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"A dynamic analysis of human welfare in a warming planet"☆

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ABSTRACT

Climate science indicates that climate stabilization requires low GHG emissions. Is this consistent with nondecreasing human welfare?

Our welfare or utility index emphasizes education, knowledge, and the environment. We construct and calibrate a multigenerational model with intertemporal links provided by education, physical capital, knowledge and the environment.

We reject discounted utilitarianism and adopt, first, the *Pure Sustainability Optimization* (or Intergenerational Maximin) criterion, and, second, the *Sustainable Growth Optimization* criterion, that maximizes the utility of the first generation subject to a given future rate of growth. We apply these criteria to our calibrated model via a novel algorithm inspired by the turnpike property.

The computed paths yield levels of utility higher than the level at reference year 2000 for all generations. They require the doubling of the fraction of labor resources devoted to the creation of knowledge relative to the reference level, whereas the fractions of labor allocated to consumption and leisure are similar to the reference ones. On the other hand, higher growth rates require substantial increases in the fraction of labor devoted to education.

1. Introduction

Since the late 1980s, scientists have become increasingly concerned with the effect of the emission of greenhouse gases (GHGs) on global temperature. The Intergovernmental Panel on Climate Change (IPCC) has now issued four reports, documenting the conjecture, expressed with increasing levels of confidence, that recent increases in global temperature are primarily anthropogenic in origin, attributable in the main, but not solely, to the burning of fossil fuels. Much has been written about strategies of mitigation of these emissions, and/or adaptation to the higher temperatures that will ensue if we extrapolate according to their present rate of growth.

In this article, we study the problem of intergenerational equity in a world that is constrained to limit GHG emissions in order to keep global temperature at an acceptably low level. We construct and calibrate a dynamic model involving economic and environmental variables. We eschew the specification of a physical model of emission-stock interactions, and consider instead a particular path for the environmental variables, which entails low emissions after 2050, and realistically appears to be feasible given present knowledge of climate dynamics. The economic variables are then endogenous in our optimization program. We develop a computational algorithm based on the turnpike property, and compute paths of resource allocation which, in a society which consists of a representative agent for each generation beginning with the present one, optimizes an objective function that sustains growth in human welfare forever, for exogenously specified rates of growth, taken to include zero as one possibility.

We show that positive rates of growth in human welfare are possible, while the first generation experiences a utility level higher than the reference level. The computed paths involve investments in knowledge at noticeably higher levels than in the past: the fraction of labor resources devoted to the creation of knowledge must be doubled, whereas the fractions of labor allocated to consumption and leisure are similar to those of the reference level.

On the other hand, higher growth rates, while also feasible, require substantial increases in the fraction of labor devoted to education. We test for the robustness of the model calibration, and find qualitatively similar results.

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We now summarize what is new about our approach, and how it contrasts with the influential works of William Nordhaus (1991, 1994, 2008a) and Nicholas Stern (2007).

The society in our model consists of an infinite set of generations, each represented by a single agent. The agents' utility function, and the set of feasible paths of resource allocation, are specified as follows.

- The representative agent's utility – welfare, standard of living or quality-of-life function – has four arguments: (i) consumption of a produced commodity, (ii) educated leisure time, which is raw leisure valued by the agent's level of education or skill, (iii) the quality of the biosphere at the time the agent lives, a public good, and (iv) the level, or stock, of human knowledge, a public good.[1]
- There are three production sectors: *commodity production* uses as inputs skilled labor, capital, accumulated human knowledge, biospheric quality, and the level of GHG emissions permitted. The *production of knowledge* is purely labor intensive, using only skilled labor and past knowledge (think corporate research and development, and university research). The *education of children* is purely labor intensive, using only the skilled labor of teachers.
- There are four conduits of intergenerational transmission: capital passes from one generation to the next, after investment and depreciation; knowledge passes in like manner, with depreciation; the stock of biospheric quality augmented by emissions of the present generation passes to the next; and adult teachers educate children who become skilled workers and consumers at the next date.
- One very important function is not explicitly modeled: the evolution of biospheric quality from emissions. One might postulate a law of motion for the process by which biospheric quality at date $t+1$ consists of biospheric quality at date t, partially rejuvenated by natural processes that absorb carbon dioxide, plus the impact of new emissions of GHGs. However, the scientific view on the nature of this law of motion is very much in flux, and so we have elected *not* to imply a false precision by inserting such a law into our model. In place of doing so, we simply take a path of emissions and concomitant atmospheric concentration of carbon dioxide computed from the popular Model for the Assessment of Greenhouse-gas Induced Climate Change (MAGICC; a previous version was used by the Intergovernmental Panel on Climate Change, 2007, IPCC AR4, Working Group I) which stabilizes the atmospheric concentration at 450 ppm CO_2, and we constrain our production sector not to emit more than is allowed on this path. That is to say: *we do not* optimize over possible paths of future emissions, because we believe the knowledge to do so does not exist at present.
- Our exercise is entirely normative: we choose the path to maximize the utility of the first generation, subject to guaranteeing a rate of growth of utility of g for all future generations. We compute this path for various values of g. The path with $g=0$ we call '*pure sustainability optimization*,' as it sustains human welfare forever at the highest possible level. The paths with $g>0$ we call '*sustainable growth optimization*'. We do not propose a rule for adjudicating among various values of g: but our calculations suggest that values of g of 2% per annum (64% per generation) are more ethically attractive than the optimal path at $g=0$.
- As our approach is purely normative, we do not propose an economic equilibrium model, nor do we attempt to predict what the path would be in the absence of policy (what is often called the *business as usual* path).
- Technological change is modeled by the presence of knowledge, accumulated through investment in R&D, as an input into commodity production. Thus knowledge can substitute for capital, labor, and emissions through the process of technological change.

[1] Many utility functions present in the literature include one or several of these arguments. Environmental amenities, in particular, often appear as arguments in natural resource models, see, e. g., Jeffrey Krautkraemer (1985).

What is the output of the model which interests us? First, we seek to understand what rates of growth of human welfare can be sustained, given the postulated constraints on emissions. Second, we wish to understand the trade-offs implied by choosing to grow at higher rates: for instance, it turns out to be feasible to support welfare growth of 64% per generation with our calibration, but the cost will be lower welfare for the first generation than it would enjoy under a 0% growth scenario. What is the magnitude of this trade-off? Third, we wish to understand how labor should be allocated among its four uses for various values of g: labor allocated to commodity production, to educating children, to research and knowledge production, and to leisure. Should we radically re-allocate labor from its present uses?

We now contrast our approach to those of Nordhaus (2008a) and Stern (2007).

- Nordhaus (2008a) also carries out a normative exercise of maximizing an intergenerational social welfare function. He does not fix a path of emissions. Instead, he proposes a law of motion of biospheric degradation, and optimizes over not only the paths of consumption, investment, and capital, but also of emissions. As we note in Section 6.2 below, his solution paths entail, for the next two centuries, emission levels substantially higher than the path that we adopt.
- The utility function of his representative agents consists only of consumption of a produced commodity. Accordingly, emissions and biospheric quality affect human welfare only indirectly, through their impact on production.
- Nordhaus proposes an exogenous path of technological change. There is no knowledge-production sector in his model. Neither is there an education sector in Nordhaus (2008a).
- Most importantly, the social welfare function in Nordhaus (2008a) is *discounted utilitarian.* He maximizes the discounted sum of generational utility levels, where the discount rate is calibrated from the rate of time impatience of existing consumers, calculated via the Ramsey equation.
- The Stern (2007) Review does not carry out a full optimization exercise. It compares only two paths: 'business as usual,' against an alternative path that cuts back severely on emissions. The criterion used to compare these two paths is discounted utilitarian. But the objective differs from Nordhaus because Stern chooses a much smaller discount rate (larger discount factor) than Nordhaus. Rather than calibrating the discount rate from the Ramsey equation – and thus from the rate of impatience of market consumers – Stern (2007) discounts future utility only because future generations might not exist, due to a small probability, at each date, of the disappearance of the human species.

There are three principal differences between our work and that of Nordhaus and Stern.

(a) We include four arguments in the utility function, not one. This is more realistic, we believe, and also provides more possibilities for substitution in order to maintain growth of human welfare.[2] But in order to isolate the role of our alternative social welfare criterion (sustainability instead of discounted utilitarianism) from that of our alternative notion of individual welfare (multivariate utility function instead of consumption) we have also performed our analysis substituting consumption for utility while maintaining the sustainability criterion.

(b) Our objective is to sustain the growth of human welfare, at some specified rate of growth, rather than maximizing the discounted sum of generational utilities. We lack the space in the present paper to argue *why* we view our approach as superior: but we refer the reader to extended discussions of

[2] While Nordhaus (2008a) claims that his 'consumption' can be interpreted as including myriad goods, this is incorrect. For the production of different goods (leisure, education, knowledge) impact very differently upon biospheric quality through their emission of GHGs. Nordhaus's aggregation would be valid only if all relevant goods impacted upon biospheric quality in the same way.

this matter in Llavador et al. (2010), and Roemer (2011). We ask the reader to note that sustaining growth is heard much more in both scientific and popular discussions than maximizing the sum of discounted utilities. While the latter has a long history in economic theory, it has little popular resonance. This, however, is not the main basis of our critique in the just mentioned papers.

(c) We do not optimize over paths of emissions, and we have explained our choice not to do so above. Macroeconomists are used to arguing over what the discount rate should be: indeed, this is the main topic of disagreement between Nordhaus and Stern (see Section 6.3 below). A discount rate does not appear in our model, and the reader may wonder why — are we avoiding an important issue? The answer is that it is possible to insert a discount rate into our model, along the lines of Stern: the utility of future generations can be discounted because they may not exist. This topic is treated in Llavador et al. (2010). It turns out, however, that the discount rate has a very different impact on the outcome of optimization in our 'sustainability of welfare growth' approach than in the discounted-utilitarian approach.

Because of the difficulty of obtaining reliable global data, we have calibrated our model with US data only. Thus, the agents in our model must be interpreted as US residents. However, the emission paths refer to global emissions, with concomitant atmospheric concentrations of CO_2. We must therefore propose a way to allocate emissions to the United States which conforms to the global emission path that we take as our constraint.

For each growth scenario we study, we calculate two optimal paths of resource allocation: the first assumes that the US continues to emit 24% of global emissions forever, and the second assumes that US emits only its per capita share of global emissions forever. Obviously, the first is an optimistic, and the second a pessimistic, path as far as the welfare of US residents is concerned. These paths, we believe, give upper and lower bounds, respectively, on what US residents can expect in the political agreement that will eventually transpire among nations to allocate rights to emit GHGs to countries. That is: it is unreasonable to suppose that the US will emit more than its present share of GHGs in the future; and even if the US is allocated permits according to a global per capita share rule, it is almost surely the case that trading in permits will result in the US emitting more than its per capita share (hence, the second scenario provides a lower welfare bound).

