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"Should we sustain? And if so, sustain what? Consumption or the quality of life? "*

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1. Introduction

The rapid growth in greenhouse gas (GHG) emissions and concomitant increase in atmospheric carbon concentration during the past century have raised, in a dramatic way, the spectre of catastrophic effects for the welfare of mankind: in the last century, the only comparable events were the two world wars and worst-case scenarios associated with nuclear proliferation. Unlike these events, the effects of increased atmospheric carbon concentration, mainly due to associated temperature increases, will occur gradually and with a long time lag. Sustainability has gained traction as an ethic and an appropriate goal, as we face the costs of using up our scarce biospheric resource, of a non-carbon-saturated atmosphere.

That ethic is quite pervasive, at least among environmentalists. Perhaps surprisingly, it has influenced economists much less: to wit, the major contributions to the economics of climate change advocate not a 'sustainabilitarian' approach, but a utilitarian one, of maximizing the discounted sum of utilities of the sequence of generations beginning with the present one. Indeed, the two most influential pieces of recent economic analysis, William Nordhaus's (2008) book and Nicholas Stern's (2007) Review, both employ versions of discounted utilitarianism, as we will examine below.

There is, however, a literature on sustainability in economics, due to Robert Solow, and built upon by John Hartwick and others. We will review that literature, briefly contrast it with the currently more popular approaches of discounted utilitarianism, and then conclude with a brief description of our own approach, which attempts to rejuvenate the sustainabilitarian research program.

Whether to sustain welfare, or to maximize a discounted sum of welfare levels, is a question concerning the social objective function. The second question we will address is what the *arguments* of that social welfare function should be: incomes of different generations, their consumptions, their utilities, or something else? In climate-change economics – and here the cue is taken from the traditional practice in growth theory – welfare is taken to be some concave function of consumption. We wish to depart from that convention here, as well. Of course, human welfare depends upon more than commodity consumption, but most climate-change economists seem to believe that representing the different kinds of inputs to human welfare as a single good is a harmless abstraction. Education and knowledge, in these models, only indirectly impact upon welfare, through their role as inputs in the production of consumption and investment goods. To the contrary, we maintain that ignoring the fact that education and knowledge contribute directly to human welfare is not harmless: for in meeting the challenge of climate change, we should entertain the possibility of shifting our consumption bundle from energy-intensive goods to other goods, such as education, leisure, and knowledge, which can be produced in an environmentally more friendly way. This possibility of substitution does not exist in (for example) the models of Nordhaus and Stern, and those analyses may thereby be unnecessarily truncating the set of techniques for adapting to climate change.

We conclude the paper by demonstrating concretely the consequences of, on the one hand, choosing the optimal path to sustain commodity consumption 'forever,' or choosing the optimal path to sustain 'welfare' forever, where welfare includes inputs of knowledge, education and biospheric quality. For both optimization programs, the feasible set of paths is the same – and education, knowledge, and GHG emissions enter in production. The two optimal paths turn out to be quite strikingly different, which is to say that a more comprehensive definition of human welfare changes dramatically the policy recommendation.

2. Social objectives

2.1. GDP, consumption and the quality of life

A large segment of economics, particularly in empirical applications, takes as the index of welfare per capita GDP (gross domestic product) or GNI (gross national income, which includes net income generated abroad) or NNI (net national income, which in addition excludes capital depreciation). Each of these GDP-related measures has two main components: consumption and investment. Consumption is related to current welfare, whereas investment is a contribution to capital, a factor for future welfare. The correlation of any GDP-related measure to consumption is necessarily imperfect: a country, like Singapore, with a high investment-to-GDP ratio has levels of consumption substantially lower than other countries with the same GDP per capita. Hence, the two components, consumption and investment, should be separated. One can then ask two distinct questions.

(i) Is the consumption component of the GDP-related measure an appropriate index of human welfare?

(ii) Is the investment component of the GDP-related measure an appropriate index of the change in capital?

The second question is treated in Section 3.3 below. The present section addresses the first one. What is an appropriate index of human welfare?

The shortcomings of the consumption component of a GDP-related measure as a welfare index have been discussed for decades: see the Sarkozy Report (Joseph Stiglitz *et al*, 2009) for a recent, comprehensive assessment. The criticisms have spurred three different approaches (a) the inclusion of additional variables besides consumption, in the welfare or utility index, in particular non-marketed goods or services (b) the measurement of subjective happiness, and (c) the capabilities approach. We refer to Marc Fleurbaey (2009) for a critical review of (b) and (c), and we focus here on (a). [1]

Population size and the level of inequality have potentially important impacts on mankind's well-being. Many of the existing social welfare indices explicitly consider inequality (e. g., the Human Development Index, HDI, by the United Nations Human Development Programme, 2011, Charles Jones and Peter Klenow, 2010, and the recommendations in Stiglitz *et al*., 2009). Population size is on occasion taken into consideration (see, e. g., Nordhaus, 2008, and Section 3.5.1 below), based on the utilitarian idea that an increase in the total number of people with sufficiently high welfare levels is desirable (Charles Blackorby and David Donaldson, 1984). But climate change economics often abstracts from population and distributional ethics: we frequently assume that the number of people in a generation is given, and ignore inequality by representing each generation by a single agent.

The main variables that have been introduced, besides consumption, in various indices of human welfare are *leisure*, the *environment*, *health* and *education*: observe that these variables may contribute very differently to carbon emissions. It should be noted that a large segment of the economic growth literature, as well as of the economic analysis of climate change literature (e. g., Nordhaus, 2008) does consider improvements in knowledge, education or the environment, but only in so far as they make possible the production of consumption goods with less labor time or produced capital. The present text focuses on their role as direct arguments in the utility function or welfare index.

For instance, Nordhaus and James Tobin's (1972) Measure of Economic Welfare (MEW) modifies GDP per capita by excluding depreciation and including leisure and non-market work, in addition to taking congestion and pollution into account. Labor economics has traditionally

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[1] The implications of the capability approach do not differ much from those of ours.

considered both consumption and leisure as arguments in the utility function, with leisure interpreted as a broad aggregate of unpaid activities, including home production, the care of children and relatives and home educational activities, in addition to free time and rest and recreation. Jones and Klenow (2010) provide a welfare index where leisure plays an important role in comparing the quality of life across countries.

Environmental economics has emphasized the quality of air and water, as well as the recreational opportunities offered by natural environments. Pollution appears as an argument in Emmet Keeler *et al*. (1972) and, as just noted, in the MEW of Nordhaus and Tobin (1972). The amenity value of the forest was stressed by Richard Hartman (1976), and that of undeveloped ecosystems by Anthony Fisher, John Krutilla and Charles Cicchetti (1972) and Scott Barrett (1992).

Health economics has constructed indices of health and longevity by means of defining QALY (quality-adjusted life years) in order to quantify the benefits of medical intervention. Life expectancy is recognized as a major factor in human welfare. As societies become wealthier, extending life becomes increasingly valuable. Robert Hall and Charles Jones (2007) estimate that, in the US, "a sixty-five year old would give up 82 percent of her consumption […] to have the health status of a 20 year old."[2] Many recent welfare indices include life expectancy as an argument, see, e. g., Jones and Klenow, 2010, and the HDI of United Nations Human Development Programme (2011).

Education and the accompanying accumulation of human capital have traditionally been considered production factors, increasing the productivity of the time worked and associated wages (see Richard Freeman, 1986). Education increases productivity both in the market and in some areas of home production (Robert Michael, 1972, 1973, Arleen Leibowitz, 1975, Reuben Gronau, 1986, Salvador Ortigueira, 1999, J. J. Heckman, 1976). As a result, highly educated women spend more time in child care and education, but less in meal preparation and doing laundry, than their less educated counterparts (Leibowitz, 1975). The literature uses the language of production and output, but, as noted by Michael (1973, p. 307), "A distinguishing characteristic of human capital is that it is embedded in an individual: it therefore accompanies

[2] Perhaps surprisingly, life expectancy seems to increase linearly with time. Hall and Jones (2007) estimate that during the last 50 years, life expectancy in the US has increased by 1.4 months per year. And Jim Oeppen and James Vaupel (2002, p. 1029) report that "female life expectancy in the record-holding country has risen for 160 years at a steady pace of almost 3 months per year."

him whenever he goes, not only in the labor market, but also in the theater, the voting booth, the kitchen and so forth." Even though the home production literature views education as affecting nonmarket productivity, he continues, "one could alternatively argue, for example, that education affects the household utility function…"

We have mentioned the Human Development Index (HDI): it was proposed by the first *United Nations Development Report* (United Nations Development Programme, 1990) and has been updated yearly (see the *United Nations Development Report*, 2010, 2011 for the latest issues: the 2010 edition introduced some changes in its composition). It aggregates three dimensions: (a) health, with life expectancy at birth as indicator, (b) education, with mean and expected years of schooling as indicators, and (c) living standards, with GNI per capita as indicator (see, e. g., United Nations Development Programme, 2010, Figure 1.1). The more recent Human Development Reports, as well as Eric Neumayer (2001) among others, emphasize three dimensions of human welfare acknowledged by the authors of the 1990 HDI but neglected in its definition, namely environmental and climate sustainability and human security, rights and freedoms. The 2011 Report introduces an 'adjusted HDI,' which reduces the HDI by a factor reflecting income inequality in the country.

