

Math Camp  
**Homework 7**

- (1) Suppose an experiment is run where four fair coins are flipped. Let

$$X = (\text{number of heads})^2 - (\text{number of tails}).$$

Compute the PMF of  $X$  and use it to compute  $P(-3 \leq X \leq 3)$ . Then compute the expectation and the variance of  $X$ .

- (2) Let the random variable  $X$  be given by the PDF

$$f(x) = \begin{cases} 1/2 & : -1 \leq x \leq 0 \\ 1/4 & : 0 \leq x \leq 2 \end{cases}$$

Compute  $P(X \leq 1)$ , then compute the expectation and the variance of  $X$ .

- (3) In lecture we discussed improper integrals where one or both bounds may be infinite. Another type of improper integral is when the bounds are real numbers, but the function blows up to infinity at one end. Compute

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

by first computing  $\int_a^1 \frac{1}{\sqrt{x}} dx$ , then taking a limit as  $a \rightarrow 0^+$ . Use the same idea to show

$$\int_0^2 \frac{1}{(x-2)^3} dx$$

is infinite. (For the second integral, you may need a  $u$ -substitution first.)

- (4) Let the random variable  $X$  be given by the PDF

$$f(x) = \begin{cases} \frac{2}{x^3} & : 1 \leq x < \infty \\ 0 & : x < 1 \end{cases}$$

Compute  $P(a \leq X \leq b)$  as a formula in terms of  $a$  and  $b$ . Then compute the expectation and variance of  $X$  (note that one or both of these might be infinite).