The paper is organized as follows. Section 2 presents the formal details of the model. Section 3 explains our strategy for calculating optimal paths. Section 4 presents the results under our postulated utility function. Section 5 compares these results with the ones obtained by substituting consumption for utility as the index of individual welfare. Section 6 discusses the relation to the literature in more detail, and Section 7 summarizes and concludes.

2. Our approach

2.1. The utility function

A large segment of the literature (e. g., Nordhaus, 2008a) postulates an individual or generational utility function with the consumption of a single, produced good as its only argument (sometimes augmented by leisure time): improvements in knowledge, education, and the environment are then important only in so far as they make possible the production of consumption goods with less labor time or capital.

In fact, both the consumption of goods and the availability of natural capital positively affect human welfare. Indeed, the spectacular increase of consumption in developed economies during the last century has undoubtedly provided a major welfare improvement (D. G. Johnson, 2000). But, in our view, two other factors have also had major impacts. First are the improvements in life expectancy, health status and infant survival, partly due to the rise in consumption, but to a large extent due to medical discoveries, and their implementation by the public health system.[3] Second is the improvement in literacy and, more generally, in the amount of education received by the average person, which has enhanced not only the productivity of labor but also utility: the contribution of leisure to utility increases as leisure time embodies higher levels of human capital, see Salvador Ortigueira (1999) and Martin Wolf (2007), as well as J. J. Heckman (1976), Philip Oreopoulos and Kjell Salvanes (2011) and Robert T. Michael (1972).[4] In Wolf's words:

> "The ends people desire are, instead, what makes the means they employ valuable. Ends should always come above the means people use. The question in education is whether it, too, can be an end in itself and not merely a means to some other end — a better job, a more attractive mate or even, that holiest of contemporary grails, a more productive economy. The answer has to be yes. The search for understanding is as much a defining characteristic of humanity as is the search for beauty. It is, indeed, far more of a defining characteristic than the search for food or for a mate. Anybody who denies its intrinsic value also denies what makes us most fully human."

Our approach follows the spirit of the Human Development Index produced by the United Nations Development Programme (UNDP) (2010), which considers three dimensions, namely (a) life expectancy, (b) education, and (c) consumption (GDP per capita). On the other hand, as we discuss in Section 2.1 below, the welfare or the consumption of a generation's children is *not* an argument in the utility function.

The first argument in the utility function is consumption. But we emphasize other factors as well:

(i) Education, which modifies the value of leisure time to the individual;

(ii) Knowledge, in the form of society's stock of culture and science, which directly increases the value of life (in addition to any indirect effects through productivity), via improvements in health and life expectancy, and because an understanding of how the world works and an appreciation of culture are intrinsic to human well-being,

(iii) An undegraded biosphere, which is valuable to humans for its direct impact on physical and mental health.[5]

Hence, consumption, educated leisure, the stock of human knowledge, and the quality of the biosphere are arguments in the utility function. The first two arguments are private goods, and the last two are public goods.

We abstract from all conflicts except for the intergenerational one and, accordingly, we assume a representative agent in each generation. We assume that a generation lives for 25 years, and we formally postulate the following utility function of Generation t, $t \geq 1$:

$$\widehat{\Lambda}\left(c_t, x_t^l, S_t^n, S_t^m\right) \equiv (c_t)^{\alpha_c}\left(x_t^l\right)^{\alpha_l}\left(S_t^n\right)^{\alpha_n}\left(\hat{S}^m - S_t^m\right)^{\alpha_m} \quad (1)$$

[3] Jim Oeppen and James Vaupel (2002), p. 1029 report that "female life expectancy in the record-holding country has risen for 160 years at a steady pace of almost 3 months per year."

[4] Increases in the human capital of the parents can also improve the quality of their child-rearing services, a component of the parents' "leisure."

[5] This is captured in the Cost-Benefit literature on global warming by the computation of the so-called "noneconomic effects."

where the exponents are positive and normalized such that $\alpha_c+\alpha_l+\alpha_m+\alpha_n=1$, and where:

c_t annual average consumption per capita by Generation t[6];
x_t^l annual average leisure per capita, in efficiency units, by Generation t[7];
S_t^n stock of knowledge per capita, which enters Generation t's utility function and production function, understood as located in the last year of life of Generation t,
S_t^m total CO_2 in the atmosphere, in GtC, which is understood as located in the last year of life of Generation t[8]; and
$\hat{S}^m$ "catastrophic" level of CO_2 in the atmosphere.

The presence of the stock of CO_2 in the utility function captures our view that environmental deterioration is a public bad in consumption (as well as in production), contrary to the modeling of Nordhaus (1994, 2008a) and Nordhaus and Joseph Boyer (2000), where it is only a public bad in production. As noted, Section 5 below discusses the implications of assuming that the level of consumption is the relevant index of individual welfare.

2.2. Optimization programs: sustainable utility levels and sustainable growth

We are concerned with *human sustainability*, which requires maintaining human welfare, rather than *green sustainability*, which may be defined as keeping the quality of the biosphere constant.[9] Human sustainability can be justified by appealing to John Rawls' (1971) Maximin principle (see Roemer, 1998, 2007).[10] It can be argued, and this is Rawls's position when justifying the (contemporaneous) *difference principle*, that it is the quality of life of each person that should enter the Maximin calculus, rather than subjective happiness, which generally includes the satisfaction that the individual derives from the welfare of other people, such as her children.

Maximizing the utility of the worst-off generation will often require the maximization of the utility of the first generation subject to maintaining that utility for all future generations, so that there is no *utility growth* after the first generation.[11] Formally, the optimization program is of the following type.

Pure sustainability optimization program

max Λ subject to $(c_t)^{\alpha_c}(x_t^l)^{\alpha_l}(S_t^n)^{\alpha_n}\left(\hat{S}^m-S_t^m\right)^{\alpha_m}\geq\Lambda,\ t\geq1,$
and subject to the feasibility conditions given by specific production relations, laws of motion of the stocks and resource constraints, and with the initial conditions given by the relevant stock values in the base year (2000).

At a solution of the Pure Sustainability Optimization Program, the path of the utility will typically be stationary, and it can be (at least asymptotically) supported by stationary paths in all the arguments of the utility function.

Alternatively, the planner may seek a positive rate of growth in the utility of future generations at the cost of reducing the utility of Generation 1. It is, however, not obvious how to justify sacrifices of the worst-off present generation for the sake of improving the already higher welfare levels of future ones.[12]

One might argue that parents want their children to have a higher quality of life than they do. Thus, welfare growth might be supported by all parents over the Pure Sustainability Optimization solution. An alternative justification for altruism towards future generations would appeal to *growth as a public good*: we may feel justifiably proud of mankind's recent gains in, say, extraterrestrial travel, or average life expectancy, and wish them to continue into the far future even at a personal cost.[13]

Indeed, there is an asymmetry in the way we feel about contemporaneous vs. temporally disjoint inequality: a person in a poor country may not wish to sacrifice her utility for the sake of improving that of a person in a *richer* country, while at the same time be willing to make some sacrifices for the welfare of unrelated, yet-to-be born individuals who will as a consequence be richer than she.

Assume that society wants to achieve a sustained rate ρ of growth in future utility: instead of maximizing the utility of the worst-off generation, it aims at the maximization of the utility of the first generation, subject to the condition that utility subsequently grows at a given rate ρ per generation. The optimization program then becomes:

Sustainable growth optimization program

max Λ subject to: $(c_t)^{\alpha_c}(x_t^l)^{\alpha_l}(S_t^n)^{\alpha_n}\left(\hat{S}^m-S_t^m\right)^{\alpha_m}\geq(1+\rho)^{t-1}\Lambda,\ t\geq1,$ for $\rho\geq0$ given, and subject again to the feasibility and initial conditions.

Note that the Pure Sustainability Optimization Program can be written in this form by letting $\rho=0$.

At a solution to this program, utility grows at a constant rate, but it is impossible to have steady positive growth of all variables because of the finite capacity $\hat{S}^m$ of the biosphere.

2.3. Economic constraints

Feasible paths are characterized by *economic constraints* and by *environmental stock-flow relations*. We adopt the following economic constraints. Recall that $t=1, 2, \ldots$ is measured in generations (25 years).

$$f\left(x_t^c,S_t^k,S_t^n,e_t,S_t^m\right)\equiv k_1\left(x_t^c\right)^{\theta_c}\left(S_t^k\right)^{\theta_k}\left(S_t^n\right)^{\theta_n}(e_t)^{\theta_e}\left(S_t^m\right)^{\theta_m}\geq c_t+i_t,\ t\geq1,\quad(2)$$

with $k_1>0$, $\theta_c>0$, $\theta_k>0$, $\theta_n>0$, $\theta_c+\theta_k+\theta_n=1$, $\theta_e>0$ and $\theta_m<0$ (Aggregate production function),

$$\left(1-\delta^k\right)S_{t-1}^k+k_2i_t\geq S_t^k,\ t\geq1,\ \text{with } k_2>0 \text{ and } \delta^k\in[0,1]\quad(3)$$

(Law of motion of physical capital),

$$(1-\delta^n)S_{t-1}^n+k_3x_t^n\geq S_t^n,\ t\geq1,\ \text{with } k_3>0 \text{ and } \delta^n\in[0,1]\quad(4)$$

(Law of motion of the stock of knowledge),

$$x_t^e+x_t^c+x_t^n+x_t^l\equiv x_t,\ t\geq1\ \text{(Allocation of efficiency units of labor)},$$

$$k_4\,x_{t-1}^e\geq x_t,\ t\geq1,\ \text{with } k_4>0\ \text{(Education production function)},\quad(5)$$

with initial conditions (x_0^e, S_0^k, S_0^n), where c_t, x_t^l, S_t^n and S_t^m have been defined in Section 2.1 above, and where:

x_t^c average annual efficiency units of labor per capita devoted to the production of output by Generation t,

[6] The unit for c_t, x_t, and S_t^n that we use in our calibrations is thousands of year-2000 dollars.
[7] See A1.5 and A1.6 in Appendix 1 in the web supplement to the present paper for the calibration of the efficiency unit of labor.
[8] To convert the CO_2 stock measured in GtC (as our S_t^m) into CO_2 ppm concentrations, multiply by 0.47. And, conversely, to convert CO_2 ppm into our S_t^m, multiply by 2.13. On the other hand, the amounts of CO_2 are often reported as $GtCO_2$: because of the molecular weights, one GtC in CO_2 corresponds to 3.67 $GtCO_2$.
[9] See Eric Neumayer (1999a) and the articles collected in Geir Asheim (2007) for the analysis of the various notions of sustainability.
[10] As is well known, Rawls himself (1971, Section 6.24) was reluctant to apply the principle to the intergenerational problem.
[11] But not always: see Silvestre (2002).
[12] Recall that we assume away intragenerational inequality, thereby depriving economic growth of a role in alleviating contemporaneous poverty. This important topic has high priority in our research agenda.
[13] See Silvestre (2007).

e_t average annual emissions of CO_2 from energy (fossil fuels and cement) in GtC by Generation t,
S_t^k capital stock per capita available to Generation t,[14]
i_t average annual investment per capita by Generation t,
x_t^n average annual efficiency units of labor per capita devoted to the production of knowledge by Generation t,
x_t^e average annual efficiency units of labor per capita devoted to education by Generation t,
x_t average annual efficiency units of time (labor and leisure) per capita available to Generation t.