2.2. Intergenerational justice

Sustaining the level of a human welfare forever can be justified by appealing to John Rawls's (1971) maximin principle (see Roemer, 1998, 2007). The ethical justification for this type of sustainability is, in our view, that the date at which a person is born is morally arbitrary. Thus every generation has a *right* to a level of well-being at least as high as that of any other generation. Similarly, Sudhir Anand and Amartya Sen (2000) justify the sustainability of human welfare by an appeal to "universalism." Our argument, that the date at which a person is born is morally arbitrary, in turn validates "universalism." Maximizing the quality of life of the worst-off generation will often require the maximization of the quality of life of the first generation subject to maintaining that quality of life for all future generations, so that there is no quality-of-life growth after the first generation (see Silvestre, 2002, for possible exceptions to this rule).

2.3. Positive growth rates

Alternatively, society may seek a positive rate of growth in the quality of life of future generations at the cost of reducing the quality of life of the present one. It is, however, not obvious how to justify sacrifices of the worst-off present generation for the sake of improving the already higher welfare levels of future ones. Recall that we assume away intra-generational inequality, thereby depriving economic growth of a role in alleviating contemporaneous poverty.

One might argue that parents want their children to have a higher quality of life than they do. Thus, welfare growth might be supported by all parents over a constant-utility path. An alternative justification for altruism towards future generations would appeal to *growth as a public good*: we may feel justifiably proud of mankind's recent gains in, say, extraterrestrial travel, or average life expectancy, and wish them to continue into the far future even at a personal cost.[3] If indeed we do put value on human development, we –the present generation– may choose *not to enforce our right* to be as well off as all future generations: we may prefer to allow future generations to become better off than we are, at some (perhaps small) cost to ourselves.

Indeed, there is an asymmetry in the way we feel about contemporaneous vs. temporally disjoint inequality: a person in a poor country may not wish to sacrifice her quality of life for the sake of improving that of a person in a *richer* country, while at the same time be willing to make some sacrifices for the welfare of unrelated, yet-to-be born individuals who will as a consequence be richer than she.

3. An overview of the literature

3.1. Steady states with reproducible resources

The economic analysis of climate change is rooted in a long tradition that analyzes the tradeoff between present consumption and the enhanced consumption possibilities in the future offered by saving. The pioneering work by Martin Faustmann (1849) sought to determine the best rate of rotation for the harvesting of a forest. Faustmann's work anticipated the literature on the sustainable yields of other renewable resources, such as fisheries, often developed by biologists and involving a large empirical component (see, e. g., H. Scott Gordon, 1954, Anthony Scott, 1955, Colin Clark, 1990). A first objective is the characterization of the dynamic path for the population of a species under various specifications of the natural growth rate and the

[3] See Silvestre (2007).

dependence of the harvest on both the harvesting effort and the size of the population, and the definition of the *maximal sustainable yield* (MSY). Second, after introducing the disutility of harvesting effort, this literature studies the best dynamic paths, either by a cost-benefit argument or by maximizing the discounted future benefits and costs using the tools of optimal control theory.

The "sustainability" of the MSY-type solutions must be understood as an attribute of the steady state: consumption, or the harvest, is less than maximal during the transition from the initial conditions to the steady state. If we postulate that a different generation lives at each date, then the earlier generations consume less than the later ones. The same observation applies to the Solow-Swan model of neoclassical growth (Robert Solow, 1956, Trevor Swan, 1956), which, contrary to the MSY approach, is often explicitly worded in terms of different generations. The emphasis continues to be on the best steady state, now characterized by the *golden rule* (which corresponds to the MSY of the renewable resource literature): in Edmund Phelps's (1961, p. 642) words: "each generation invests on behalf of the future generations that share of income that […] it would have had past generation invest on behalf of it." Again, consumption is less than maximal during the transition from the initial conditions to the steady state. The golden rule can be viewed as intergenerationally equitable only among the generations that come to be after the steady state is reached.

3.2. Maximin: the Solow-Hartwick theory

Solow (1974) pioneered the analysis of Rawls's maximin principle in the context of intergenerational equity. As is well known, Rawls himself (1971, Section 44) was reluctant to apply the principle to the intergenerational problem, but Solow (1974, p. 30) decided to be "*plus rawlsien que le Rawls*." Solow (1974) introduced both an exhaustible natural resource and produced capital. Identifying the utility of a generation with its consumption of a single produced good, he worked out the maximin paths of the variables for a Cobb-Douglas technology. In order to maintain equal, maximal utility across all generations, the stock of produced capital has to increase to compensate for the depletion of the nonrenewable resource. Solow did not use the term "sustainability," but of course his objective can be translated as sustaining utility.

Harold Hotelling's (1931) maximization, by variational methods, of the discounted sum of benefits from the depletion of an exhaustible resource yields the well-known *Hotelling rule*, by which the rate of change in the marginal product of the extracted amount must equal the interest, or discount, rate. He considered the exhaustible resource as the only productive input, which of course ruled out any sort of sustainability: in his words (p. 139) "the indefinite maintenance of a steady state of production is a physical impossibility." But the Hotelling rule turned out to play a role in Solow's Maximin model: John Hartwick (1977) showed that Solow's (1974) paths for produced capital and for the depletion of the exhaustible resource could be characterized by a Hotelling-type rule (translated as the equality between the marginal product of the extracted amount and the marginal product of produced capital) together with what is now known as the Hartwick Rule:

"INVESTMENT IN PRODUCED CAPITAL
= MARGINAL PRODUCT OF EXTRACTED AMOUNT TIMES THE EXTRACTED AMOUNT."

The Hartwick rule can be paraphrased as

"INVESTMENT IN PRODUCED CAPITAL = RENTS OBTAINED BY EXTRACTION,"

i. e.,

"NET INVESTMENT IN PRODUCED CAPITAL
= VALUE OF NET DEPLETION OF THE EXHAUSTIBLE RESOURCE,"

or

"SUM OF NET INVESTMENTS IN ALL FORMS OF CAPITAL = 0."

The precise relation between an appropriately defined index of the sum of net investments and maintaining consumption involves some subtleties: see Avinash Dixit *et al*. (1980) and Geir Asheim *et al*. (2003), but a formal argument shows that the Hotelling Rule and the Hartwick Rule together imply stationary consumption in a simple continuous-time model. Let $S^k(t)$ denote the stock of produced capital at t, $m(t)$ be the flow of extraction of the exhaustible resource, $f(S^k, m)$ the production function, and $c(t)$ consumption. Postulate the law of motion of produced capital:

$$\frac{dS^k}{dt} = f(S^k, m) - c,$$

and define

Hotelling Rule: $\dfrac{\partial f}{\partial S^k} = \dfrac{\dfrac{d}{dt}\left(\dfrac{\partial f}{\partial m}\right)}{\dfrac{\partial f}{\partial m}}$;

Hartwick Rule: $\dfrac{dS^k}{dt} = \dfrac{\partial f}{\partial m} m.$

The differentiation, with respect to time, of the law of motion of capital yields

$$\frac{d^2S^k}{dt^2} = \frac{\partial f}{\partial S^k}\frac{dS^k}{dt} + \frac{\partial f}{\partial m}\frac{dm}{dt} - \frac{dc}{dt}, \tag{3.1}$$

whereas that of the Hartwick Rule yields

$$\frac{d^2S^k}{dt^2} = \frac{d}{dt}\left(\frac{\partial f}{\partial m}\right)m + \frac{\partial f}{\partial m}\frac{dm}{dt} = \frac{\partial f}{\partial S^k}\frac{\partial f}{\partial m}m + \frac{\partial f}{\partial m}\frac{dm}{dt} \qquad \text{[by Hotelling Rule]}$$

$$= \frac{\partial f}{\partial S^k}\frac{dS^k}{dt} + \frac{\partial f}{\partial m}\frac{dm}{dt}, \qquad \text{[by Hartwick Rule]}$$

which together with (3.1) implies that $dc/dt = 0$.

