

Math Camp
Homework 1

(1) Write the following sets as lists of elements:

(a) $A = \{x \in \mathbb{N} \mid x > 3 \text{ and } x < 9\}$

(b) $B = \{a \in \mathbb{N} \mid a < 30 \text{ and } a = b^2 + 2 \text{ for some } b \in \mathbb{N}\}$

(c) $C = \{t \in \mathbb{Q} \mid t^2 = 16\}$

(d) $D = \{(x, y) \in \mathbb{R}^2 \mid x \in \mathbb{Z}, y \in \mathbb{Z}, \text{ and } x^2 + y^2 = 5\}$

(2) Let $A = \{1, 2\}$, $B = \{2, 3, 4, 5\}$, $C = \{3, 4, 5, 6, 7\}$.

Compute $A \cap B$, $A \cap C$, and $A \cup (B \cap C)$.

(3) Find the range of each of the following functions f .

(a) $f : \{0, 1, 2, 3\} \rightarrow \mathbb{Z}$ defined by $f(x) = x^2 - 2x$

(b) $f : [-2, 2] \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 5$

(c) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + 3$

(d) $f : (-\infty, 3) \cup (3, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{x-3}$

(Hint: there is only one real number that is not in the range. One approach is to write $f(x) = a$ and try to solve for x in terms of a by clearing denominators. You'll be able to do this for all $a \in \mathbb{R}$ with one exception.)

(4) Write an equation, in $y = mx + b$ form, of the line that passes through the points $(2, 4)$ and $(4, 0)$ in \mathbb{R}^2 .

(5) Let $A = [2, 5]$ and $B = [3, 10)$. Write $A \cup B$, $A \cap B$, and $A \cap \overline{B}$ using interval notation.

(6) Let the functions $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$, $g(x) = 2x + 4$, and $h(x) = \frac{1}{2}x - 2$.

(a) Compute $f \circ g$ and $g \circ f$.

(b) Show that g and h are inverse functions, i.e., show that $g \circ h = x$ and $h \circ g = x$.

(7) Are the following functions injective? Surjective? Explain.

(a) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 3$

(b) $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(x) = 2x + 3$