We call emissions e_t and concentrations S_t^m *environmental variables*, whereas the remaining variables are called *economic*.

The following remarks compare our technology to some common in the growth literature.

Remark 1. The labor input in production, x_t^c, is measured in efficiency units of labor, which may be viewed as the number of labor-time units ("hours") multiplied by the amount of human capital embodied in one labor-time unit (as is customary since Hirofumi Uzawa, 1965 and Robert Lucas, 1988). Hence, because we assume that $\theta_c + \theta_k + \theta_n = 1$, our production function displays decreasing returns to "capital" when construed to consist of physical and human capital. But returns would be constant if we broadened the notion of "capital" to include also the stock of knowledge.

Remark 2. We assume that the production of new knowledge requires only efficiency labor (dedicated to R&D, or to "learning by not doing"), but that knowledge depreciates at a positive rate.

These assumptions are in line with a large segment of the growth literature.

Remark 3. Our education production function, $x_t = k_4\, x_{t-1}^e$, states that the education of a young generation requires only efficiency labor of the previous generation. If we normalize to unity the total labor–leisure time available to Generation t, then x_t can be interpreted as the amount of human capital per time unit in Generation t. Because our model is generational (t is a generation), instead of being an infinitely lived consumer (for whom t is just a moment in her life), our education production function cannot be interpreted in exactly the same manner as in many existing models of investment in human capital, which, in addition, are often cast in continuous time. More specifically, our formulation displays the following features. (i) As in Uzawa (1965) and Lucas (1988), we do not include physical capital as an input in the production of education. (ii) We interpret the labor input in the production of education as that of teachers, rather than students. This departs from the interpretations by Lucas (1988) and Sergio Rebelo (1991), but it agrees with the comments in Uzawa (1965) and Robert Barro and Xavier Sala-i-Martin (1999). (iii) We see the education of a generation as a social investment, in line with Lucas (1988). Also, we adopt a broad view of educational achievement, which in particular bestows the ability to adapt to new technologies, as emphasized by Claudia Golding and Lawrence Katz (2008).

Remark 4. We have postulated Cobb–Douglas utility and production functions, which implies an elasticity of substitution of one between "natural" and "manmade" variables. This type of substitutability is no doubt controversial (Neumayer, 1999b; Reyer Gerlagh and B.C.C. van der Zwaan, 2002). But, because we exogenously adopt the values for the environmental stocks and flows (as discussed in the following section), the implications of substitutability are less drastic in our analysis than in models that aim at the endogenous determination of both natural and manmade variables.

[14] The unit for S_t^k and i_t that we use in our calibrations is also thousands of year-2000 dollars.

2.4. Environmental stocks and flows

Anthropogenic greenhouse gas (GHG) emissions have caused atmospheric concentrations with no precedents in the last half a million years (see, e.g., Pierre Friedlingstein and Susan Solomon, 2005). The unparalleled behavior of GHG concentrations has motivated a growing literature that tries to predict the relationship among the paths of emissions, concentrations and global temperature changes. Following a large segment of literature, we focus on CO_2 emissions and concentrations.[15]

Most of the more recent and detailed physical models have no steady states, in the strict sense, with positive emissions. But if emissions are steady at low enough levels, then the stock of GHG eventually grows very slowly, experiencing minor increases in a scale of thousands of years. The stocks of GHG are then said to be "stabilized" even though, strictly speaking, they are not constant in the very long run. Here we assume a constant "long term" value of the stock of GHG, where "constant" is a simplification of "stabilized," and where the "long term" scale refers to a few hundreds, but not thousands, of years.

Because of the complexity of the climate models proposed and the lack of a canonical physical model of the current state of climatology, we shun false precision and do not attempt to specify the set of feasible flow-stock sequences $((e_t, S_t^m))_{t=1}^{\infty}$. Accordingly, we do not try to compute optimal paths for emissions and the environmental stock. Instead, we stipulate a target of about 450 ppm in long run CO_2 concentrations ($S^{m^*} = 954.1$), and adopt a path of CO_2 emissions based on the MAGICC (Model for the Assessment of Greenhouse-gas Induced Climate Change, http://www.cgd.ucar.edu/wigley-magicc) for the Emission Scenario WRE 450 and with the default ("best estimate") set of parameters. For corroboration purposes, we have checked that the Bern model (Joos et al., 1996, 2001) yields similar concentration levels for the emissions path that we adopt.[16]

These paths involve increasing emissions in the near future, and reduced emissions in the more distant future. We adopt this general pattern, but we simplify the path by postulating only three levels of emissions and stock, which average over each generation the abovementioned lifetime paths for emissions, while taking as stock those values dated at the end of the life of the generation. The algorithm described in Section 3 below motivates our choice of a two-generation interval to reach the target stabilization level.

We adopt the initial values $\left(\bar{e}_{2000}^W, \bar{S}_{2000}^m\right) = (6.58, 772.6)$ at year 2000 (World Resources Institute, 2009), where e^W stands for annual world emissions in GtC.[17] Our postulated (emission, stock) pairs are:

$(e_1^W, S_1^m) = (7.69, 882)$ for Generation 1,
$(e_2^W, S_2^m) = (6.05, 936.1)$ for Generation 2, and
$(e_t^W, S_t^m) = (e^{W^*}, S^{m^*}) = (4.14, 954.1)$ for Generation t, $t \geq 3$,

as recorded in the first and last columns of Table 1. See also Fig. 3 below.

As noted in the introduction, we have calibrated our economic model with US data due to the difficulty of obtaining reliable world data (see the following section for the numerical values). But our CO_2 emission paths refer to world emissions. We must therefore allocate

[15] The long-term effects of non-CO_2 GHG emissions have been addressed by Marcus Sarofim et al. (2005).

[16] We are indebted to Max Tavoni for help in the specification of the emission paths from MAGICC.

[17] We take $\bar{S}_{2000}^m = 772.6$ GtC (or 363 ppm) as the year 2000 atmospheric CO_2 stock (for reference, the pre-industrial level was approximately 595.5GtC in 1850) from the *CAIT Indicator Framework Paper* (World Resources Institute, 2009). To compare with the Stern (2007) Review, note that our 6.58 GtC translates into 24.15 $GtCO_2$ annual world emissions from energy (fossil fuels and cement). Once we include CO_2 emissions from land use change (7.62 $GtCO_2$) and from other Kyoto gases (9.72 $GtCO_2e$), the figure for total emissions (41.49 $GtCO_2e$) is consistent with the 42 $GtCO_2e$ total GHG emissions in 2000 reported in the Stern Review (2007, page 170).

Table 1
Our postulated paths for the environmental variables.

	World CO_2 emissions (GtC)	US CO_2 emissions (GtC) Scenario 1 ($e^{US} = 0.24 \times e^{World}$)	US CO_2 emissions (GtC) Scenario 2 ($e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}$)	Stock of CO_2 in (world) atmosphere (GtC)
Year 2000	$\bar{e}^W_{2000} = 6.58$	$\bar{e}^{US}_{2000} = 1.6$	$\bar{e}^{US}_{2000} = 1.6$	$\bar{S}^m_{2000} = 772.6$
Generation 1	$e^W_1 = 7.69$	$e^{US1}_1 = 1.85$	$e^{US2}_1 = 0.27$	$S^m_1 = 882$
Generation 2	$e^W_2 = 6.05$	$e^{US1}_2 = 1.45$	$e^{US2}_2 = 0.19$	$S^m_2 = 936.1$
Generation t, $t \geq 3$	$e^{W^*} = 4.14$	$e^{US1^*} = 0.99$	$e^{US2^*} = 0.13$	$S^{m^*} = 954.1$

emissions to the United States in line with the global emission path that we adopt. To do so, we consider two alternative scenarios.

The first scenario maintains the share of US emissions at its year-2000 share. The US accounted for 1.6 GtC in that year, representing 24% of all energy (fuel and cement) emissions (World Resources Institute, 2009). Hence, our Scenario-1 future US emissions are given by the 24% of (e^W_1, e^W_2, e^{W^*}): they are displayed in the second column in Table 1.

The second scenario assumes that the US emits its per capita share of the global emissions e^W_1, e^W_2 and e^{W^*}. We use the United Nations projections for world population, and compute the emissions per capita for the total emissions e^W_1, e^W_2 and e^{W^*} as 0.96, 0.46 and 0.45, respectively (in tC per capita). Keeping the US population constant at year 2000 level (284,257 thousand people), we obtain the Scenario 2 values of total US displayed in the third column in Table 1.

These two scenarios represent upper and lower bounds for the welfare of the US representative agent: we conjecture that, even if emission permits were distributed on a per capita basis to the various countries, the US would end up purchasing rights permits from other countries. Hence, Scenario 2 provides a lower bound on the welfare of the representative US citizen.

2.5. The calibration of parameters and initial values

As noted, we draw on US data in order to calibrate the parameters of the utility function, output and education production functions and the laws of motion for physical capital and knowledge, as well as the benchmark, year-2000 values of economic stocks and flows. Appendix 1 in the web supplement to the present paper details our calibration procedures, which yield the values displayed in Tables 2 and 3. The values in Table 2, as well as those for $\bar{S}^k_{2000}$, $\bar{S}^n_{2000}$ and $\bar{x}^e_{2000}$ from Table 3, will enter the computational algorithm described in the following section.

3. Computational strategy and algorithm

Our computational strategy is based on the Ray Optimization Theorem below, in the spirit of turnpike theory: see our companion paper Llavador et al. (2010) for a turnpike theorem in a simpler model. Consider a pair (e^*, S^{m^*}) such that the constant sequence $((e^*, S^{m^*}))_{t=1}^{\infty}$ is an environmentally feasible flow-stock path, and the following optimization program.