The Hartwick rule, in its interpretation

“SUM OF NET INVESTMENTS IN ALL FORMS OF CAPITAL = 0,”

plays a central role in the sustainability literature, as we discuss next.

3.3. Weak and strong sustainability

The Club of Rome Report (Meadows *et al*., 1972) and the Report of the United Nations Conference on the Human Environment, Stockholm, 1972 (see the United Nations Environment Programme UNEP, www.unep.org, where the term “sustainable development” appears) conveyed an increasing concern about the destructive effects of economic activity, which could jeopardize the quality of life of future generations. This motivated the call for sustainability (Brundlandt Report: World Commission on Environment and Development, 1987, and Rio Summit, 1992, see United Nations Department of Economic and Social Affairs, Earth Summit Agenda 21, http://www.un.org/esa/dsd/agenda21/res_agenda21_00.shtml), understood in the oftquoted terms of the Brundlandt report: "sustainable development is development that meets the needs of the present generation without compromising the ability of future generations to meet its own needs." In academia, this gave rise to a critique of the mainstream, neoclassical economic theories of growth and development, and the flourishing of environmental economics. But environmental economics was often based on discounted utilitarianism (see Section 3.4 below), whereas the new sustainability approach adopts the long run view.

As we noted, the Solow-Hartwick theory has a legitimate claim to the title of sustainability, because it focusses on maintaining human quality of life forever, even though it narrowly defines utility as only consumption. The role of undamaged natural environments as a direct factor in human quality of life could easily be modeled by augmenting the list of arguments in the utility function, including, for instance, natural resources, as discussed in Section 2.1 above. The resulting extension of the Solow-Hartwick model to a world where environmental stocks enter the utility function is often identified with *weak sustainability* (Maite Cabeza Gutés, 1996, Neumayer, 2010, p. 21).

But while the Solow-Hartwick theory focuses on sustaining utility, in accordance with the Maximin social welfare criterion, the sustainability approach, even in its weak form, favors focusing upon the "capacity for utility" rather than "utility" (Neumayer, 2010, p. 8). The distinction, to some extent semantic, gives the Hartwick rule, appropriately generalized, a central role*: sustainability is understood as maintaining capital as an end in itself*. *Weak* and *strong* sustainability then interpret "maintaining capital" in different ways. Weak sustainability stays closer to the Hartwick rule, defined as the non-negativity of the sum of investments in all kinds of capital, including both human and natural, renewable and exhaustible. Strong sustainability emphasizes the lack of substitutability among various forms of capital, and advocates the maintaining the physical stocks of some forms of natural capital.

The weak sustainability approach motivates the empirical measure of the sum of net investments in all forms of capital, leading to the Genuine Savings (GS) index (or "genuine investment") compiled by the World Bank on a country-by-country basis. The index has received attention: for instance, it is used by Kenneth Arrow *et al.* (2004) to conclude that "we find reason to be concerned that consumption is excessive."

Any attempt at aggregating the changes in various forms of capital faces the challenge of pricing them. Market prices are patently inadequate, given the severity of the non-internalized negative environmental externalities (see, e. g., Stern, 2008). Hartwick's (1977) appeal to the Hotelling rule amounts to basing the price of the exhaustible resource on Hotelling's (1931) maximization of the sum of discounted sum of future benefits. In a parallel manner, the sustainability literature attempts to derive prices from optimization problems where the objective function is sum of discounted utilities (see, e. g., Neumayer, 2010, Section 5.1.1): the above mentioned papers by Dixit *et al*. (1980) and Asheim *et al*., (2003) show the difficulties involved.

As noted, GDP-type measures combine consumption and investment. Weak sustainability indicators such as Genuine Savings aim at correcting the shortcomings of their investment component. The desire to simultaneously correct for the shortcomings of both their investment and their consumption component leads to measures such as the Index of Sustainable Economic Welfare (ISEW), also known as Genuine Progress Indicator (GPI), the evolution of which has been computed, and compared to that of the GDP per capita, for a variety of countries, see Neumayer (2010, Section 5.2).

As noted, strong sustainability involves a deeper departure from pre-existing economic analysis and denies the *substitutability* between natural and produced capital (Neumayer, 1999, 2010, Reyer Gerlagh and B. C. C. van der Zwaan, 2002). While weak sustainability, or the Solow-Hartwick model, aims at preserving an aggregate of the stocks of produced and natural capital, strong sustainability advocates bequeathing to future generations either an aggregate of stocks of natural capital, or, better yet, physical stocks of certain forms of natural capital. The need for aggregating the various forms of produced capital or even some forms of natural capital remains, but the approach is less precisely defined than weak sustainability, and neither explicitly deals with future paths of the economic variables, nor appeals to optimization. Neumayer (2010, Ch. 6) offers detailed arguments as well as a discussion of the construction of the various quantitative indicators of strong sustainability.

3.4. Discounted utilitarianism

The neoclassical theory of optimal economic growth (Tjalling Koopmans, 1965, David Cass, 1965) adopts the normative criterion, already used by Hotelling (1931) and, reluctantly, by Frank Ramsey (1928), of maximizing the discounted sum of utilities, which in discrete time can be expressed as

$$\sum_{t=1}^{T} u(c_t)\frac{1}{(1+\delta)^{t-1}}, \tag{3.2}$$

where:

- c_t is consumption per capita at date t, and δ is the discount rate.

- The single-date utility function u may be specified as the *identity* (linear) function, so that utility is synonymous with consumption, or as *strictly concave* function of consumption. In the latter case, a popular functional form is:

$$u(c_t) = \frac{c_t^{1-\eta} - 1}{1-\eta},\ \ \eta > 0, \eta \neq 1, \tag{3.3}$$

which becomes $u(c_t) = \ln c_t$ for $\eta = 1$.

- Population growth is often taken into account, but the analysis is conducted in per capita terms, and, thus, the absolute size of population does not play any role.

- The length T of the planning horizon is modeled either as large and finite, or as infinite. Each of these modeling choices presents idiosyncratic analytical challenges: a finite horizon has the problem of the arbitrariness of any terminal conditions, e. g., how much capital to leave at the end. On the other hand, some mathematical implications of the infinite-horizon model puzzle our intuitions, anchored in a finite world.

Expression (3.2) can be interpreted in the following ways.

Long-lived consumer. A representative consumer is postulated, who lives for all dates, has a single-date (or instantaneous) utility function with consumption as its argument, and discounts the future. The discount factor $\frac{1}{1+\delta}$ (or the discount rate δ) reflects the consumer's marginal rate of intertemporal substitution, or subjective rate of time preference: a more impatient consumer has a larger δ, and attaches little value to a unit of consumption made available far into the future. Expression (3.2) is then the utility function of this consumer. The discount rate δ is sometimes identified with the "market interest rate," e. g., by Hotelling (1931), who nevertheless feels compelled to justify it: he first refers to the productivity of capital (a fallacious argument, since δ should then appear in the technological constraints, and not in the objective function) and to the fact that (p. 145) "future pleasures are uncertain in a degree increasing with their remoteness in time."

Dynasty. In this interpretation, a consumer lives for a short number of dates, but derives utility from the utility, or the consumption, of her descendants. For example, the first member of the dynasty may care about her own *consumption* and her child's *utility*, discounted; the child in turn cares about her consumption and her child's utility. By expanding this iterated expression, the first member of the dynasty must care about the discounted sum of consumptions of all her descendants, where the discount factor decreases exponentially with time. The discount rate δ reflects each generation's discounting of its child's utility. Note that both in this interpretation and in the previous one, the maximization of (3.2) is equivalent to the maximization of the utility of the present (first) generation, either because it is the only one, or because the utilities of future generations count only to the extent that they affect that of the first generation: under both interpretations, the present generation has the role of an hegemon.[4]

Classical utilitarianism. Utilitarianism, in its original, undiscounted form (Jeremy Bentham, 1789, John Stuart Mill, 1848, Henry Sidgwick, 1874), proposed the maximization of the sum of the individual utilities, unweighted because "each individual must count as one, and none as more than one." [5] On the other hand, utilitarians postulated that marginal utility is decreasing in consumption. Thus, in the intergenerational context, the classical utilitarian maximandum can be viewed as (3.2) with $\delta = 0$ and a strictly concave utility function u, perhaps of the form (3.3).[6] (Again, each date t corresponds to a different generation.) It should be noted that, when $T = \infty$, undiscounted utilitarianism may face the problem of the divergence of (3.2).

