Optimal Unilateral Carbon Policy*

Samuel Kortum† and David A. Weisbach‡

December 5, 2024

Abstract

Climate policy by a coalition of countries can shift activities—extraction, production, and consumption—to regions outside the coalition. We build a stylized general-equilibrium model of trade and carbon externalities to derive a coalition's optimal Pareto-improving policy in such an environment. It can be implemented through: (i) a tax on fossil-fuel extraction at a rate equal to the global marginal harm from carbon emissions, (ii) a tax on imports of energy and goods, and a rebate of the tax on exports of energy but not goods, all at a lower rate per unit of carbon than the extraction tax rate, and (iii) a goods-specific export subsidy. This combination of taxes and subsidies exploits international trade to expand the policy's reach. It promotes energy efficient production and eliminates leakage by taxing the carbon content of goods imports and by encouraging goods exports. It controls the energy price in the non-taxing region by balancing supply-side and demand-side taxes. We use a quantitative version of the model to illustrate the gains achieved by the optimal policy and simpler variants of it. Combining supply-side and demand-side taxes generates first-order welfare improvements over current and proposed climate policies.

Keywords: carbon taxes, border adjustments, leakage, climate change
JEL Codes: F18, H23, Q54

---

*This paper is a revision of our Cowles Foundation Discussion Paper No. 2311 (2021). We thank Costas Arkolakis, Lorenzo Caliendo, Arnaud Costinot, Jonathan Eaton, Cecilia Fieler, Matthew Grant, Hajin Kim, Joshua Macey, Gib Metcalf, Joseph Shapiro, and Michael Wang for valuable suggestions. Earlier drafts were presented at (in chronological order): the Yale Trade Lunch (October 2019), the Virtual International Trade and Macro Seminar, the Harvard-Berkeley-Yale Virtual Seminar on the Economics of Climate Change and Energy Transition, the Berkeley Agricultural and Resource Economics Seminar, the 2d International Trade Dynamics Workshop, the Chinese Economists Society 2020 North American Virtual Conference, University of Texas at Austin, Boston University, the Ron Jones Lecture at the University of Rochester, and the Cowles Lunch at Yale. Michael Barresi, Jingze Dong, Adaolisa Ezekobe, Tanay Kondiparthy, and Yujia Yao have provided excellent research assistance. We are grateful for financial support from the Tobin Center for Economic Policy and the Becker-Friedman Institute.

†Yale University, Department of Economics, samuel.kortum@yale.edu

‡The University of Chicago Law School, d-weisbach@uchicago.edu

1 Introduction

Current approaches to address climate change vary widely by country and are likely to continue to do so for the indefinite future. Widely varying climate policies may create incentives to shift the location of taxed or regulated activities to jurisdictions with lax policies. Often expressed in terms of leakage—the increase in emissions abroad because of climate policies at home—these effects have the potential to undermine the effectiveness of domestic climate policies.

To better understand the design of regionally differentiated climate policies, we consider the simplest model that captures the core elements of the problem. Our model includes extraction and trade of fossil-fuel energy and an intensive and extensive margin of trade in goods produced with fossil fuels. We solve the model to find the optimal allocation when one region of the world imposes a climate policy and the rest of the world is passive, and then find taxes and subsidies that implement this allocation.

The result of our analysis is a policy that resembles those currently in place or that have been proposed, but that also differs in important ways. In particular, the optimal unilateral policy imposes a carbon price on the supply of fossil fuel energy as well as on the demand for fossil fuels or goods produced with fossil fuels. By doing so, the policy balances supply-side and demand-side effects on the world energy market. It includes a border tax on the carbon content of goods imports that leaves the extensive margin of imports the same as in a no-policy world. It does not, however, include a rebate of taxes for exports that would leave the extensive margin unchanged. Instead, it expands the range of low-carbon exports through subsidies. With these features the policy eliminates carbon leakage.