Program $E[\rho, e^*, S^{m^*}]$. Given (ρ, e^*, S^{m^*}), Max Λ_1 subject to

$$c_t^{\alpha_c}\left(x_t^l\right)^{\alpha_l}\left(S_t^n\right)^{\alpha_n}\left(\hat{S}^m - S^{m^*}\right)^{\alpha_m} \geq \Lambda_1(1+\rho)^{t-1}, t \geq 1,$$

$$k_1\left(x_t^c\right)^{\theta_c}\left(S_t^k\right)^{\theta_k}\left(S_t^n\right)^{\theta_n}\left(e^*\right)^{\theta_e}\left(S^{m^*}\right)^{\theta_m} \geq c_t + i_t, \quad t \geq 1,$$

$$\left(1-\delta^k\right)S_{t-1}^k + k_2 i_t \geq S_t^k, \quad t \geq 1,$$

$$\left(1-\delta^n\right)S_{t-1}^n + k_3 x_t^n \geq S_t^n, \quad t \geq 1,$$

$$x_t^e + x_t^n + x_t^l + x_t^c \equiv x_t, \quad t \geq 1,$$

$$k_4 x_{t-1}^e \geq x_t, \quad t \geq 1,$$

with initial conditions (x_0^e, S_0^k, S_0^n).

Recall that ρ is the rate of growth of the utility per generation. It will be convenient to denote by g the rate of growth of the economic variables, again per generation.

Theorem 1. Ray Optimization Theorem. *Assume constant returns to scale in production in the sense that* $\theta_c + \theta_k + \theta_n = 1$. *Given* $(g, e^*, S^{m^*}) \in [0, k_4 - 1) \times \Re_{++} \times \left(0, \hat{S}^m\right)$, *there is a ray* $\Gamma(g, e^*, S^{m^*}) \equiv \left\{(x^e, S^k, S^n) \in \Re^3_+ : (S^k, S^n) = x^e\left(q^k(g, e^*, S^{m^*}), \ q^n(g)\right)\right\}$, *such that if* $(x_0^e, S_0^k, S_0^n) \in \Gamma(g, e^*, S^{m^*})$, $(x_0^e, S_0^k, S_0^n) \neq 0$, *then the solution path to Program* $E[\rho, e^*, S^{m^*}]$ *satisfies:*

(i) $(x_t^e, S_t^k, S_t^n) = (1+g)^t (x_0^e, S_0^k, S_0^n)$, $t \geq 1$, and hence $(x_t^e, S_t^k, S_t^n) \in \Gamma(g, e^*, S^{m^*})$, $t \geq 0$;

$$c_1 = p^c(g)\, q^k\left(g, e^*, S^{m^*}\right) x_0^e,$$

$$i_1 = p^i(g)\, q^k\left(g, e^*, S^{m^*}\right) x_0^e;$$

(ii)
$$x_1^l = \nu^l(g)\, q^n(g)\, x_0^e,$$
$$x_1^n = \nu^n(g)\, q^n(g)\, x_0^e,$$
$$x_1^c = \nu^c(g)\, q^n(g)\, x_0^e;$$

(iii) $\left(c_t, i_t, x_t^l, x_t^n, x_t^c\right) = (1+g)^{t-1}\left(c_1, i_1, x_1^l, x_1^n, x_1^c\right), t \geq 1.$

Table 2
Calibrated values for functional parameters.

Parameter	Value
α_c	0.319
α_l	0.637
α_n	0.016
α_m	0.028
k_1	20.487
k_2	13.118
k_3	649.34
k_4	35.45
θ_c	0.6667
θ_k	0.2778
θ_n	0.0556
θ_m	−0.0363
θ_e	0.0910
δ^k	0.787
δ^n	0.787
$\hat{S}^m$	2654.32

Table 3
Benchmark year-2000 magnitudes.

Variable	Value	Units
$\bar{c}_{2000}$	27.78	Thousands of 2000 dollars per capita.
$\bar{i}_{2000}$	6.83	Thousands of 2000 dollars per capita.
$\bar{y}_{2000}$	34.61	Thousands of 2000 dollars per capita.
$\bar{x}_{2000}$	1.396	1950-efficiency units per capita.
$\bar{x}^e_{2000}$	0.0461	1950-efficiency units per capita.
$\bar{x}^c_{2000}$	0.3916	1950-efficiency units per capita.
$\bar{x}^n_{2000}$	0.0230	1950-efficiency units per capita.
$\bar{x}^l_{2000}$	0.9353	1950-efficiency units per capita.
$\bar{S}^k_{2000}$	73.65	Thousands of 2000 dollars per capita.
$\bar{S}^n_{2000}$	15.64	Thousands of 2000 dollars per capita.

Utility grows at rate ρ, were $1+\rho=(1+g)^{1-\alpha_m}$, and all other variables grow at rate g, except for emissions and concentrations, which remain constant at (e^*, S^{m^*}).

Proof. Appendix 2 in the web supplement to the present paper, where the various proportionality factors (q, p, ν) are computed in terms of the parameters of the model.

In particular, it is important to observe that, for $g=\rho=0$, whenever the initial endowments (x_0^e, S_0^k, S_0^n) lie in $\Gamma(0, e^*, S^{m^*})$, the solution to Program $E[0, e^*, S^{m^*}]$ is stationary over time.

We conjecture that a turnpike theorem, analogous to the one in Llavador et al. (2010), is true for Program $E[\rho, e^*, S^{m^*}]$ for any g, and so, if we begin with an endowment vector off the ray $\Gamma(g, e^*, S^{m^*})$, then the optimal solution will converge to the ray $\Gamma(g, e^*, S^{m^*})$. Hence, in the long run, the solution will be almost a steady-state path. Motivated by this conjecture, we now construct feasible paths which begin at the actual year-2000 endowment values $\left(\overline{x}_{2000}^e, \overline{S}_{2000}^k, \overline{S}_{2000}^n\right)$ and reach the ray $\Gamma(g, e^*, S^{m^*})$ in two generations, taking as given the values (e_1^{USj}, S_1^m), (e_2^{USj}, S_2^m) and (e^{USj^*}, S^{m^*}), $j=1, 2$, reported in Table 1.[18]

More precisely, for various rates of growth $\rho\geq 0$ of utility (or associated rates of growth g of the economic variables), we construct feasible paths $(\Lambda_1, \Lambda_2, \ldots)$ such that the ratio Λ_t/Λ_{t-1} of utility growth experienced by the later generations $t\geq 2$ is $1+\rho$, and analyze the implications of these sustained growth factors for the utility Λ_1 of Generation 1. The reference level of utility, Λ_0, is determined by the year-2000 values of the relevant arguments,

We proceed in two steps. First, we solve the optimization problem for (endogenous) initial conditions guaranteeing that the optimal solution is a steady state (i. e., all economic variables grow at the same, predetermined rate, while environmental variables stay constant). Second, we go from the historical initial conditions to the steady state path in two generations, while keeping the rate of growth of the utility for all generations after the first one at the predetermined rate.

The utility of Generation t is given by $c_t^{\alpha_c}\left(x_t^l\right)^{\alpha_l}\left(S_t^n\right)^{\alpha_n}\left(\hat{S}^m-S_t^m\right)^{\alpha_m}$. For fixed S_t^m, if all variables grow at a rate g, then utility will grow at rate ρ where $1+\rho=(1+g)^{1-\alpha_m}$. A balanced growth solution requires here three growth rates: g for the variables $(S^n, x^n, x^e, x^c, x^l)$, γ for the variables i, c and S^k, and ρ for utility. But ρ and γ are functions of g: so there is one independently chosen growth rate. For $\theta_c+\theta_k+\theta_n=1$, we have that $g=\gamma$.

We computationally solve Program G below for the chosen $(\rho, e_1, S_1^m, e_2, S_2^m, e^*, S^{m^*})$.

Program G. Given $(\rho, e_1, S_1^m, e_2, S_2^m, e^*, S^{m^*})$, Max Λ_1 subject to

$$c_1^{\alpha_c}\left(x_1^l\right)^{\alpha_l}\left(S_1^n\right)^{\alpha_n}\left(\hat{S}^m-S_1^m\right)^{\alpha_m}\geq\Lambda_1,$$

$$c_2^{\alpha_c}\left(x_2^l\right)^{\alpha_l}\left(S_2^n\right)^{\alpha_n}\left(\hat{S}^m-S_2^m\right)^{\alpha_m}\geq(1+\rho)\Lambda_1,$$

$$\left(x_2^e, S_2^k, S_2^n\right)\in\Gamma\left(g, e^*, S^{m^*}\right),$$

$$k_1\left(x_1^c\right)^{\theta_c}\left(S_1^k\right)^{\theta_k}\left(S_1^n\right)^{\theta_n}\left(e_1\right)^{\theta_e}\left(S_1^m\right)^{\theta_m}\geq c_1+i_1,$$

$$k_1\left(x_2^c\right)^{\theta_c}\left(S_2^k\right)^{\theta_k}\left(S_2^n\right)^{\theta_n}\left(e_2\right)^{\theta_e}\left(S_2^m\right)^{\theta_m}\geq c_2+i_2,$$

$$\left(1-\delta^k\right)S_0^k+k_2 i_1\geq S_1^k,$$

$$\left(1-\delta^k\right)S_1^k+k_2 i_2\geq S_2^k,$$

$$(1-\delta^n)S_0^n+k_3x_1^n\geq S_1^n,$$

$$(1-\delta^n)S_1^n+k_3x_2^n\geq S_2^n,$$

$$k_4x_0^e\geq x_1^e+x_1^n+x_1^l+x_1^c,$$

$$k_4x_1^e\geq x_2^e+x_2^n+x_2^l+x_2^c,$$

for the initial conditions $\left(x_0^e, S_0^k, S_0^n\right)=\left(\overline{x}_{2000}^e, \overline{S}_{2000}^k, \overline{S}_{2000}^n\right)$ as given in Table 3.

Using the Global Optimization Package (GO v. 8.0) for *Mathematica* 8, we compute the numerical solution paths to Program G for our calibrated parameter values. We perform this calculation for three sustained growth rates of utility, namely $\hat{\rho}=0.00$ (no growth), $\hat{\rho}=0.01$ and $\hat{\rho}=0.02$, where $\hat{\rho}$ is the rate of growth of utility expressed in *per annum* terms, with corresponding rates of growth per generation (defined by $1+\rho=(1+\hat{\rho})^{25}$) equal to $\rho=0.00$, $\rho=0.28$ and $\rho=0.64$, respectively.