Utilitarianism with uncertainty. Suppose that there is an exogenous probability of p that each generation t of humans will be the last one, should the species have lasted until date t. Thus, the probability that the species lasts exactly T generations is $p(1-p)^{T-1}$. Suppose the social planner is a utilitarian, who desires to *maximize the total utility of all generations that exist.* A prize, in the von Neumann-Morgenstern terminology, is of the form 'the human species lasts exactly T generations and enjoys the sequence of consumptions $(c_1, c_2, ..., c_T)$.' Denote

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[4] See Roemer (2011).
[5] See Jon Elster (2008).
[6] The strict concavity of a (differentiable) utility functions is equivalent to the property of decreasing marginal utility.

this prize by $(T; c_1, ..., c_T)$. As a utilitarian, the von Neumann-Morgenstern utility function of the planner, which is defined on prizes, is given by:

$$U(T; c_1, ..., c_T) = \sum_{t=1}^{T} u(c_t).$$

Now the planner must choose an infinite sequence of generational utilities to maximize his expected utility, which is:

$$\sum_{T=1}^{\infty} p(1-p)^{T-1} U(t; c_1, ..., c_T). \tag{3.4}$$

A straightforward calculation shows that maximizing (3.4) is equivalent to maximizing:

$$\sum_{T=1}^{\infty} (1-p)^{t-1} u(c_t).$$

Again, we have the discounted utilitarian formula where the discount factor is now $1-p$, or the associated discount *rate* is $\delta = \dfrac{p}{1-p}$.

Of these four interpretations, three maximize a discounted sum of generational utilities (all but classical utilitarianism). One sees, however, that the appropriate discount rate may be very different across interpretations. With the 'long-lived consumer' approach, one might choose a discount rate by estimating actual rates of time preference of living humans. But the discount rate that must be used under the last approach has nothing to do with subjective rates of time preference. Moreover, even though we assumed that the probability p in the last interpretation is exogenous, that can be modified: it would be more appropriate to generate the probability of human extinction endogenously, as associated with choice of path.

<u>Axiomatic justifications</u>. Koopmans (1960) provided an axiomatic justification for the discounted-utilitarian social welfare function. Assuming that a preference order on infinite consumption streams obeys certain axioms, he proved that it must be representable by a discounted-utilitarian social welfare function like (3.2) for *some* choice of *u* and *some* choice of δ. But we believe this theorem is not salient for the current debate for two reasons. First, the axioms are not particularly intuitive or compelling, so the theorem does not provide a persuasive *argument* for discounted utilitarianism. Second, the theorem gives no instruction concerning

what the discount rate, or the utility function, should be. Since the choice of the discount rate has been focal in recent discussions, Koopmans's theorem cannot be brought to bear.

3.5. The Nordhaus model

3.5.1. Discounted utilitarianism with inequality aversion

The work of Nordhaus and his collaborators is central in the economics of climate change (Nordhaus, 2008, Nordhaus and Joseph Boyer, 2000). Their analysis is based on maximizing the objective function

$$\sum_{t=1}^{T} L_t \frac{1}{1-\eta}(c_t)^{1-\eta} \frac{1}{(1+\delta)^t}, \tag{3.5}$$

where L_t is the number of people in generation t. Nordhaus (2008, pp. 33, 60) calls the δ and η of (3.5) "central" and "unobserved normative parameters," reflecting "the relative importance of the different generations." The parameter δ is a "pure social time discount rate:" as just observed, a high δ means that the welfare of a generation born far into the future counts very little in the social welfare function. The second one represents "the aversion to inequality of different generations." Informally speaking, if the rates of growth turn out to be negative, then δ and η push in opposite directions, a high δ favoring the earlier generations and a high η favoring the later, less well off, ones. But for positive rates of growth, when the latter generations are better off, high values of either δ or η favor the earlier generations. This is the case for the paths proposed by Nordhaus (2008).

Ignoring the term L_t, (3.5) is formally (3.2) with the utility function of (3.3). But the interpretation of the parameter η is quite different in Nordhaus (2008) than in the utilitarian maximization, discounted or undiscounted. For classical utilitarianism, η embodies the decrease of individual marginal utility, whereas for Nordhaus η is an inequality-aversion parameter of the social welfare function. In fact, as $\eta \to \infty$, $\sum_{t=1}^{T} \frac{1}{1-\eta}(c_t)^{1-\eta}$ tends to the Maximin social welfare function $\min_t c_t$.

3.5.2. Nordhaus's parameter calibration

Nordhaus (2008) calibrates η and the annual discount rate $\hat{\delta}$ as follows. He adopts the "Ramsey equation"

$$\hat{r} = \hat{\delta} + \eta\,\hat{g}, \tag{3.6}$$

where $\hat{r}$ is the real per year rate of interest on capital and $\hat{g}$ is the per year rate of growth of consumption. Equation (3.6) corresponds to the first-order condition for the solution of the Ramsey Program: an infinitely lived consumer under continuous time maximizes $\int_0^\infty u(c(t))e^{-\delta t}dt$ subject to the constraint $dS^k/dt \le \hat{f}(S^k(t)) - c(t) - \delta^k S^k(t)$, where S^k is produced capital, which depreciates at rate δ^k, and $\hat{f}$ is the production function.

Nordhaus's (2008) calibration method consists in inferring $\hat{r}$ and $\hat{g}$ from "observed economic outcomes as reflected by interest rates and rates of return on capital" and choosing $\hat{\delta}$ and η subject to the Ramsey equation, which gives one degree of freedom. More specifically, Nordhaus (2008, p. 178) takes an "observed" value of $\hat{r} = 0.055$ (and, implicitly, a predicted growth rate of $\hat{g} = 0.02$.)[7] Equation (3.6) is then satisfied by any $(\hat{\delta}, \eta)$ pair satisfying $\hat{\delta} = 0.055 - 0.02\eta$, in particular by the values $(\hat{\delta}, \eta) = (0.015, 2)$ chosen by Nordhaus (2008).

We remark that the constrained optimization of (3.5), with η and $\hat{\delta}$ given, yields endogenous $\hat{r}$ and $\hat{g}$: write them $\hat{r}^N(\hat{\delta}, \eta)$ and $\hat{g}^N(\hat{\delta}, \eta)$. However, there is no reason why (3.6) has to hold, i. e., why $\hat{r}^N(\hat{\delta}, \eta) = \hat{\delta} + \eta\,\hat{g}^N(\delta, \eta)$, because the Ramsey Program is different: in particular, it does not incorporate any environmental variables. In other words, the Kuhn-Tucker conditions for solving program (3.5) will not include the Ramsey equation, but some possibly much more complicated version of it. Hence, even if $\hat{r} = \hat{r}^N(\hat{\delta}, \eta)$, appealing to the Ramsey equation in order to justify high values for either η or $\hat{\delta}$ (or both) is not justified.

In addition, Nordhaus (2008) takes $\hat{r}$ to coincide with observed historical yearly returns on capital. But there is no reason why the historically observed $\hat{r}$ must equal $\hat{r}^N(\hat{\delta}, \eta)$. It might be argued that the historical $\hat{r}$ is actually the one that is endogenously obtained at the solution of the Ramsey Program given $(\hat{\delta}, \eta)$, call it $\hat{r}^R(\hat{\delta}, \eta)$, in which case the Ramsey equation

[7] Elsewhere in the book he refers to a $\hat{r}$ of 0.04 (pp. 9-11) and to a $\hat{g}$ of 0.013 (p. 108).

$\hat{r}^R(\hat{\delta},\eta)=\hat{\delta}+\eta\hat{g}^R(\hat{\delta},\eta)$ could be used as a restriction on the admissible $(\hat{\delta},\eta)$ pairs. This could make sense if the historical past were populated by a single infinitely lived consumer, unconcerned with the environment, in which case the parameters $(\hat{\delta},\eta)$ would be "positive," rather than "normative." But Nordhaus considers a world of many distinct generations, with "normative" parameters $(\hat{\delta},\eta)$, and interprets the rates of return observed in the market as depending on these "normative" parameters, in particular on the aversion, by past and current market participants, to inequality among generations. In sum, Nordhaus's use of the discounted utilitarian formula is not based on any of the four justifications discussed above.