To compare the optimal policy to existing or commonly proposed policies, we also solve the model with restrictions on the choice of instruments, so that the constrained optimum is similar to policies currently in place (e.g., a price on emissions from domestic production). We then compute a parameterized version of the model for both the optimal policy and the various constrained policies, allowing us to understand their quantitative differences. In our core calibration we assume that OECD countries impose a carbon policy and the rest of the world does not, although we consider a number of other possibilities. We also test the sensitivity of the results to uncertain parameters.

The paper proceeds as follows. The remainder of this section describes our model structure as it relates to the prior literature and previews our key results. Section 2 lays out the model. Section 3 solves the problem of a planner designing an optimal carbon policy for one region with the other region behaving as in a competitive equilibrium. Section 4 derives a set of taxes and subsidies that implement this unilaterally optimal policy. Section 5 derives the taxes that the planner would impose when further constrained in the activities it can control. We explore the quantitative implications of these policies in Section 6, using a calibrated version of the model. Section 7 extends the analysis to include a renewable energy sector. Section 8 concludes.

1.1 Model Structure and Prior Literature

Markusen (1975) provides a starting point for analyzing unilateral climate policies. He considers a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. (A natural interpretation of this good in our context is fossil fuels.) One of the countries, Home, imposes taxes or subsidies to address pollution while the other country, Foreign, is passive. Home's optimal policy in this setting is a Pigouvian tax on the extraction of fossil fuels at a rate equal to domestic marginal harm from the embedded carbon. This tax is combined with either a tariff on imports of fossil fuels (if Home is an importer) or a subsidy on exports (if Home is an exporter), which together can be thought of as a border adjustment. The border adjustment shifts a portion of the tax downstream from extraction of fossil fuels to their use or consumption. A key but often overlooked aspect of Markusen's model is that the border adjustment would typically be at a lower rate than the underlying Pigouvian tax, so that only a portion of the tax is shifted downstream.

The intuition for this result, developed in Hoel (1994) in the climate context, is that the optimal policy combines a tax on the supply of fossil fuels and on the demand for them. By choosing the mix of supply-side and demand-side taxes, Home is able to control the price of the fossil fuels in Foreign. Weisbach et al. (2023) show that the optimal mix depends on the relative supply and demand elasticities for fossil fuels in Foreign, echoing the intuition of Ramsey taxation.

A limitation of Markusen's model for studying climate change is that with only two sectors and no explicit representation of how fossil fuels are used in production of other tradable goods, it cannot capture leakage caused by the shifting of such production to regions with lax policies. This type of leakage, however, is thought to be central to the design of climate policy.[1]

[1]Typical estimates of leakage (as a fraction of policy-induced reductions at home) are in the range of

To allow taxes to affect the location of production and hence generate leakage, there must be production of goods using fossil fuels and trade in those goods, which we represent using Dornbusch, Fisher, Samuelson (1977). The model must also include extraction and trade in energy to allow the carbon policy in one region to affect energy prices elsewhere. To steer clear of terms-of-trade considerations unrelated to climate, we restrict policies to those that do not make Foreign worse off, as in Böhringer, Lange, and Rutherford (2014). To simplify the model, we assume that all energy is a single type of fossil fuel. (In Section 7 we extend the model to multiple types of fossil fuels and clean energy.)

A number of prior models include many of these features. There is a large body of work using CGE models to study regionally-differentiated carbon taxes.[2] These models include a high level of detail and allow the modeler to consider many different policies and outcomes. They require, however, that the modeler choose the policies to study, which means that they cannot tell us which policies are optimal, nor do they provide transparent intuitions for their results. To our knowledge, all of these models constrain the set of policy choices under consideration. Most, consider taxes on production and border adjustments that fully shift those taxes downstream to consumption. We are not aware of any that examine Markusen's recommendations from 50 years ago of combining prices on both sides of the market, that is taxes on extraction as well as on production and consumption.