4. Results

Tables 4–6 describe the obtained paths of utility and of the economic variables, as well as the fractions of labor devoted to the various ends, in the two scenarios for US emissions. To facilitate interpretation, we report unit-free relative, rather than absolute, values, often relative to the year-2000 reference values of Table 3 above.[19] Some of the information in these tables is summarized in Table 7 and depicted in Fig. 1. Recall (see Section 2.4 above) that we postulate a path of total emissions aimed at stabilizing CO_2 concentrations at 450 ppm, and consider two scenarios: Scenario 1 ($e^{US}=0.24\times e^{World}$), in which the US is responsible for 24% of all emissions (its share of total emissions in 2000); and Scenario 2 ($e_{\text{per capita}}^{US}=e_{\text{per capita}}^{World}$), in which total emissions are allocated on a per capita basis.

Our computations yield the following results.

Result 1. *Utility can be sustained forever at a level substantially higher than the year-2000 reference level (24% higher in Scenario 1, or 15% higher in Scenario 2)*

See the first column of Table 4(a). The utility of the first generation jumps to 24% (resp. 15%) above that of the year-2000 reference level in the first (resp. second) scenario on US emissions, and stays there forever. This fact is illustrated by the two horizontal lines in both graphs in Fig. 1: the lower, dotted line, with ordinate equal to 1, corresponds to the year-2000 reference level, while the continuous horizontal line with circular dots gives the sustained level of utility for all generations $t\geq 1$. As expected, the lower US emissions of Scenario 2 yield smaller increases in the utility of the US representative agents.

Result 2. *Moderate growth rates can be achieved at the cost of a small reduction in the utility of the first generation, which stays well above the year-2000 reference level*

A tradeoff between the utility of the first generation and the subsequent growth rates must indeed be expected. But our analysis shows that its magnitude is quite small: Generation 1's sacrifice for the sake of a higher growth rate is tiny for reasonable growth rates.

[18] Why two generations? Our optimization program can allow for the convergence to the ray to occur at generations later than Generation 2. Relaxing a constraint cannot hurt, but we have performed the computations for three and four generations, with very little modification of the results. Allowing stocks to converge to the ray in four generations instead of two improves the common utility of all generations by only about one tenth of one percent. We have also maximized the utility of Generation 1 subject to its stocks reaching the ray. Note that Generation 1's investment in knowledge (which affects the utility of Generation 1 both directly and indirectly through production) and Generation 1's investment in physical capital (which affects the utility of Generation 1 only indirectly through production) create intergenerational public goods. It turns out that, even for a zero-growth target, when Generation 1 maximizes its own utility subject to the stock proportionality dictated by the ray, it invests so heavily as to make the utility of the future generations higher that its own, a feature formally similar to the one discussed in Silvestre (2002).

[19] Absolute values can easily be computed from Tables 3–6.

Table 4
$\hat{\rho} = 0.00$ (sustainable utility, no growth).

(a). Paths for economic variables

Gen	Utility Λ_t/Λ_0	Generational utility growth Λ_t/Λ_{t-1}	Consumption $c_t / \bar{c}_{2000}$	Generational consumption growth c_t/c_{t-1}	Investment $i_t / \bar{i}_{2000}$	Stock of capital $S_t^k / \bar{S}_{2000}^k$	Stock of knowledge $S_t^n / \bar{S}_{2000}^n$
	Scenario 1 ($e^{US} = 0.24 \times e^{World}$)						
2000	1.00	–	1.000	–	1.000	1.000	1.000
1	1.24	1.24	1.484	1.484	2.098	2.766	2.747
2	1.24	1.00	1.450	0.977	1.568	2.496	2.743
3	1.24	1.00	1.408	0.971	1.615	2.496	2.743
4	1.24	1.00	1.408	1.000	1.615	2.496	2.743
Gen	Scenario 2 ($e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}$)						
2000	1.00	–	1.000	–	1.000	1.000	1.000
1	1.15	1.15	1.169	1.169	1.602	2.162	2.740
2	1.15	1.00	1.129	0.966	1.219	1.944	2.765
3	1.15	1.00	1.096	0.971	1.257	1.944	2.765
4	1.15	1.00	1.096	1.000	1.257	1.944	2.765

(b). The allocation of labor

Gen	Efficiency units of labor $\frac{x_t}{\bar{x}_{2000}}$	Labor allocation (% of total efficiency units) $\frac{x_t^c}{x_t}\frac{c_t}{c_t + i_t}$ % in consumption	$\frac{x_t^c}{x_t}\frac{i_t}{c_t + i_t}$ % in investment	$\frac{x_t^n}{x_t}$ % in knowledge	$\frac{x_t^e}{x_t}$ % in education	$\frac{x_t^l}{x_t}$ % in leisure
	Scenario 1 ($e^{US} = 0.24 \times e^{World}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2150	0.0748	0.0374	0.0279	0.6444
2	1.1573	0.2201	0.0585	0.0322	0.0287	0.6604
3	1.1781	0.2196	0.0619	0.0316	0.0282	0.6587
4	1.1781	0.2196	0.0619	0.0316	0.0282	0.6587
Gen	Scenario 2 ($e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2155	0.0726	0.0373	0.0281	0.6460
2	1.1667	0.2201	0.0584	0.0323	0.0287	0.6604
3	1.1876	0.2196	0.0619	0.0316	0.0282	0.6587
4	1.1876	0.2196	0.0619	0.0316	0.0282	0.6587

Table 7 (obtained from Tables 4, 5 and 6) displays the relevant ratios. As stated in Result 1, utility can be sustained forever while the utility of the first generation is 1.24 (Scenario 1) and 1.15 (Scenario 2) times the year-2000 reference level. The first and second rows of Table 7 show that, in order to subsequently maintain a 1% growth rate per year (28% per generation), the utility of the first generation must be 0.8% lower than the no-growth value. In other words, a maintained growth rate of 28% per generation can be reached at the cost of a less than 1% reduction of the utility of the first generation relative to the sustainable (no growth) path. Similarly, the first and third rows of Table 7 show that a maintained growth rate of 2% per year (64% per generation) can be reached at the cost of a less than 2% reduction of the utility of the first generation relative to the no growth path.[20]

Fig. 1 shows the utility paths computed under the different growth targets. Note that they stay well above the year-2000 reference level. It is not possible at the scale of the graph to distinguish among the three values of the utility of the first generation (for annual growth rates of 0, 1% and 2%, respectively), all clustered close to 1.23 (resp. 1.14) in Scenario 1 (resp. 2).

How are these utility paths implemented? Labor time is, in the reference year 2000, allocated to the various ends as indicated in the first row of Table 4(b) (or 5(b), or 6(b)). Table 8 indicates how these fractions should be modified in the proposed solutions. We observe the following features.

Result 3. *The most important change required by the implementation of the proposed utility paths is the doubling of the reference fraction of labor devoted to the creation of knowledge, whereas the fractions of labor allocated to consumption and to leisure are similar to those of the reference year 2000*

The largest change displayed in Table 8 occurs in the fraction of labor allocated to knowledge, which must be about twice the year-2000 reference level. The fraction of time devoted to investment in physical capital must be slightly higher, whereas the fractions of labor time devoted to the production of the consumption good and to leisure is slightly lower than the year 2000 reference values.[21]

Now we turn to the implications of increasing the growth rates, evidenced by comparing the rows of Table 8.

Result 4. *Higher growth rates require substantial increases in the fraction of labor devoted to education (of the order of a 30% increase for each additional 1% of annual growth). They also require moderate increases in the fraction of labor devoted to investment in physical capital and in knowledge. These increases are compensated by minor decreases in the fractions of labor devoted to consumption and leisure*

We have tested for the robustness of our results in several ways. In addition to our calibrated values, we also considered catastrophic

[20] Using the envelope theorem, and knowledge of the shadow prices (derived in Appendix 2 in the web supplement to the present paper) for the sustainability program, we can compute that, if $\Lambda(g)$ is the value of the g-sustainable program, then $\Lambda'(0) = -\Lambda(0)/(k_4 - 1)$. From this it follows that, to a first order approximation, $\Lambda(0)/\Lambda(0.28) = 1.00819$, consistent with the estimate in Table 7 of 0.8%. The derivation is available from the authors.

[21] Compare with Charles queryJones and Williams (1998), who estimate that the socially optimal investment in R&D is two to four times present investment.

Table 5
$\hat{\rho} = 0.01$ (1% annual growth or 28% generational growth).

(a). Paths for economic variables

Gen	Utility Λ_t/Λ_0	Generational utility growth Λ_t/Λ_{t-1}	Consumption $c_t / \bar{c}_{2000}$	Generational consumption growth c_t/c_{t-1}	Investment $i_t / \bar{i}_{2000}$	Stock of capital $S_t^k / \bar{S}_{2000}^k$	Stock of knowledge $S_t^n / \bar{S}_{2000}^n$
	Scenario 1 ($e^{US} = 0.24 \times e^{World}$)						
2000	1.00	1.00	1.000	–	1.000	1.000	1.000
1	1.23	1.23	1.471	1.471	2.078	2.741	2.723
2	1.58	1.28	1.855	1.261	2.192	3.251	3.549
3	2.02	1.28	2.328	1.255	2.883	4.199	4.584
4	2.60	1.28	3.007	1.292	3.724	5.424	5.921
	Scenario 2 ($e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}$)						
2000	1.00	1.00	1.000	–	1.000	1.000	1.000
1	1.14	1.14	1.159	1.159	1.586	2.143	2.716
2	1.46	1.28	1.444	1.246	1.706	2.531	3.577
3	1.88	1.28	1.813	1.255	2.245	3.270	4.621
4	2.41	1.28	2.342	1.292	2.900	4.224	5.969

(b). The allocation of labor

Gen	Efficiency units of labor $\frac{x_t}{\bar{x}_{2000}}$	Labor allocation (% of total efficiency units) $\frac{x_t^c}{x_t}\frac{c_t}{c_t+i_t}$ % in consumption	$\frac{x_t^c}{x_t}\frac{i_t}{c_t+i_t}$ % in investment	$\frac{x_t^n}{x_t}$ % in knowledge	$\frac{x_t^e}{x_t}$ % in education	$\frac{x_t^l}{x_t}$ % in leisure
	Scenario 1 ($e^{US} = 0.24 \times e^{World}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2131	0.0740	0.0370	0.0363	0.6389
2	1.5075	0.2165	0.0629	0.0340	0.0371	0.6495
3	1.9802	0.2161	0.0658	0.0334	0.0364	0.6483
4	2.5580	0.2161	0.0658	0.0334	0.0364	0.6483
	Scenario 2 ($e^{US}_{\text{per capita}} = e^{World}_{\text{per capita}}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2136	0.0719	0.0369	0.0366	0.6404
2	1.5196	0.2165	0.0629	0.0340	0.0371	0.6495
3	1.9961	0.2161	0.0658	0.0334	0.0364	0.6483
4	2.5785	0.2161	0.0658	0.0334	0.0364	0.6483

$\hat{S}^m$stock levels that correspond to temperature differences with respect to the preindustrial level in the range of 5.5 °C to 10 °C (see the calibration of $\hat{S}^m$in A1.2 of the web supplement to the present paper). We have also considered lower and higher values for α_m/α_c, θ_e, and θ_m. Unsurprisingly, the sustainable level of utility increases with the catastrophic level of carbon concentration in the atmosphere ($\hat{S}^m$) and with the elasticities of output to emissions (θ_e) and to the quality of the environment (θ_m), and decreases with the relative weight of the environment in utility (α_m).Yet our qualitative conclusions continue to hold under these changes. Finally, we have also considered different values of parameters associated with the educational technology (k_4), and we have found that we can sustain forever levels of utility above the 2000 reference value, even for unrealistically low values of k_4.[22]

5. Utility, or just consumption?[23]

As noted above, our work departs from a large body of the literature along several dimensions, in particular: (a) the definition of the individual utility function, which includes education, knowledge and CO_2 stocks in addition to consumption, and (b) the notion of social welfare, based on sustainability instead of discounted utilitarianism. Our main result has been the possibility of sustaining human welfare while keeping emissions at a path that stabilizes CO_2 concentrations at 450 ppm. The question arises: what if we adopted consumption as our index of the individual standard of living? Would the sustainability result still hold? How different would the paths be?

