Math Camp

Homework 2

(1) Solve the following equations:

(a) $3x + 5 = 5x + 11$

Subtracting 11 and $3x$ from both sides, we have $-6 = 2x$, so $x = -3$.

(b) $x^2 + 8x = 20$

Subtracting 20, we can write $x^2 + 8x - 20 = 0$. This factors as $(x + 10)(x - 2) = 0$, so the solutions are $x = 2$ and $x = -10$.

(c) $x^3 - 9x = 0$

This factors as $x(x + 3)(x - 3) = 0$, so the solutions are $x = 0$, $x = -3$, and $x = 3$.

(d) $\dfrac{x}{4} = \dfrac{5}{x+1}$

Multiplying to clear denominators gives $x^2 + x = 20$. Rearranging and factoring, we get $x = -5$, $x = 4$.

(e) $\dfrac{1}{x} + \dfrac{1}{x+3} = \dfrac{1}{2}$

Multiply both sides by x and $x + 3$ to get $x + 3 + x = \frac{1}{2}(x)(x + 3) = \frac{1}{2}x^2 + \frac{3}{2}x$. Multiplying by 2, we have $4x + 6 = x^2 + 3x$, or $x^2 - x - 6 = 0$. This factors as $(x - 3)(x + 2) = 0$, so the solutions are $x = 3$ and $x = -2$.

(f) $x + \sqrt{x+5} = 7$ (Hint: rearrange to square a square root. Check your answers.) Squaring $\sqrt{x+5} = 7 - x$, we have $x + 5 = 49 - 14x + x^2$. Rearranging, $x^2 - 15x + 44 = 0$, which factors as $(x - 4)(x - 11) = 0$. So the solutions are presumably $x = 4$ and $x = 11$, but only $x = 4$ works to solve the original equation.

(2) Use rules of exponents to write $\sqrt{\frac{a^8}{a^3}}$ as a raised to a single power.

This gives us $\sqrt{a^5} = a^{5/2}$.

(3) Simply the expression $\dfrac{x^5 - x^3}{x^2 + x}$ by canceling common factors.

$$\frac{x^5 - x^3}{x^2 + x} = \frac{x^3(x+1)(x-1)}{x(x+1)} = x^2(x-1).$$

(4) Solve the equations. Your answer may be in terms of e or a natural log if appropriate.

(a) $e^{2x+5} = e^3 e^{x-2}$

Taking natural logs, $2x + 5 = 3 + x - 2$. So $x = -4$.

(b) $(e^{x+1})^2 = 5e^{x-1}$

We have $e^{2x+2} = 5e^{x-1}$. Taking logs, $2x + 2 = \ln(5) + x - 1$, so $x = \ln(5) - 1$.

(c) $\ln(2x + 1) = 3$

$2x + 1 = e^3$, so $x = \frac{e^3 - 1}{2}$.

(d) $\log_3(x^2) - \log_3(x^3) = 2$

Combining the logs gives $\log_3(1/x) = 2$. So $1/x = 9$ and $x = 1/9$.

(e) $\log_2(x) + \log_2(x + 3) = 2$ (Hint: check your answers)

Combining the logs gives $\log_2(x^2 + 3x) = 2$. So $x^2 + 3x = 4$. The solutions are ostensibly 1 and -2, but -2 doesn't work because $\log_2(-2)$ is not defined. Thus $x = 1$ is the only solution.

(5) Solve the following systems of equations:
(a)

$$x_1 + 3x_2 = 16$$
$$2x_1 + 2x_2 = 12$$

We have $x_1 = 16 - 3x_2$, so the second equation becomes $2(16 - 3x_2) + 2x_2 = 12$. This can be solved to give $x_2 = 5$, and so $x_1 = 1$.

(b)

$$x^2 + y^2 = 8$$
$$x + y = 0$$

We have $x = -y$, so $(-y)^2 + y^2 = 8$, and $y^2 = 4$. So $y = 2$ or $y = -2$, and in each case $x = -y$. Thus there are two solutions: either $x = 2$ and $y = -2$, or $x = -2$ and $y = 2$.

(c)

$$2a + b + 2c = 9$$
$$b + c = -1$$
$$3c = 6$$

This system is easy to solve, as long as we solve for c, then b, then a. First, $c = 2$. So $b + 2 = -1$, and $b = -3$. So $2a + (-3) + 2(2) = 9$, and $a = 4$.