Hoel (1996), Keen and Kotsogiannis (2014), and Staiger (2022) also include many of these features. Those models, however, restrict the choice of policies the planner is allowed to consider. In particular, they do not allow the planner to control emissions intensity, a common feature of climate policy throughout the world (e.g., the Corporate Average Fuel Economy standards, appliance efficiency standards, and many building codes). Moreover, Keen and Kotsogiannis do not allow Home to control import or export margins, which restricts its ability to choose how it engages in trade. Kruse-Andersen and Sørensen (2022), written contemporaneously with this paper, also includes most of these features but focuses on a case where the planner is not permitted to use border

5% to 25% on an economy-wide basis. For example, Böhringer et al. (2012) commissioned 12 modeling groups to study leakage using common datasets and scenarios. They find leakage in the range of 5% to 19% for a tax in Kyoto Protocol Annex B countries that reduces emissions by about 9.5%. A number of studies examine energy-intensive and trade-exposed industries, where leakage is expected to be the highest (e.g., Fowlie et al (2016)).

[2]Branger and Quirion (2014), Elliott et al. (2013), Zhang (2012), and Metz et al. (2007) all provide surveys of these studies.

adjustments, where the planner separately values reductions in leakage (i.e., not just because leakage reduces the effectiveness of the carbon policy), and where the planner must meet an emissions target considering only Home's emissions. Some of their results are similar to those found here including a subsidy for exports.

To find the optimal unilateral policy, we give a planner control over all activities in Home. These include domestic energy extraction, the use of energy for production in Home (including both intensive and extensive margins), imports and exports (again including intensive and extensive margins), and domestic consumption. The planner cannot *directly* control Foreign extraction, Foreign production of goods for consumption there, and Foreign's choice of what exports to buy from Home. Each activity outside of the planner's control generates a wedge between Foreign's valuation of these items and the planner's valuation, which we describe next.

1.2 Key Results

While we provide explicit results later, it is useful to preview the central intuitions:

1. The optimal unilateral policy implies a set of wedges, all defined to be positive: (i) the wedge between the Foreign energy price and the planner's marginal valuation of extracting a unit of energy (*extraction wedge*); (ii) the wedge between the planner's marginal valuation of a unit of energy and the Foreign energy price (*consumption wedge*); and (iii) wedges, for each good that Home exports, between the shadow cost of producing and delivering it and its marginal utility to Foreign consumers (*export wedges*).

2. The planner balances the extraction wedge (on the supply side) and the consumption and export wedges (on the demand side), taking account of Foreign responses to a change in the energy price. By balancing supply and demand-side effects this way, the planner limits Foreign's incentives to either increase extraction or to increase production or consumption.

3. There is no import wedge: Home's import margin for goods remains unchanged from the no-policy equilibrium and Foreign producers supplying imports must use the same low energy intensity as Home producers.

4. Home's exports are produced using the same energy intensity as goods consumed domestically. Moreover, the export margin expands relative to the no-policy

equilibrium. The policy crowds out Foreign production (for consumption there), replacing it with goods produced with lower energy intensity in Home.

5. The taxes and subsidies that generate this policy in a decentralized equilibrium match up with these wedges: the effective tax on domestic extraction equals the extraction wedge, the tax on energy embodied in domestic production and domestic consumption equals the consumption wedge, and the subsidies for exported goods equal the corresponding export wedges.

6. The policy can be implemented as follows: (i) a domestic carbon tax on the extraction of energy at a nominal rate equal to the global marginal harm from emissions, i.e., at the full Pigouvian rate; (ii) a border adjustment on imports and exports of energy, at a rate equal to the consumption wedge (we call it a "partial border adjustment" because it is at a lower rate than the underlying nominal extraction tax); (iii) the same partial border adjustment on the energy content of goods imports, but not exports; and (iv) subsidies to expand a range of low-carbon exports from Home, equal to export wedges.[3] While the nominal extraction tax equals the Pigouvian rate, the partial border adjustment shifts some of that tax downstream to production or consumption, leaving the effective extraction tax equal to the extraction wedge.