In order to address this issue we have repeated our optimization but substituting consumption for utility as given by (1) above.[24] A first observation is that straight consumption maximization would require devoting 24 h daily to the production of output. In order to make the results comparable with the ones in Sections 2–4 above, we now assume that the fraction of the time devoted to leisure is the one along the optimal path in the model of Sections 2–4.

Our sustainability results do carry over to the new specification. Of course, we should expect higher levels of consumption than in Sections 2–4 above. Fig. 2 and Table 9 compare the values for consumption, the stock of knowledge and education along the paths and for the steady states of the sustainable human welfare solution (zero growth, Scenario 1) in the two models. Consumption is indeed higher when the objective is consumption rather than utility, but, perhaps surprisingly, not by much (about 7% higher).

Remarkably, when the objective is utility instead of consumption, the steady-state stock of knowledge is over twice as large, and steady-state education is over four times as large. Thus, the notion of individual welfare does have consequences for the allocation of resources. We do believe that our utility function is a better index of human welfare than just consumption or GDP per capita: our analysis then recommends substantially deeper investments in knowledge and education than in models where these variables only affect productivity.

[22] For example, for a value of k_4 equal to 31, utility can be sustained forever at a level 8.6% higher than the year 2000 reference level under Scenario 1, and at a level 0.9% higher under Scenario 2.

[23] We are indebted to a referee and to the editor of the Journal for suggesting this extension.

[24] This requires among other things the computation of the new turnpike ray.

Table 6
$\hat{\rho}=0.02$ (2% annual growth or 64% generational growth).

(a). Paths for economic variables

Gen	Utility Λ_t/Λ_0	Generational utility growth Λ_t/Λ_{t-1}	Consumption $c_t/\bar{c}_{2000}$	Generational consumption growth c_t/c_{t-1}	Investment $i_t/\bar{i}_{2000}$	Stock of capital $S_t^k/\bar{S}_{2000}^k$	Stock of knowledge $S_t^n/\bar{S}_{2000}^n$
	Scenario 1 ($e^{US}=0.24\times e^{World}$)						
2000	1.00	1.00	1.000	–	1.000	1.000	1.000
1	1.22	1.22	1.455	1.455	2.053	2.710	2.692
2	2.00	1.64	2.362	1.624	2.976	4.197	4.558
3	3.28	1.64	3.821	1.618	5.008	6.986	7.587
4	5.38	1.64	6.359	1.664	8.335	11.627	12.628
Gen	Scenario 2 ($e^{US}_{\text{per capita}}=e^{World}_{\text{per capita}}$)						
2000	1.00	1.00	1.000	–	1.000	1.000	1.000
1	1.13	1.13	1.146	1.146	1.566	2.118	2.685
2	1.85	1.64	1.839	1.604	2.315	3.268	4.595
3	3.04	1.64	2.975	1.618	3.899	5.439	7.648
4	4.99	1.64	4.951	1.664	6.490	9.053	12.729

(b). The allocation of labor

Gen	Efficiency units of labor $\frac{x_t}{\bar{x}_{2000}}$	Labor allocation (% of total efficiency units) $\frac{x_t^c}{x_t}\frac{c_t}{c_t+i_t}$ % in consumption	$\frac{x_t^c}{x_t}\frac{i_t}{c_t+i_t}$ % in investment	$\frac{x_t^n}{x_t}$ % in knowledge	$\frac{x_t^e}{x_t}$ % in education	$\frac{x_t^l}{x_t}$ % in leisure
	Scenario 1 ($e^{US}=0.24\times e^{World}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2108	0.0731	0.0366	0.0471	0.6318
2	1.9547	0.2128	0.0659	0.0352	0.0477	0.6384
3	3.3063	0.2125	0.0685	0.0345	0.0470	0.6375
4	5.5031	0.2125	0.0685	0.0345	0.0470	0.6375
Gen	Scenario 2 ($e^{US}_{\text{per capita}}=e^{World}_{\text{per capita}}$)					
2000	1.0000	0.2251	0.0554	0.0165	0.0330	0.6700
1	1.1699	0.2113	0.0710	0.0365	0.0475	0.6333
2	1.9703	0.2128	0.0659	0.0352	0.0477	0.6384
3	3.3327	0.2125	0.0685	0.0345	0.0470	0.6375
4	5.5470	0.2125	0.0685	0.0345	0.0470	0.6375

6. Relation to the literature

6.1. Discounted utilitarianism

A large fraction of the literature on climate change adopts the discounted-utilitarian normative criterion. But we find discounted utilitarianism ethically unacceptable, at least for the low pure-time discount factors typically used, which put a weight on the utility of future generations much lower than that of the present generation. The only ethical justification for putting a lower weight on the welfare of future generations in the utilitarian calculus should be based on a positive probability of extinction of mankind. As argued in the Stern Review, this rationale would perhaps support a discount rate of $\hat{\delta}=0.001=0.1\%$ *per annum*, associated with a 0.905 probability of mankind's surviving 100 years.[25] Of course, a rigorous development of this idea requires an explicit model of uncertainty: see Llavador et al. (2010).

The low time discount factors frequent in the literature are mathematically expedient, because they make the sum of discounted utilities finite.[26] But only with extremely low factors would this be the case for the economy modeled and calibrated here, as can be argued as follows.

Denote by $\hat{P}[e^*, S^{m^*}]$ the set of feasible paths according to the constraints of Program $E[\rho, e^*, S^{m^*}]$ of Section 3 above, for some fixed endowment vector (x_0^e, S_0^k, S_0^n). (This set is independent of the value of ρ.)

The associated *Discounted-Utilitarian Program*, with a discount factor of φ, is:

$$\textbf{Program } DU\left[\varphi, e^*, S^{m^*}\right] : \max \sum_{t=1}^{\infty} \varphi^{t-1}\Lambda_t(\pi) \text{ s.t. } \pi \in \hat{P}\left[e^*, S^{m^*}\right],$$

where $\Lambda_t(\pi)$ is the utility at date t along the path π. We have:

Corollary to Theorem 1. *Program DU*$[\varphi, e^*, S^{m^*}]$ *diverges if* $\varphi k_4^{1-\alpha_m}>1$.

Table 7
The utility of the first generation (first column) relative to the year-2000 reference level Λ_0, and the sacrifice of the first generation to sustain subsequent positive growth rates (second column). The tildes denote the solution for the corresponding variable as a function of $\hat{\rho}$.

	Utility of first generation $\frac{\tilde{\Lambda}_1(\hat{\rho})}{\Lambda_0}$	Sacrifice of first generation $\frac{\tilde{\Lambda}_1(0)-\tilde{\Lambda}_1(\hat{\rho})}{\tilde{\Lambda}_1(0)}$
	Scenario 1 ($e^{US}=0.24\times e^{World}$)	
$\hat{\rho}=0.00$ (Sustainable, No growth)	1.24	0.000
$\hat{\rho}=0.01$ ($\rho=0.28$)	1.23	0.008 = 0.8%
$\hat{\rho}=0.02$ ($\rho=0.64$)	1.22	0.019 = 1.9%
	Scenario 2 ($e^{US}_{\text{per capita}}=e^{World}_{\text{per capita}}$)	
$\hat{\rho}=0.00$ (Sustainable, No growth)	1.15	0.000
$\hat{\rho}=0.01$ ($\rho=0.28$)	1.14	0.008 = 0.8%
$\hat{\rho}=0.02$ ($\rho=0.64$)	1.13	0.019 = 1.9%

[25] A discount rate δ defines a discount factor φ by $\varphi=1/(1+\delta)$. Hence, a discount rate of 0.1% yields a discount factor of 0.99001 *per annum*. See Sections 6.2 and 6.3 below for the discussion of other discount factors used in the literature.

[26] See Llavador et al. (2010) for a discussion of how a discounted utilitarian would choose paths when the discounted-utilitarian program diverges.

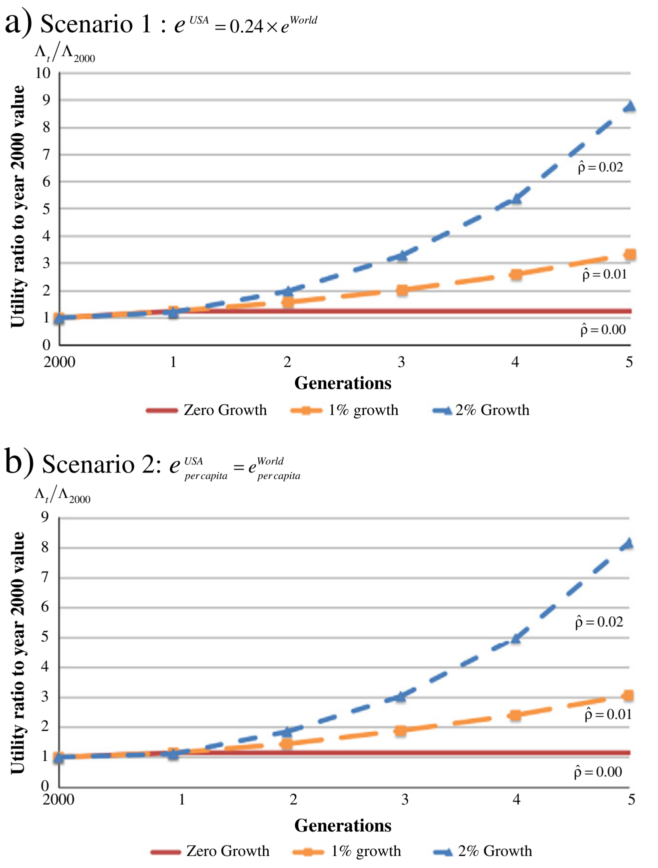

Fig. 1. Utility of generations $t = 1$ to 5 for alternative rates $\hat{\rho}$ of per *annum* growth in utility. All variables grow at a rate slightly higher than $\hat{\rho}$, with the exception of emissions and the CO_2 concentrations, which follow the path described in Table 1 above.