3.6. Cost-Benefit analysis: The Stern Review

Cost-Benefit analysis underpins the recommendations of the Stern Review, in turn based on the reports of the Intergovernmental Panel on Climate Change (IPCC TAR, 2001) and on Christopher Hope (2006). The Stern Review does not attempt to solve an optimization program: it is rather a cost-benefit analysis arguing that the "costs of inaction are larger than costs of action." Assuming a path of growth for the GDP, and starting from a Business as Usual (*laissez-faire*) hypothesis on the path of GHG emissions, it considers alternative policies that reduce emissions in the present, and eventually stabilize carbon concentration in the atmosphere. The Review argues that, properly discounted, the benefits of strong, early action on climate change outweigh the costs.

It should be noted that discount rates has different roles in Cost-Benefit Analysis and discounted-utilitarianism optimization. If the consumer or planner uses the pure time discount rate δ to weight the utilities of the various generations in the utilitarian maximand, Cost-Benefit Analysis uses the consumption discount rate $\delta+\eta\tilde{g}$ to evaluate the changes in future consumption streams due to a particular (marginal) investment project, relative to a reference consumption path that exogenously grows at a rate $\tilde{g}$. The project passes the Cost Benefit test if the discounted sum of the consumption streams is positive.[8] The Stern Review chooses a pure

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[8] The objective function is (Stern, 2007, p. 51), $\int_0^\infty u(c(t))e^{-\delta t}dt$, where $u(c)=\frac{1}{1-\eta}c^{1-\eta}$ is the utility function of generation t, and δ is the pure time discount rate above. Consider a reference consumption stream $\{c(t),t\in[0,\infty)\}$, that grows at rate $\tilde{g}$, with $c(0)=1$, and a "small" project that will modify the consumption

time discount rate of $\hat{\delta} = 0.001$ per annum, together with $\eta = 1$ and $\tilde{g} = \frac{\dot{c}}{c} = 0.013$ (1.3 % per annum), yielding a *consumption discount rate* of 0.014 per annum. *The key point, however, is that the discount rate used here derives not from the subjective rate of time preference of an agent, but from a postulated probability p that each generation of humans will be the last one.* In other words, Stern is using the discounted utilitarian formula associated with the von Neumann-Morgenstern utilitarian social planner who is maximizing the discounted sum of utilities due to the uncertain length of tenure of the human species. Because Stern postulates an annual probability of extinction of $p = 0.001$, he calculates an associated discount rate of $\hat{\delta} = \frac{p}{1-p} = 0.001$ per annum. This implies that the utility of individuals a century from now will be discounted by about 10%.

When commentators on the Stern review suggest higher consumption discount rates (Arrow, 2007, Nordhaus, 2007, Martin Weitzman, 2007: see the debate in the *Postscripts to the Stern Review* available at www.sternreview.org.uk, as well as the issue of *World Economics* 7 (4), October-December 2006, and the subsequent Simon Dietz *et al*., 2007) they have in mind the long tradition in growth theory that identifies the discounted utilitarian program with the utility of a long-lived consumer, and hence they want a discount rate approximating empirical rates of time preference. But Stern is not using that model, and his choice of a small discount rate is perfectly consistent, if we believe that the probability of species extinction is small. Indeed, a choice of annual probability of extinction of $p = 0.001$ is probably too *large* by at least an order of magnitude. In particular, Stern's choice of p implies that with probability one-half, the human species will survive only another 700 years. This seems too pessimistic. With a choice of $p = 0.0001$, the species will survive 7000 years with probability one-half. We suspect that Stern

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stream by $\{\Delta c(t), t \in [0,\infty)\}$. Up to a first-order approximation, the change in the value of the objective function is $\int_0^\infty u'(c(t))\Delta c(t)e^{-\delta t}\,dt = \int_0^\infty c(t)^{-\eta}\Delta c(t)e^{-\delta t}\,dt = \int_0^\infty (e^{\tilde{g}t})^{-\eta}\Delta c(t)e^{-\delta t}\,dt = \int_0^\infty e^{-(\eta\tilde{g}+\delta)t}\Delta c(t)\,dt$, i. e., the discount rate on consumption, defined as the relative change in the discount factor, is $\eta\tilde{g} + \delta$. The project passes the cost-benefit test if $\int_0^\infty e^{-(\eta\tilde{g}+\delta)t}\Delta c(t)\,dt > 0$.

was cautious, and chose an overly pessimistic value for p, because he anticipated criticism over the discount rate.

However, because the Stern Review does not solve a full optimization program, its recommendations are in principle open to the criticism, voiced by the critics of the Review, that the consumption discount rate should reflect the rates of return of the available investment alternatives: even if, using a consumption discount rate of 0.014, carbon emission reductions pass the Cost-Benefit test, future generations could conceivably be better off if the current generation avoided incurring the costs of GHG reductions and invested instead in other intergenerational public goods. In defense of the Review, Dietz *et al*. (2007, p. 137) argue that "it is hard to know why we should be confident that social rates of return would be, say, 3% or 4% into the future. In particular, if there are strong climate change externalities, then social rates of return on investment may be much lower than the observed private returns on capital over the last century, on which suggestions of a benchmark of 3% or 4% appear to be based."

Finally, one must evaluate the debate between Nordhaus and Stern by deciding which model is ethically superior. We think Nordhaus's model is best justified by the model of an infinitely lived consumer, because of his calibration of the discount rate from the subjective rate of time preference of living consumers. Stern is taking the viewpoint of a utilitarian planner facing uncertainty. Given this choice, we prefer Stern, because we see no reason that the fortunes of future generations *should* be determined by the preferences of the present generation – in particular, by its degree of impatience! That they *will* be so determined is, perhaps, a politically realistic statement, but the Stern approach, of postulating a von Neumann-Morgenstern planner, is ethically superior, in treating each generation equally --'each as one, and none as more than one.' That future utilities are discounted is not due to the hegemony of the present generation, but to the unalterable fact that time's arrow points in only one direction, that of increasing uncertainty.

Although we prefer Stern's model to Nordhaus's, on ethical grounds, we do not rest with Stern, but substitute a sustainabilitarian planner for Stern's utilitarian one.[9] To this topic we now turn.

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[9] Despite their analytical differences, both Stern and Nordhaus have exerted a positive intellectual leadership in the public debate on global warming, see Nordhaus (2012) for a recent contribution.

4. A model with education, knowledge and greenhouse gas emissions

We here report on our work on sustainability in the context of GHG emissions and global warming (Llavador *et al*., 2010, 2011*a*, *b*, Roemer, 2011).

4.1. The quality-of-life function

As in the literature discussed in Section 3 above, our work abstracts from all conflicts except for the intergenerational one and we assume a representative agent in each generation. Our approach is purely normative, and we do not propose an economic equilibrium model, nor do we attempt to predict what the path would be in the absence of policy.

Contrary to the continuous-time models mentioned in Section 3 above, we work with discrete time, with one date, or period, per generation, and we assume that a generation lives for 25 years. We consider an infinite number of generations, indexed by $t \geq 1$.

More precisely, we formally postulate the following *quality-of-life*, *standard-of-living*, *welfare* or *utility* function of Generation t, $t \geq 1$:

$$(c_t)^{\alpha_c}(x_t^l)^{\alpha_l}(S_t^n)^{\alpha_n}(\hat{S}^m - S_t^m)^{\alpha_m}, \tag{4.1}$$

where the exponents are positive and normalized such that $\alpha_c + \alpha_l + \alpha_m + \alpha_n = 1$, and where:

c_t = annual average consumption per capita by Generation t;

x_t^l = annual average leisure per capita, in efficiency units, by Generation t;

S_t^n = stock of knowledge per capita, which enters Generation t's quality-of-life function as well as the production function (see below), understood as located in the last year of life of Generation t;

S_t^m = total CO_2 in the atmosphere, in GtC, which is understood as located in the last year of life of Generation t; and

$\hat{S}^m$ = "catastrophic" level of CO_2 in the atmosphere.

