Avg\_Revenue_{Si} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \end{array} \right) \\ & - c_S \end{aligned}$$

Step 5b: Write M_C as a functions of P_C and M_S as a function of M_C and $\overline{P}_S$.

Price Index Sector C

$$P_C^{1-\sigma_C} = P_{H,C}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C}$$

The domestic component is given by:

$$\begin{aligned}
P_{C,H}^{1-\sigma_C} &= N_{Cf} Avg_Price_{Cf} + N_{Ci} Avg_Price_{Ci} \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg_Price_{Cf} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Price_{Ci} \end{pmatrix} M_C \\
&= \begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Cf} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg_Price}_{Ci} P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} \end{pmatrix} M_C
\end{aligned}$$

We can therefore write $P_{C,H}$ as:

$$P_{C,H}^{1-\sigma_C} = P_C^{(1-\sigma_C)(1-\delta_C)\lambda_C} b_C^1 M_C,$$

Where

$$b_C^1 \equiv \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \widetilde{Avg\_Price}_{Ci}$$

The foreign component is given by:

$$P_{F,C}^{1-\sigma_C} = (\epsilon\tau_a\tau_c)^{1-\sigma_C}.$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}(\epsilon\tau_a\tau_c)^{1-\sigma_C}}{\tau_a},$$

$$\begin{aligned}
&\Rightarrow (\epsilon\tau_a\tau_c)^{1-\sigma_C} = \frac{\tau_a \times Exports}{D_{H,C}} \\
&= \frac{\tau_a \times N_{Cf} Avg_Exports_{Cf}}{D_{H,C}} \\
&= \frac{\tau_a \times Avg_Exports_{Cf}}{\exp(\sigma_C \times d_{H,C})} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \\
&= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)} \frac{\tau_a \times Avg_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C.
\end{aligned}$$

Where we have used

$$\exp(\sigma_C \times d_{H,C}) = \Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} (P_C)^{\lambda_C(1-\delta_C)(\sigma_C-1)}.$$

Therefore:

$$P_{F,C}^{1-\sigma_C} = (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}\, b_C^2 M_C,$$

Where

$$b_C^2 \equiv \frac{\tau_a \times Avg\_Exports_{Cf}}{\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}}} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}.$$

Rewriting:

$$\begin{aligned} P_C^{1-\sigma_C} &= P_{C,H}^{1-\sigma_C} + P_{F,C}^{1-\sigma_C} \\ &= (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}\, b_C^1 M_C + (P_C)^{\lambda_C(1-\delta_C)(1-\sigma_C)}\, b_C^2 M_C \end{aligned}$$

So that:

$$P_C^{1-\sigma_C} = (b_C M_C)^{\frac{1}{(1-\lambda_C(1-\delta_C))}} \tag{S.25}$$

where

$$b_C \equiv b_C^1 + b_C^2.$$

Price Index Sector S

$$
\begin{aligned}
P_S^{1-\sigma_S} &= N_{Sf} Avg\_Price_{Sf} + N_{Si} Avg\_Price_{Si} \\
&= \begin{pmatrix} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\ +\frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \end{pmatrix} M_S \\
&= \begin{pmatrix} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \\ +\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si} P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} \end{pmatrix} M_S
\end{aligned}
$$

$$
\Rightarrow P_S^{1-\sigma_S} = P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S} b_S M_S
$$

Where

$$
b_S \equiv \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \widetilde{Avg\_Price}_{Si}
$$

Given that $P_S = \overline{P}_S$ is fixed, we can also write M_S as a function of P_C and model parameters.

$$
M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S P_C^{(1-\sigma_S)(1-\delta_S)\lambda_S}}, \tag{S.26}
$$

and using (S.25):

$$
M_S = \frac{\overline{P}_S^{1-\sigma_S}}{b_S \left(b_C M_C\right)^{\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}}. \tag{S.27}
$$

Step 5c:Write X_C^{int} as a function of M_C and M_S.

$$
\begin{aligned}
X_C^{int} = \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} &\left(Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right) M_C \\
+ \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} &\left(Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right) M_S \\
= d_C M_C + d_S M_S &
\end{aligned}
$$

Where

$$
d_C = \lambda_C \frac{(1-\delta_C)(\sigma_C-1)}{\sigma_C} \left(Avg\_Revenue_{Cf} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} + Avg\_Revenue_{Ci}\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \right)
$$

$$
d_S = \lambda_S \frac{(1-\delta_S)(\sigma_S-1)}{\sigma_S} \left(Avg\_Revenue_{Sf} \frac{\varrho_{Si}^{change}\omega_{Si}+\varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} + Avg\_Revenue_{Si}\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \right)
$$

Step 5d: Pin down P_C using the equation defining Φ_C, and obtain M_C and M_S.

We can now express aggregate income I as a function of P_C using equations (S.24), (S.25) and (S.26) and solve for P_C. Remember that:

$$\exp\left(d_{H,C}\right) = \left(\frac{\zeta I + X_C^{int}}{P_C^{1-\sigma_C}}\right)^{\frac{1}{\sigma_C}}$$

Using the formula defining Φ_C and manipulating, we obtain:

$$P_C^{-\lambda_C(1-\sigma_C)(1-\delta_C)}\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \exp\left(\sigma_C \times d_{H,C}\right) = \frac{\zeta\left(a_C M_C + a_S M_S\right) + d_C M_C + d_S M_S}{P_C^{1-\sigma_C}},$$

which leads to

$$\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} = \frac{\left(\zeta a_C + d_C\right) M_C + \left(\zeta a_S + d_S\right) M_S}{b_C M_C},$$

which allows us to solve for M_C

$$M_C = \frac{1}{b_C}\left(\frac{\left(\zeta a_S + d_S\right)\overline{P}_S^{1-\sigma_S}}{b_S\left(\Phi_C^{\frac{\sigma_C-1}{\Lambda_C}} - \frac{\left(\zeta a_C + d_C\right)}{b_C}\right)}\right)^{\frac{1}{1+\frac{(1-\sigma_S)(1-\delta_S)\lambda_S}{(1-\sigma_C)(1-(1-\delta_C)\lambda_C)}}},$$

and then for M_S using (S.27).

Step 5e: Now that we have values of M_C and M_S, we obtain masses of firms N_{kj}.

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C$$

$$N_{Cf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S$$

Step 5f: Obtain aggregate posted vacancies V_{kj}.

Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$V_{Cf} = N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^v}$$

$$V_{Ci} = N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^v}$$

$$V_{Sf} = N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^v}$$

$$V_{Si} = N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^{\upsilon}}$$

Step 5g: Save the values for P_C and $P_{F,C}$:

$$P_C^{1-\sigma_C} = (b_C M_C)^{\frac{1}{(1-(1-\delta_C)\lambda_C)}}$$

$$P_{F,C}^{1-\sigma_C} = (P_C)^{-\lambda_C(1-\delta_C)(\sigma_C-1)} b_C^2 M_C$$