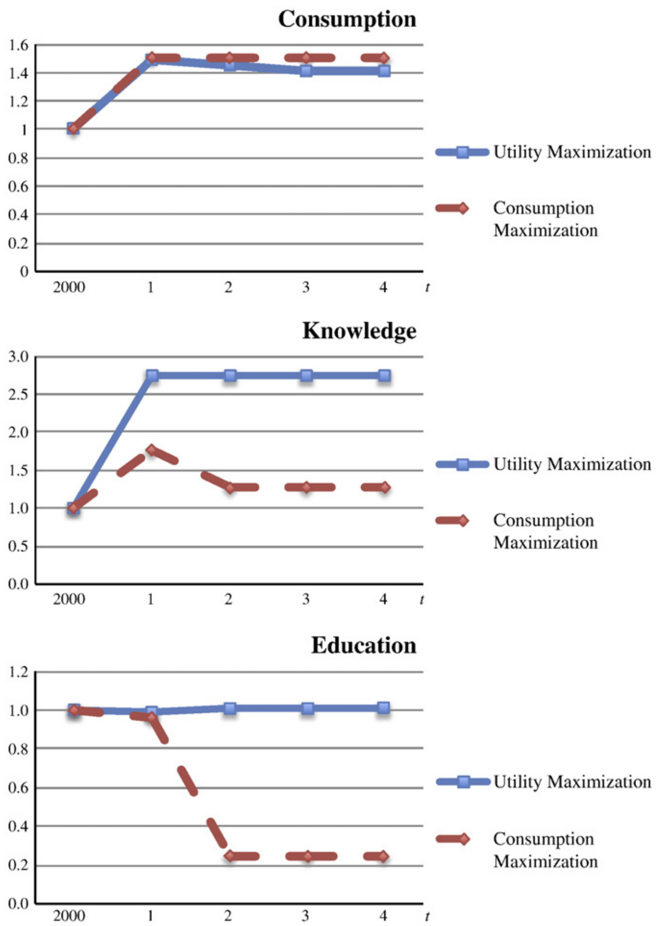

Fig. 2. Comparison of the paths for consumption, stock of knowledge and education (zero growth) prescribed by consumption maximization vs. utility maximization.

Proof. By Theorem 1, for any $g < k_4 - 1$ there is a ray $\Gamma(g, e^*, S^{m^*})$ such that, from any initial endowment vector on this ray, the balanced growth path where the economic variables grow at rate g is feasible. For any $g < k_4 - 1$, we can construct a path which, in a finite number of dates, moves from the given endowment vector (x_0^e, S_0^k, S_0^n) to some point on this ray. We then complete the path by appending the balanced growth path just referred to. Again by Theorem 1, utility grows by a factor of $1 + \rho$ at each date, after the initial section of the path, where $1 + \rho = (1 + g)^{1 - \alpha_m}$. But g may be chosen so that $1 + g$ is arbitrarily close to k_4. Hence, the terms of the discounted-utilitarian objective will grow by a factor arbitrarily close to $\varphi k_4^{1 - \alpha_m}$: in particular, g can be chosen so that this factor is greater than one, by the premise, which proves the corollary. □

If we take $1 - p = 0.975$ per generation of 25 years, as does the Stern Review, then the discounted utilitarian program will diverge as long as $(1 - p)k_4^{1 - \alpha_m} > 1$. But this inequality surely holds with our calibration of the parameters.

It is notable that the 'power' of the technology, in the sense of whether or not Program $DU[\varphi, e^*, S^{m^*}]$diverges, depends only on the technological parameter k_4, associated with the educational technology, not on any parameters associated with the other two production functions. In a simpler model than the one here, studied in Llavador et al. (2010), we attempt to explain in an intuitive way why this is the case, and we shall not repeat that argument here. The fact depends upon the constant-returns technology, that labor is the single input in the production of skilled labor, and upon the constant-returns utility function. In particular, the last

Table 8
Comparison between steady state and year-2000 values of the allocation of labor for the various growth rates. (Scenario 1: the figures for Scenario 2 are very similar). Again, the tildes denote the solution for the corresponding variable as a function of $\hat{\rho}$.

	Consumption $\dfrac{\dfrac{\tilde{c}_t(\hat{\rho})}{\tilde{c}_t(\hat{\rho}) + \tilde{i}_t(\hat{\rho})}\dfrac{\tilde{x}^c_t(\hat{\rho})}{\tilde{x}_t(\hat{\rho})}}{\dfrac{\tilde{c}_{2000}}{\tilde{c}_{2000} + \tilde{i}_{2000}}\dfrac{\tilde{x}^c_{2000}}{\tilde{x}_{2000}}}$	Investment in phys. capital $\dfrac{\dfrac{\tilde{i}_t(\hat{\rho})}{\tilde{c}_t(\hat{\rho}) + \tilde{i}_t(\hat{\rho})}\dfrac{\tilde{x}^c_t(\hat{\rho})}{\tilde{x}_t(\hat{\rho})}}{\dfrac{\tilde{i}_{2000}}{\tilde{c}_{2000} + \tilde{i}_{2000}}\dfrac{\tilde{x}^c_{2000}}{\tilde{x}_{2000}}}$	Knowledge $\dfrac{\tilde{x}^n_t(\hat{\rho}) / \tilde{x}^n_{2000}}{\tilde{x}_t(\hat{\rho}) / \tilde{x}_{2000}}$	Education $\dfrac{\tilde{x}^e_t(\hat{\rho}) / \tilde{x}^e_{2000}}{\tilde{x}_t(\hat{\rho}) / \tilde{x}_{2000}}$	Leisure $\dfrac{\tilde{x}^l_t(\hat{\rho}) / \tilde{x}^l_{2000}}{\tilde{x}_t(\hat{\rho}) / \tilde{x}_{2000}}$
$\hat{\rho} = 0$ (No growth)	0.975	1.118	1.916	0.855	0.983
$\hat{\rho} = 0.01$ ($\rho = 0.28$)	0.960	1.189	2.022	1.104	0.968
$\hat{\rho} = 0.02$ ($\rho = 0.64$)	0.944	1.237	2.093	1.423	0.952

Table 9
Utility maximization vs. consumption maximization.

	Steady-state values (fractions over year-2000)		
	Consumption	Stock of knowledge	Efficiency units in education
Utility maximization (Sections 2-4)	1.408	2.743	1.007
Consumption maximization (Section 5)	1.506	1.272	0.245

fact requires that leisure be measured in quality units, an assumption we strongly defend. As long as the assumption that the educational technology uses only educated labor as an input is approximately true, we believe this result is robust. We are reminded of Goldin and Katz (2008), who argue that the power of the American growth performance in the twentieth century was fundamentally due to universal education.

6.2. Nordhaus's optimization

Nordhaus (2008a,b) proposes particular paths for CO_2 emissions, CO_2 concentrations and consumption per capita based on an optimization program with objective function

$$\sum_{t=1}^{T} L_t \frac{1}{1-\eta}(c_t)^{1-\eta}\frac{1}{(1+\delta)^t} \tag{6}$$

(for $\eta = 1$, ln c replaces $(c_t)^{1-\eta}/(1-\eta)$), where L_t is the number of people in generation t.[27] He calls the δ and η "central" and "unobserved normative parameters," reflecting "the relative importance of the different generations." The parameter δ is a "pure social discount rate:" a high δ means that the welfare of a generation born into the future counts very little in the social welfare function, while η represents "the aversion to inequality of different generations." (Nordhaus, 2008a, p. 33, 60).[28] Informally speaking, if the rates of growth turn out to be negative, then δ and η pull in opposite directions, a high δ favoring the earlier generations, and a high η favoring the later, less well-off ones. But for positive rates of growth, which makes later generations better off, high values of either δ or η favor the earlier generations: this is the case in the paths proposed by Nordhaus (2008a). Note that the Pure Sustainability Optimization objective function of our Section 2.2 above could be viewed as a limit case of (6) for $L_t = 1$, $\delta = 0$ and $\eta \to \infty$. Nordhaus (2008a) chooses $\eta = 2$ and $\delta = (0.015)^{10}$, corresponding to a per year rate of $\hat{\delta} = 0.015$.[29]

The paths for emissions and concentrations proposed as optimal by Nordhaus differ markedly from the ones that we postulate: Fig. 3(a) (emissions) and (b) (concentrations) illustrates. Recall that we take as given a conservative path that drops emissions to low levels by 2050 and stabilizes atmospheric CO_2 concentration at about 450 ppm by 2050. In

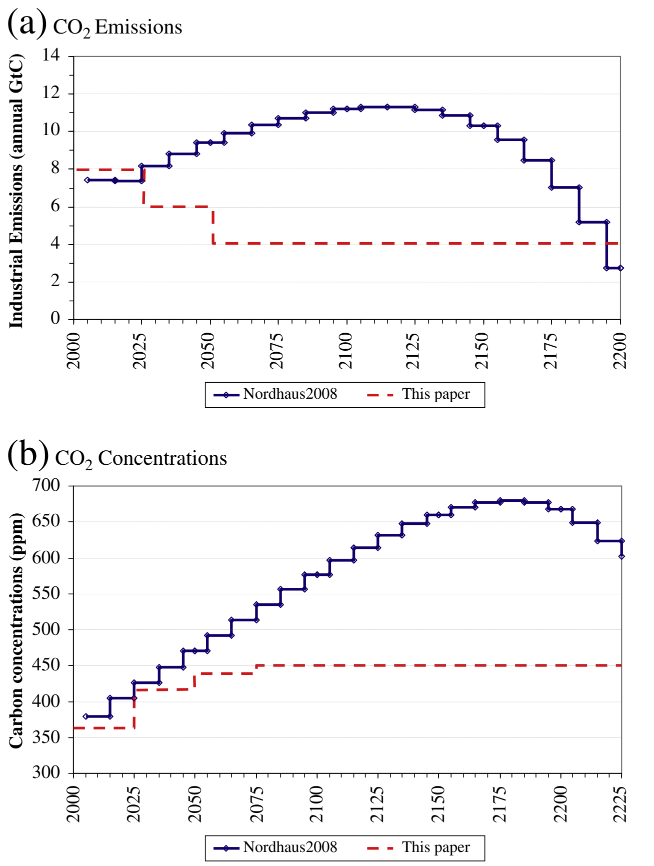

Fig. 3. Comparison of paths for the environmental variables proposed by Nordhaus (2008a,b) with the ones postulated in the present paper. The paths for Nordhaus "Optimal" are computed by running the program GAMS with data provided in Nordhaus (2008b). The curve labeled "Optimal" of Figs. 5–6 in Nordhaus (2008a) displays emissions only for the period 2005–2105, where they coincide with those of Fig. 2(a) here (except that there the emissions are per decade, and here per year). Similarly, the curve labeled "Optimal" of Figs. 5–7 in Nordhaus (2008a) displays concentrations only for the period 2005–2205, where they coincide with those of Fig. 3 (b) here.

striking contrast, Nordhaus (2008a, b) proposes as optimal a path where emissions keep increasing past the end of the 21st century. Nordhaus (2008a, b) proposed values for 2100 are about 11 GtC in emissions, with concentrations at 586.4 ppm at 2100 and at a peak of about 680 ppm in 2180.