Section 2.1 above has referred to the various indices of human welfare and their arguments. We of course keep consumption, the increase of which in advanced countries has significantly improved welfare (D. G. Johnson, 2000). Next, we list leisure, which, as indicated in Section 2.1 above, has a long tradition in economics. But we consider leisure in efficiency units, i. e., *education-enhanced* leisure, not only because education improves home productivity (such as in the quality of child-rearing services, a component of the parents leisure) but also in the sense of “human capital accompanying the person to the theater” (Michael, 1973, p. 307, quoted in Section 2.1 above). In Martin Wolf’s (2007) words:

> “The ends people desire are, instead, what makes the means they employ valuable. Ends should always come above the means people use. The question in education is whether it, too, can be an end in itself and not merely a means to some other end – a better job, a more attractive mate or even, that holiest of contemporary grails, a more productive economy. The answer has to be yes. The search for understanding is as much a defining characteristic of humanity as is the search for beauty. It is, indeed, far more of a defining characteristic than the search for food or for a mate. Anybody who denies its intrinsic value also denies what makes us most fully human.”

Next, we consider the stock of knowledge, which has the character of a public good. (Education or human capital, on the contrary, is primarily a private good which generates positive externalities.) Knowledge, in the form of society’s stock of culture and science, directly increases the value of life, because an understanding of how the world works and an appreciation of culture are intrinsic to human well-being. Moreover, medical discoveries are important factors in the improvements in health status, life expectancy, and infant survival (which are not explicit in our specification). Last, the preservation of natural environments is valuable to humans for its direct impact on physical and mental health. Because of our emphasis on climate change, we explicitly list the stock of CO_2 as an argument the quality-of-life function, reflecting our view that environmental deterioration is a public bad in consumption, as well as in production.
Our quality-of-life function follows the spirit of the Human Development Index produced by the United Nations Development Programme (2011) which considers, as noted in Section 2.1 above, three dimensions, namely (a) life expectancy, (b) education, and (c) consumption. As just mentioned, health can be thought of as at least partially represented by knowledge. Our quality-of-life function shares with the 1990 Human Development Report its abstraction from security, rights and freedoms, but it explicitly includes as an argument the quality of the biosphere.

4.2. Technology and intertemporal links: capital, knowledge, education and emissions

We model commodity production as using as inputs skilled labor, produced capital, accumulated human knowledge, biospheric quality, and GHG emissions. For simplicity, we assume that production of knowledge is purely labor-intensive using only skilled labor and past knowledge (think corporate research and development, and university research). There are four conduits of intergenerational transmission: capital passes from one generation to the next, after investment and depreciation; knowledge passes in like manner, with depreciation; the stock of biospheric quality diminished by emissions passes to the next generation. The fourth conduit is education: the education effort of one generation increases the efficiency of the labor time of the next one. Even though the total time, in hours, available to a generation for work and leisure is constant, the number of *efficiency units* of labor-leisure time available to a generations increases with the accumulated investment in education. The education production function plays an important role in the model and is discussed in more detail below.

Formally, feasible paths in our model are constrained by *the production function* and by the *environmental stock-flow relations*, modeled as follows.

$$\tilde{f}(x_t^c, S_t^k, S_t^n, e_t, S_t^m) \equiv k_1 (x_t^c)^{\theta_c} (S_t^k)^{\theta_k} (S_t^n)^{\theta_n} (e_t)^{\theta_e} (S_t^m)^{\theta_m} \geq c_t + i_t, t \geq 1, \qquad (4.2)$$

with

$k_1 > 0, \theta_c > 0, \theta_k > 0, \theta_n > 0,\ \theta_c + \theta_k + \theta_n = 1, \theta_e > 0$ and $\theta_m < 0$ (Aggregate production function),

$(1-\delta^k) S_{t-1}^k + k_2 i_t \geq S_t^k, t \geq 1$, with $k_2 > 0$ and $\delta^k \in [0,1]$ (Law of motion of produced capital),

$(1-\delta^n) S_{t-1}^n + k_3 x_t^n \geq S_t^n,\ t \geq 1$, with $k_3 > 0$ and $\delta^n \in [0,1]$ (Law of motion of the stock of knowledge),

$x_t^e + x_t^c + x_t^n + x_t^l \equiv x_t,\ t \geq 1$ (Allocation of efficiency units of labor),

$k_4\, x_{t-1}^e \geq x_t,\ t \geq 1$, with $k_4 > 0$ (Education production function), (4.3)

with initial conditions (x_0^e, S_0^k, S_0^n), where c_t, x_t^l, S_t^n and S_t^m have been defined in Section 4.1 above, and where:

x_t^c = average annual efficiency units of labor per capita devoted to the production of output by Generation t,

e_t = average annual emissions of CO_2 from energy (fossil fuels and cement) in GtC by Generation t,

S_t^k = stock of produced capital per capita available to Generation t,

i_t = average annual investment per capita by Generation t,

x_t^n = average annual efficiency units of labor per capita devoted to the production of knowledge by Generation t,

x_t^e = average annual efficiency units of labor per capita devoted to education by Generation t,

x_t = average annual efficiency units of time (labor and leisure) per capita available to Generation t.

We call emissions e_t and concentrations S_t^m *environmental variables*, whereas the remaining variables are called *economic*.

The education production function (4.3) plays a significant role in our analysis, and deserves some comparative comments with the literature. Note that the production of education is purely labor intensive, using only the skilled labor x_t^e of the preceding generation: x_t^e can be interpreted as labor time multiplied by the amount of human capital embodied in one time-unit of labor, as in Hirofumi Uzawa (1965) and Robert Lucas (1988). Because we assume that $\theta_c + \theta_k + \theta_n = 1$ in (4.2), our production function displays decreasing returns to "capital" when construed to consist of physical and human capital. But returns would be constant if we broadened the notion of "capital" to include also the stock of knowledge.

As in Uzawa (1965) and Lucas (1988), for simplicity we do not include produced capital as an input in the production of education. This contrasts with Sergio Rebelo (1991) and Robert Barro and Xavier Sala-i-Martin (1999, p. 179). Adapting the notation of Barro and Sala-i-Martin, their "human capital production function" is,

$$dH / dt = B[(1-v^k)S^k]^{\tilde{\eta}}[(1-v^H)H]^{1-\tilde{\eta}} - \tilde{\delta}H, \qquad (4.4)$$

where H is the amount of human capital, $(1-v^k)S^k$ is the amount of produced capital used in education, $(1-v^H)$ is the fraction of human capital used in education, and B, $\tilde{\eta}$, and $\tilde{\delta}$ are parameters, the last one being the human-capital depreciation factor.

We interpret the labor input in the production of education as that of teachers, rather than students. This departs from the interpretations by Lucas (1988) and Rebelo (1991), but it agrees with the comments in Uzawa (1965) and Barro and Sala-i-Martin (1999), e. g., the latter write (p. 179) "… a key aspect of education [is that] it relies heavily on educated people as an input."

We see the education of a generation as a social investment, in line with Lucas's (1988, p. 19) dictum "…a general fact that I will emphasize again and again: that human capital accumulation is a *social* activity, involving *groups* of people, in a way that has no counterpart in the accumulation of produced capital." Also, we adopt a broad view of educational achievement, which in particular bestows the ability to adapt to new technologies, as emphasized by Claudia Goldin and Lawrence Katz (2008).

Our education production function (4.3) can be viewed as a generational version of (4.4) for the parameter values $\tilde{\eta}=0$ and $\tilde{\delta}=1$ (since, in our model, all adults die at the end of each date), giving:

$$H_t - H_{t-1} = B[(1-\nu^H)H_{t-1}] - H_{t-1}, \text{ i. e., } H_t = B(1-\nu^H)H_{t-1},$$

which is precisely (4.4) under the notational correspondence $H_t \leftrightarrow x_t$, $(1-\nu^H) \leftrightarrow \frac{x^e_{t-1}}{x_{t-1}}$ and $B \leftrightarrow k_4$.

One very important intertemporal link is not explicitly modeled, namely the evolution of the stock of GHG from emissions. One might postulate a law of motion for the process by which biospheric quality at date $t+1$ consists of biospheric quality at date *t,* partially rejuvenated by natural processes that absorb carbon dioxide, plus the impact of new emissions of GHGs. However, the scientific view on the nature of this law of motion is very much in flux, and so we have elected *not* to imply a false precision by inserting such a law into our model. In place of doing so, we simply take a path of emissions and concomitant atmospheric concentration of carbon dioxide computed from the popular Model for the Assessment of Greenhouse-gas Induced Climate Change (MAGICC; a previous version was used by the IPCC AR4, Working Group I, see Meehl *et al*., 2007) which stabilizes the atmospheric concentration at 450 ppm CO_2, and we constrain our production sector not to emit more than is allowed on this path. That is to say: *we do not* optimize over possible paths of future emissions. In a sense, our approach is dual to the Cost-Benefit method. The latter takes as given a path for the economic variables, and recommends a path for the environmental variables (based on a cost-benefit criterion in the spirit of discounted utilitarianism). We, on the contrary, take as given a path for the environmental variables, and recommend paths for the economic variables based on human sustainability.

