

Math Camp
Homework 8

- (1) Let X be a random variable with PDF given by

$$f(x) = \begin{cases} 0 & : x < 2 \\ ax & : 2 \leq x \leq 3 \\ 0 & : x > 3 \end{cases}$$

Compute the expectation, the variance, and the CDF of X . (You'll need to find a , which can be done by using the fact that the total integral of the PDF is 1.)

- (2) Let X be a uniform random variable on the interval $[2, 4]$. That is, the PDF of X is given by

$$f_X(x) = \begin{cases} 0 & : x < 2 \\ 1/2 & : 2 \leq x \leq 4 \\ 0 & : x > 4 \end{cases}$$

- (a) Compute the CDF F_X of X .

(b) Let Y be a random variable defined by $Y = X^2$. Compute the CDF F_Y of Y . (Hint: $F_Y(x) = P(Y \leq x) = P(X^2 \leq x) = P(X \leq \sqrt{x})$. The range of Y should be $[4, 16]$.)

(c) Compute the PDF f_Y of Y by taking the derivative of what you found in (b). Check your answer by showing that $\int_4^{16} f_Y(x) dx = 1$.

- (3) In a certain city, 90% of taxis are yellow, and the remaining 10% are blue. Suppose that all taxis are equally likely to be involved in an accident. For any accident involving a taxi of either color, an eyewitness correctly reports the color of the taxi 80% of the time, and reports the other possible color 20% of the time.

If an accident involving a taxi occurs and an eyewitness reports that the taxi is blue, what is the probability that the taxi is blue?