Math Camp

Homework 6

(1) Use a Riemann sum with 4 subintervals of equal width and right-hand endpoints as sample points to approximate each of the following integrals:

(a)

$$\int_0^4 2^x \, dx$$

$(1)(2^1) + (1)(2^2) + (1)(2^3) + (1)(2^4) = 2 + 4 + 8 + 16 = 30$

(b)

$$\int_1^9 \frac{1}{x} \, dx$$

$(2)(\frac{1}{3}) + (2)(\frac{1}{5}) + (2)(\frac{1}{7}) + (2)(\frac{1}{9}) \approx 1.5746$

(2) Compute the following definite integrals using the Fundamental Theorem of Calculus:

(a)

$$\int_0^3 5x^4 + 12x^3 - x^2 + 2 \, dx = x^5 + 3x^4 - \frac{x^2}{2} + 2x\Big|_0^3 = 3^5 + 3(3^4) - \frac{3^2}{2} + 2(3) - 0$$

(b)

$$\int_1^2 \frac{4 + x^2}{x^4} \, dx = \int_1^2 4x^{-4} + x^{-2} = 4\frac{x^{-3}}{-3} - x^{-1}\Big|_1^2 = \frac{4}{-24} - \frac{1}{2} - \left(\frac{4}{-3} - 1\right)$$

(c)

$$\int_2^6 \frac{3}{x} \, dx = 3(\ln(6) - \ln(2)) = 3\ln(3)$$

(d)

$$\int_{-1}^1 e^{4x} \, dx = \frac{1}{4}\left(e^4 - e^{-4}\right)$$

(3) Use either substitution or integration by parts to compute the indefinite integrals.

(a)

$$\int \frac{3x^2 + 1}{x^3 + x} \, dx$$

Let $u = x^3 + x$ so that $du = 3x^2 + 1 \, dx$. The integral becomes

$$\int \frac{1}{u} \, du = \ln|u| + C = \ln|3x^2 + 1| + C.$$

(b)

$$\int xe^{-x^2}\,dx$$

Let $u = -x^2$ so that $du = -2x\,dx$, and $x\,dx = -\frac{1}{2}\,du$. We have

$$\int -\frac{1}{2}e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C$$

(c)

$$\int x^2 e^x\,dx$$

Let $u = x^2$ and $dv = e^x\,dx$. Then $du = 2x\,dx$ and $v = e^x$. Integrating by parts,

$$\int x^2 e^x\,dx = x^2 e^x - \int 2xe^x\,dx.$$

Now we need to integrate by parts again: let $u = 2x$ and $dv = e^x\,dx$. Then $du = 2\,dx$ and $v = e^x$. So we get

$$x^2 e^x - \int 2xe^x\,dx = x^2 e^x - 2xe^x + \int 2e^x\,dx = x^2 e^x - 2xe^x + 2e^x + C.$$

(d)

$$\int \frac{2e^{2x} + e^x}{\sqrt{e^{2x} + e^x + 1}}\,dx$$

Set $u = e^{2x} + e^x + 1$. Then $du = 2e^{2x} + e^x$. So

$$\int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{2e^{2x} + e^x + 1} + C.$$

(e)

$$\int x^2 \cos(x)\,dx$$

We integrate by parts twice as in part (c). First let $u = x^2$ and $dv = cos(x)\,dx$, so that $du = 2x\,dx$ and $v = sin(x)$. So

$$\int x^2 \cos(x)\,dx = x^2 \sin(x) - \int 2x \sin(x)\,dx.$$

Then take $u = 2x$ and $dv = \sin(x)\,dx$ to get

$$x^2 \sin(x) - \int 2x \sin(x)\,dx = x^2 \sin(x) - 2x(-\cos(x)) + \int -2\cos(x)\,dx = x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C.$$

(f)

$$\int \cos(x)\sin(x)\sqrt{(\sin(x))^2 + 1}\,dx$$

Let $u = \sin(x)^2 + 1$. Then $du = 2\sin(x)\cos(x)\,dx$ We get

$$\int \frac{1}{2}\sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2} = \frac{1}{3}(\sin(x)^2 + 1)^{3/2} + C.$$

(g)

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

Let $u = \sqrt{x}$, so $du = \frac{1}{2\sqrt{x}}\, dx$.

$$\int 2e^u \, du = 2e^u + C = 2e^{\sqrt{x}} + C$$