We have postulated Cobb-Douglas utility and production functions, which implies an elasticity of substitution of one between "natural" and "man-made" variables. As discussed in Section 3.3 above, this type of substitutability is controversial. But, because, as just discussed, we exogenously adopt the values for the environmental stocks and flows, the implications of substitutability are less drastic in our analysis than in models that aim at the endogenous determination of both natural and man-made variables.

4.3. Sustaining the quality of life

We are concerned with *human sustainability*, which requires maintaining human welfare, as discussed in Section 2.2 above. Formally, we consider the following optimization program

Pure Sustainability Optimization Program:

$$\max \Lambda \ \text{ subject to } (c_t)^{\alpha_c} (x_t^l)^{\alpha_l} (S_t^n)^{\alpha_n} (\hat{S}^m - S_t^m)^{\alpha_m} \geq \Lambda, \quad t \geq 1,$$

and subject to the feasibility conditions given by specific production relations, laws of motion of the stocks and resource constraints, and with the initial conditions given by the relevant stock values in the base year (2000). At a solution of the Pure Sustainability Optimization Program, the path of quality of life will typically be stationary, as noted, and it can be (at least asymptotically) supported by stationary paths in all the arguments of the quality-of-life function.

The main result of our calibrated model shows that human sustainability is achievable: it is possible to sustain forever human welfare at a level higher than the reference level (year 2000) while keeping emissions at a path that stabilizes CO_2 concentrations at 450 ppm. (Llavador *et al.*, 2011). Figure 1 below depicts the implied paths for some of the economic variables.

4.4. Sustaining consumption

Our work departs from a large body of the literature in the definition of the individual quality-of-life function, which includes education, knowledge and CO_2 stocks in addition to consumption. The question arises: what if we conventionally adopted consumption as our index of the individual standard of living? Would the sustainability result still hold? How different would the paths be?

We have repeated our optimization but substituting consumption for quality of life as given by (4.1) above, and assuming that the fraction of the time devoted to leisure is the same as in the solution of the Pure Sustainability Optimization Program.

Our sustainability results carry over to the new specification. Of course, we should expect higher levels of consumption than in (4.3). Figure 1 and Table 1 compare the values for consumption, the stock of knowledge and education along the paths and for the steady states of the sustainable human welfare solution in the two models. Consumption is indeed higher when the objective is consumption rather than quality of life, but, perhaps surprisingly, not by much (about 7% higher).

	Steady-state values (fractions over year-2000)			
	Consumption	Stock of knowledge	Efficiency units in education	Quality of Life
Quality-of-life maximization (Sections 2-4)	1.41	2.74	1.01	1.24
Consumption maximization (Section 5)	1.51	1.27	0.25	0.51

Table 1. Quality-of-life vs. consumption maximization.

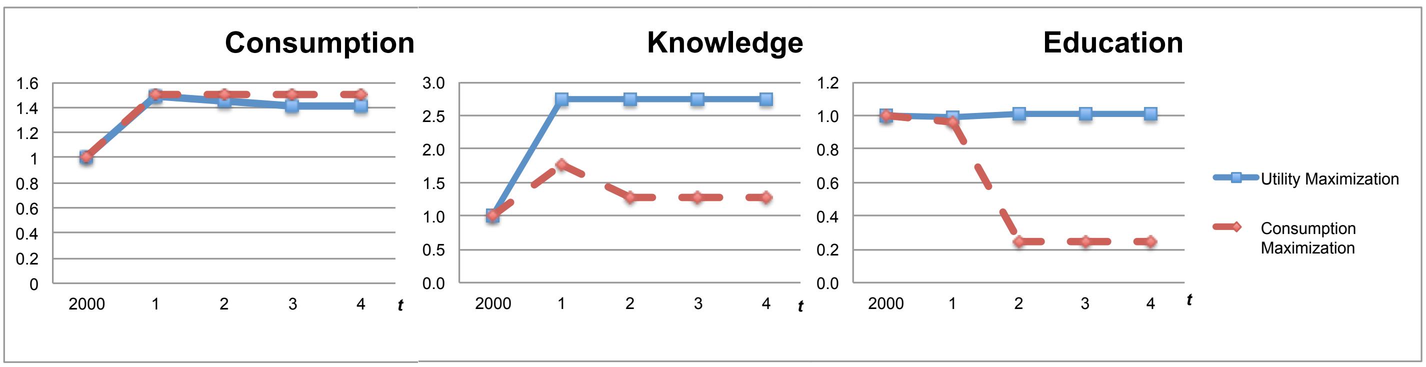

Figure 1. Comparison of consumption, stock of knowledge and education paths for quality of life maximization and consumption maximization.

Remarkably, when the objective is quality of life instead of consumption, the steady-state stock of knowledge is over twice as large, and steady-state education is over four times as large. Thus, the notion of individual welfare has dramatic consequences for the allocation of resources. We believe that our quality-of-life function is a better index of human welfare than just consumption: we therefore recommend substantially deeper investments in knowledge and education than would be optimal in models where these variables only affect productivity.

As just noted, maximizing consumption instead of the quality of life yields a higher value for consumption, but not by much. So if the "true" social welfare index were consumption, a planner who "mistakenly" maximized the quality of life would be making a relatively small error. What about the converse? In other words, if (as we believe) our quality of life function, as given by (4.1) provides an appropriate welfare index , but the public policy aims at maximizing consumption, what would be the loss in the quality of life? The answer is given in the last column of Table 1: the steady-state level of quality of life achieved by its maximization is about two and a half times the level of quality of life reached under consumption maximization. Figure 2 compares the paths for the quality of life under the two maximization objectives.

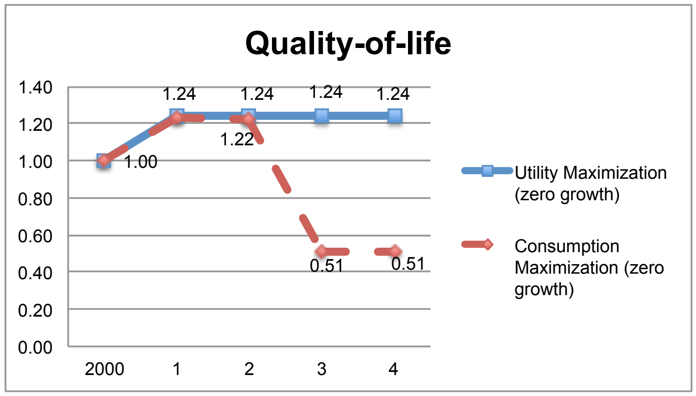

Figure 2. Comparison of quality of life paths induced by quality of life maximization and consumption maximization.

The comparison reinforces the case for sustaining quality of life. Suppose we are uncertain about the appropriate objective we should adopt: quality of life or consumption. If we choose quality of life when we should have chosen consumption, we pay a relatively small cost. On the other hand, if we sustain consumption when quality of life is the appropriate objective, the steady-state level of quality of life falls considerably.

Our model is relatively complex, and does easily offer intuitive reasons for the just discussed asymmetry between consumption and the quality of life. In order to clarify the result, the following section (more formal than the rest of the text, and which can be skipped by the reader less interested in the mathematical development) provides a stripped-down model which derives a similar result, although with less striking numbers. In any event, the analysis in the following section validates the robustness of our results comparing the maximization of the quality of life with that of consumption.

