

Math Camp
Homework 3

(1) Compute $\lim_{a \rightarrow 3} \frac{a^2 + a - 12}{a^2 - 9}$.

(2) Let $f(t) = \frac{2t^2 - 8t + 8}{t^2 + 3t - 10}$. Compute $\lim_{t \rightarrow 2} f(t)$ and $\lim_{t \rightarrow \infty} f(t)$.

(3) Let $f(x) = \frac{e^{-x} + 2e^x}{2e^{-x} + e^x}$. Compute $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

(4) (a) Estimate $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$ with the aid of a calculator or computer.

(b) Compute the limit by multiplying by $\frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x}$ and simplifying the numerator.

(5) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} x + 1 & : x \leq 0 \\ 1 - x & : 0 < x < 2 \\ 2 & : x = 2 \\ x^2 - 5 & : 2 < x < 3 \\ 1 & : x \geq 3 \end{cases}$$

Where is f continuous? (Your answer could take the form of a union of intervals.)

(6) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^2 + 3 & : x \leq 1 \\ ax + b & : 1 < x < 4 \\ 4 - \frac{8}{x} & : x > 4 \end{cases}$$

Find the unique values of a and b that make f continuous on all of \mathbb{R} .

(7) Use the limit definition of the derivative to compute the derivative of:

(a) $f(x) = x^4$

(b) $g(x) = \frac{1}{x^2}$

(8) Use your work in (7) to answer the following:

(a) Find the tangent line to the graph of $y = x^4$ at the point $(2, 16)$.

(b) Find the tangent line to the graph of $y = \frac{1}{x^2}$ at the point $(1, 1)$.

(9) Suppose you know that $f(x) = x^3 - 12x$ has derivative $f'(x) = 3x^2 - 12$. Find all intervals where f is increasing and all intervals where f is decreasing.