A striking feature of Nordhaus (2008a) is that the path for per capita consumption (his only variable in the individual utility function) is virtually identical (at least for the 21st century) in the "optimal" and in the "baseline" (*laissez faire*) paths, see his Fig. 5.9. Yet he claims (p. 82) that the value of the objective function at the optimal solution is 3.37 trillions of 2005 US$ higher than at the baseline solution. We conjecture that this puzzle may be partially explained by population growth, which increases the value of the objective function for a given level of consumption per capita, together with minute differences in consumption per capita. Because of the little difference between the optimal and nonoptimal paths of consumption per capita, we conjecture that his rate of growth in consumption per capita is basically driven by his postulated exogenous growth in total factor productivity.

6.3. Cost-benefit analysis: the Stern Review

Cost-Benefit analysis underpins the recommendations of the Stern Review, in turn based on the Third Assessment Report (TAR) of the

[27] The objective function is given in Nordhaus (2008a, p. 205), with each period $t = 1$, 2,.... understood as a decade (instead of our 25-year generations). His notation is different. The optimization is numerically solved by the General Algebraic Modeling System (GAMS) program, see Nordhaus (2008b).

[28] The parameter η could also be interpreted, following the classical utilitarians and the discounted utilitarian approach discussed in the previous section, as an index of the concavity of a common, cardinal and interpersonally unit-comparable utility function displaying decreasing marginal utility, see Roemer (1998) for definitions. Expression (6), setting $L_t = 1$, is often used as the representation of the preferences of a long-lived consumer, with the discount rate δ reflecting the consumer's rate of intertemporal substitution: a more impatient consumer has a larger δ and attaches a lower value to a unit of consumption made available to her in the future. But Nordhaus (2008a,b) does not adopt these interpretations.

[29] The latter is half the value adopted in Nordhaus and Boyer (2000), see Nordhaus (2008a, p. 50).

United Nations Intergovernmental Panel on Climate Change (2001) and on Christopher Hope (2006). The Stern Review does not attempt to solve an optimization program: it is rather a Cost-Benefit analysis arguing that the "costs of inaction are larger than costs of action." Assuming a path of growth for GDP, and starting from a Business as Usual (*laissez-faire*) hypothesis on the path of GHG emissions, it considers alternative policies that reduce emissions in the present, and eventually stabilize carbon in the atmosphere. The Review argues that, properly discounted, the benefits of strong, early action on climate change outweigh the costs.

It should be noted that discount rates have different roles in Cost-Benefit Analysis and in discounted-utilitarianism optimization. Discounted utilitarianism (see Section 6.1) uses the pure time discount rate δ to weight the utilities of the various generations in the utilitarian maximand, whereas Cost-Benefit Analysis uses the consumption discount rate $\delta+\eta\tilde{g}$ to evaluate the changes in future consumption streams due to a particular (marginal) investment project, relative to a reference consumption path that exogenously grows at a rate $\tilde{g}$. The project passes the Cost-Benefit test if the discounted sum of the consumption streams is positive. As noted above, the Stern Review uses a pure time discount rate of $\hat{\delta} = 0.001$ (based on the survival justification), together with $\eta=1$ and $\tilde{g} = \dot{c}/c = 0.013$ (1.3% per annum), yielding a *consumption discount rate* of 0.014. Its commentators suggest higher consumption discount rates (Kenneth Arrow, 2007; Nordhaus, 2007, Martin Weitzman, 2007: see the debate in the *Postscripts to the Stern Review* available at www.sternreview.org.uk, as well as the issue of *World Economics* 7(4), October–December 2006, and the subsequent Simon Dietz et al., 2007).[30]

Because the Stern Review does not solve an optimization program, its recommendations are in principle open to the objection, voiced by the critics of the Review, that the consumption discount rate should reflect the rates of return of the available investment alternatives: even if, using a consumption discount rate of 0.014, carbon emission reductions pass the Cost-Benefit test, future generations could conceivably be better off if the current generation avoided incurring the costs of GHG reductions and invested instead in other intergenerational public goods. In defense of the Review, Dietz et al. (2007, p. 137) argue that "it is hard to know why we should be confident that social rates of return would be, say, 3% or 4% into the future. In particular, if there are strong climate change externalities, then social rates of return on investment may be much lower than the observed private returns on capital over the last century, on which suggestions of a benchmark of 3% or 4% appear to be based."

As we have shown, the discounted utilitarian program with the Stern Review's discount factor diverges on the set of feasible paths that we have proposed in this article. Because the Stern Review only calculates discounted utility for a small number of generations, it need not address this issue. This again shows the limitations of the Cost-Benefit method.

Our approach is in a sense dual to Cost-Benefit analysis. The latter takes as given a path for the economic variables, and recommends a path for the environmental variables (based on a cost-benefit criterion in the spirit of discounted utilitarianism). We, on the contrary, take as given a path for the environmental variables, and recommend paths for the economic variables (based on the criteria of sustainable utility and sustainable growth).

[30] Nordhaus discounts the utility of future generations by the time-rate of discount that he deduces for today's market consumer, from the Ramsey equation, which he takes to be $\delta=0.015$ per annum. This leads to a discount factor applied to the utility of those alive a century from now of $1/(1+\delta)^{100}=1/1.015^{100}=0.225$. Stern discounts the utility of those a century from now (who may not exist) according to the probability of extinction of the human species; he applies a discount factor of $(1-p)^4=(0.975)^4=0.904$. If we adopt Stern's probability-of-extinction, we do not discount the utility of those a century from now at all: that is, our discount factor applied to the utility of those a century from now is unity.

7. Summary and conclusions

Our analysis departs from the literature in three dimensions: (a) the concept of utility, (b) social welfare criterion, and (c) method.

For (a), we adopt a comprehensive notion of utility, in the spirit of the Human Development Index, that emphasizes the following three factors in addition to the conventional consumption and leisure.

(i) Education, which modifies the value of leisure time to the individual, besides enhancing her productivity;
(ii) Knowledge, in the form of culture and science, which directly improves the living experience, besides raising total factor productivity; and
(iii) The quality of the environment, which, because of the importance of climate change, we interpret as depending on the concentration of greenhouse gases in the atmosphere.

For (b), we consider two criteria. First, Pure Sustainability Optimization, which aims maximizing the utility that can be sustained for all generations. Second, Sustainable Growth Optimization, where we fix positive rates of growth, with the justification that growth has the character of a public good, and maximize the utility of the first generation subject to achieving the given, constant rate of utility growth for all subsequent generations. These objectives stand in sharp contrast to the conventional criterion of maximizing the discounted sum of utilities, which we find ethically unjustifiable, at least for the discount factors typically used.

As for (c), our method is inspired by optimization, but, given the current uncertainties in climate science, we do not attempt to compute an optimal path for environmental variables: we take instead as given a conservative path for the environmental variables, and propose paths for the economic variables based on the criteria of Pure Sustainability Optimization and Sustainable Growth Optimization. Ideally, for the Pure Sustainability Optimization Program, we would like to approach paths where all variables are stationary, whereas for the Sustainable Growth Optimization Program we would like to approach balanced-growth paths, where all variables grow at the same rate. But we cannot confidently adopt a reasonably simple model of emission-stock interaction. In addition, our formulation does not allow the quality of the atmosphere to improve without limit. Accordingly, our computations fix emissions and concentrations at levels that allow for stabilization after two generations. The resulting dynamic optimization programs defy explicit analytical solutions, and our approach has been computational. We have devised computational algorithms inspired by the turnpike property for constructing feasible and desirable, although not necessarily optimal, paths in the more complex and interesting models.

In more detail, we have adopted a simplified path for world emissions and concentrations aiming at stabilizing the concentration of CO_2 in the atmosphere at 450 ppm. Our simplified version assumes that we jump to a steady state in two generations, after which emissions are maintained at a low level and the concentration of CO_2 in the atmosphere is stabilized. We have calibrated our economic model with US data, and consider two scenarios for the path of future US CO_2 emissions, which imply upper and lower bounds on the utility of US representative generational agents. We have then computed solutions for the economic variables, by an algorithm that mimics the turnpike method.

Our main result is the feasibility of sustaining utility levels higher than the year 2000 reference value, even when maintaining a positive rate of growth for all successive generations. Not surprisingly, higher rates of sustained growth require a lower utility for the first generation, but the tradeoff is small, and the first generation reaches a utility higher than the reference value for reasonable rates of growth.

Pure Sustainability Optimization maximizes the utility level sustained for all generations, and corresponds to a zero rate of growth. Achieving this kind of human sustainability under the postulated environmental path requires particular forms of behavior for the economic variables. The most important change is doubling the fraction of labor resources devoted to the creation of knowledge, whereas the fractions of labor allocated to consumption and leisure are similar to those of the reference year 2000. On the other hand, higher growth rates require substantial increases in the fraction of labor devoted to education, together with moderate increases in the fractions of labor devoted to knowledge and the investment in physical capital.

As a final exercise, and in order to separate the effects of our feature (a), the concept of the utility, from that of (b), the social welfare criterion, we have repeated our optimization but substituting consumption for utility. Our sustainability results do carry over to the new specification. Interestingly, when the objective is utility instead of consumption, at the steady state solution the consumption level is modestly lower whereas the stock of knowledge is twice as large, and education is four times as much. Our results then support substantially deeper investments in knowledge and education than the ones obtained in models where knowledge, education and the environment only affect productivity and the index of human welfare is just consumption or GDP per capita.

Appendix A. Supplementary data

Supplementary data to this article can be found online at doi:10.1016/j.jpubeco.2011.05.017.

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