4.5. A stripped-down model comparing two social objectives: quality of life vs. consumption

Consider an economy producing a consumption commodity, using as inputs labor and knowledge (there is no capital good). 'Quality-of-life' is a function $c^{\alpha}S^{1-\alpha}$ of consumption (*c*) and the stock of knowledge (*S*), and knowledge is produced each period according to a linear production function of labor. Consider the following program:

$$\begin{aligned}&\max \lambda c+(1-\lambda)c^{\alpha}S^{1-\alpha}\\&\text{subject to}\\&c\le k(x_c)^{\theta}S^{1-\theta} \qquad \text{(i)}\\&1\ge x_c+x_n \qquad \text{(ii)}\\&S\le k_3x_n \qquad \text{(iii)}\end{aligned}$$

where x_c (resp. x_c) denotes the amount of labor denoted to the production of the consumption good (resp. knowledge) and the weight $\lambda\in[0,1]$ is fixed. Constraint (i) is the commodity production function, constraint (ii) is the labor constraint, and constraint (iii) is the knowledge production function. We may solve the constraints (which will be binding at the solution) and rewrite the program as:

$$\begin{aligned}&\max \lambda c+(1-\lambda)c^{\alpha}S^{1-\alpha}\\&\text{subject to}\\&c\le k(1-\frac{S}{k_3})^{\theta}S^{1-\theta}.\end{aligned} \tag{4.5}$$

There are two Kuhn-Tucker conditions characterizing the solution to (4.5): the first is the constraint in (4.5), which must be binding, and the second one is the dual constraint that, after some algebra, can be written:

$$\left(1-\frac{S}{k_3}\right)(1-\lambda)(1-\alpha)U+(\lambda c+(1-\lambda)U\alpha)\left(1-\frac{S}{k_3}-\theta\right)=0, \tag{4.6}$$

where $U\equiv c^{\alpha}S^{1-\alpha}$.

Now let $\lambda=1$, and so (4.5) is *maximizing consumption* only. Condition (4.6) reduces to $c\left(1-\frac{S}{k_3}-\theta\right)=0,$ i. e.,

$$\hat{S}=k_3(1-\theta).$$

On the other hand, let $\lambda=0$; then (4.5) is maximizing *quality-of-life*, and (4.6) reduces to

$$S^{*}=k_3(1-\alpha\theta).$$

It follows that the ratio of optimal knowledge in the quality-of-life formulation to optimal knowledge in the consumption-only formulation is

$$\frac{S^*}{\hat{S}} = \frac{1-\alpha\theta}{1-\theta}. \tag{4.7}$$

If we take (reasonably) $\alpha = 2/3$ and $\theta = 0.9$, then this ratio is 4, a large number.

The expression for consumption is, from the constraint in (4.5)

$$c = k\left(1-\frac{S}{k_3}\right)^{\theta} S^{1-\theta}.$$

Hence, $$\frac{c^*}{\hat{c}} = \frac{\left(1-\frac{S^*}{k_3}\right)^{\theta}(S^*)^{1-\theta}}{\left(1-\frac{\hat{S}}{k_3}\right)^{\theta}\hat{S}^{1-\theta}} = \frac{(1-1+\alpha\theta)^{\theta}\,k_3^{1-\theta}\,(1-\alpha\theta)^{1-\theta}}{(1-1+\theta)^{\theta}\,k_3^{1-\theta}\,(1-\theta)^{1-\theta}} = \alpha^{\theta}\left(\frac{1-\alpha\theta}{1-\theta}\right)^{1-\theta}, \tag{4.8}$$

which for $\alpha = \frac{2}{3}$ and $\theta = 0.9$ gives $\frac{c^*}{\hat{c}} = 0.80.$

On the other hand, using (4.8) and (4.7), the quotient for the quality-of-life levels is

$$\frac{\Lambda^*}{\hat{\Lambda}} = \left(\frac{c^*}{\hat{c}}\right)^{\alpha}\left(\frac{S^*}{\hat{S}}\right)^{1-\alpha} = \alpha^{\alpha\theta}\left(\frac{1-\alpha\theta}{1-\theta}\right)^{(1-\theta)\alpha}\left(\frac{1-\alpha\theta}{1-\theta}\right)^{1-\alpha} = \alpha^{\alpha\theta}\left(\frac{1-\alpha\theta}{1-\theta}\right)^{1-\alpha\theta},$$

which for $\alpha = 2/3$ and $\theta = 0.9$ gives $\frac{\Lambda^*}{\hat{\Lambda}} = 1.37.$

This computation may provide some understanding for why changing the objective function from ‘consumption only’ to ‘quality of life’ can have a large effect on the production of knowledge and on the quality of life, but only a rather small reduction in consumption.

4.6. Sustaining growth

We return to our full-fledged model. As discussed in Section 2.3 above, the present generation may desire that future generations enjoy a steady growth in their quality of life. Instead of maximizing the quality of life of the worst-off generation, it aims at the maximization of the quality of life of the first generation, subject to the condition that quality of life subsequently grows at a given rate g per generation, as it can be simply formalized by the following optimization program.

Sustainable Development Optimization Program (rate g)

$$\max \Lambda \text{ subject to: } (c_t)^{\alpha_c}(x_t^l)^{\alpha_l}(S_t^n)^{\alpha_n}(\hat{S}^m - S_t^m)^{\alpha_m} \geq (1+g)^{t-1}\Lambda, \quad t \geq 1$$

and subject to the feasibility and initial conditions.

In words, the solution to the Sustainable Development Optimization Program (rate g) is the feasible path which maximizes the quality of life of the first generation ($t = 1$) subject to guaranteeing a rate of growth of g in quality of life, per generation, forever. By setting $g = 0$, the Sustainable Development Optimization Program becomes the Pure Sustainability Optimization Problem of Section 4.3 above. Our main result on this question is that moderate growth rates g can be achieved at the cost of a small reduction in the utility of the first generation, which stays well above the year-2000 reference level (Llavador *et al*. 2011a)

4.7. Sustainability and uncertainty

We have discussed in Section 3.4 how uncertainty can be incorporated into the utilitarian ethic. In like manner, we can incorporate it into the sustainabilitarian ethic. Let p be the probability (as in Stern) that each generation of humans is the last one. Let the social planner be a sustainabilitarian, with expected-utility preferences. Her von Neumann-Morgenstern utility function on the prize $(T;\Lambda[1],...,\Lambda[T])$, where $\Lambda[t]$ is the quality of life for generation t, is:

$$V^{Sus}(T;\Lambda[1],...,\Lambda[T]) = \min_{1\leq t\leq T}(\Lambda[1],...,\Lambda[T]).$$

Hence, she maximizes expected utility:

$$\sum_{T=1}^{\infty} p(1-p)^{T-1}V^{Sus}(T;\Lambda[1],...,\Lambda[T]). \tag{4.9}$$

This is a potentially difficult program to solve. But Llavador, Roemer, and Silvestre (2010) prove that if p is sufficiently small, then the solution to the maximization of (4.5) is identical to the solution of the Pure Sustainability Optimization Program. For our parameterization of that program, the Stern value of $p = 0.001$ per annum is sufficiently small.

Therefore, the path that we describe in Section 4.4 is also the optimal path of (4.9).

5. Conclusions

We summarize our main points.

1. *Sustainability* has a long history in economic analysis. In contemporary climate-change economics, however, it has in large part been replaced by *discounted utilitarianism.* We argue that the two most prominent recent works, those of Nordhaus (2008) and Stern (2007), are based upon different justifications of discounted utilitarianism, and this explains their choice of different discount rates. Of the two approaches, we prefer Stern's, as it is based upon a model where each generation is treated equally, whereas in the Nordhaus model the fortunes of future generations depend upon subjective preferences of the first generation concerning how to treat their descendants.

2. But we also object to Stern's approach, as it is based upon the decision of a utilitarian social planner: we replace this planner with one whose goal is to sustain human welfare at the highest possible level. Our approach rejuvenates earlier work by Solow, Hartwick and others.

3. Nevertheless, we depart from *that* earlier work in expanding the conception of human welfare to include not only commodity consumption, but also education, leisure, and two public goods – the stock of knowledge and biospheric quality. Our concept of welfare, or quality of life, is inspired by many authors: in particular, by the Human Development Reports, and the human development index which they have used, and by many economists, who have included one or all of these arguments in conceptions of human welfare.

4. We argue that the expansion of the concept of welfare beyond consumption is not a gratuitous generalization, because it renders possible responding to the climate-change challenge by changing the consumption bundle of agents, from energy-intensive commodities to less intensive ones, like knowledge, education, and leisure.

5. We insert into the model uncertainty with regard to the length of existence of the human species, in the manner of Stern (2007), and explain why our optimal sustainable paths for the model with an infinitely lasting human species are, indeed, the optimal paths for that model, as well.

6. We report on our results, published in several papers, showing that welfare can be sustained forever at levels higher than present levels, while on a production path that reduces

GHG emissions to levels which converge to atmospheric carbon concentrations of 450 ppm. We emphasize the need to keep GHG emissions on track, given the available scientific knowledge.

7. Our results are encouraging by showing that this is possible to drastically reduce GHG emissions while maintaining the quality of life across generations, but our work shows that only moderate growth rates can be sustained, suggesting slow-growth policies.

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