1.3 Comparison to Existing and Proposed Policies

Most existing carbon prices, such as the EU Emissions Trading System (ETS) or California's Cap and Trade Program apply to emissions from domestic production (though the base tends to be incomplete and not include many items such as motor fuels). The EU ETS also includes a border tax on imports (the Carbon Border Adjustment Mechanism or CBAM) but does not remove the carbon price on exports, which means that it effectively applies to emissions from domestic production and emissions associated with domestic consumption (again, on an incomplete base).

The optimal policy includes a CBAM-like system for imports, as in the EU ETS (but not California). Nevertheless, it has two key differences with the EU ETS: (1) the optimal policy splits the carbon price between the supply of fossil fuel (extraction),

[3]The subsidy for exports is reminiscent of the export policy suggested by Fisher and Fox (2012) as it offers export rebates without removing the domestic carbon tax. It differs from Fisher and Fox, however, in that the policy seeks to expand the export margin rather than merely to maintain it.

and the demand for fossil fuel (production and consumption) and (2) like the EU ETS, it leaves the price on exports, but it subsidizes exports to expand the trade margin beyond the business-as-usual margin. Because the EU ETS has no export policy, the export margin will likely contract rather than expand.

Our simulations in Part 6 show that the first of these differences–imposing part of the carbon price on extraction–can make the policy substantially more effective. Moreover, as we discuss in Weisbach and Kortum (2023), imposing part of the tax on extraction involves only ministerial changes to many carbon tax proposals. In particular, rather than imposing a price directly on production, as is done in the EU and California, recent carbon tax proposals in the United States impose a price on domestic extraction and include border border adjustments on imports and exports of energy to shift the tax downstream to production. Lowering the border adjustment rate–that is changing a single number–would leave a portion of the price on extraction, thereby splitting the tax between the supply and demand for fossil fuels, as in the optimal policy. Adding a CBAM-like mechanism would further make these proposals resemble the optimal policy.

2 Basic Model

We divide the world into separate regions, Home and Foreign, treated like two countries. A planner optimizes Home's carbon policy knowing that Foreign responds to market prices with no policy of its own.

Regions are endowed with energy deposits, E and E^*, and labor, L and L^* ($*$ means Foreign). Production takes place in two stages. First, energy is extracted from deposits using labor and is traded at a global price p_e. Second, final goods are produced by combining labor and energy and are then traded subject to frictions. Services, the numéraire, are provided with labor and freely traded. Labor is perfectly mobile across sectors within a region.[4] Energy is used in production and also consumed directly.

As in Dornbusch, Fisher, and Samuelson (1977) (henceforth DFS) goods come in a continuum, $j \in [0, 1]$, with regions specializing in production based on comparative advantage. Welfare in each region is increasing in consumption of energy, goods, and services while decreasing in global energy use, the source of the global externality motivating Home's carbon policy.

[4]What we call labor should be interpreted as a combination of labor and capital, employed in each of the three sectors. In our quantitative application we associate this combination with value added.

2.1 Preferences

Home welfare is additively separable in consumption of services, C_s, consumption of individual goods, c_j, direct consumption of energy, C_e^d, and a climate externality:

$$U = C_s + \int_0^1 u(c_j)dj + v(C_e^d) - \varphi Q_e^W, \tag{1}$$

with u and v increasing concave functions. The parameter φ governs the cost to Home of damages from global carbon emissions, Q_e^W. Foreign preferences, U^*, take the same form, with u^*, v^*, and φ^*.[5] The global social cost of carbon is $\varphi^W = \varphi + \varphi^*$.

2.2 Technology

Deposits of energy differ by the labor required to extract it, with quantity $E(a)$ attainable in Home at a unit labor requirement below a. Tapping all deposits with $a \leq a(Q_e)$ yields $Q_e = E(a(Q_e))$. Foreign extraction from deposits $E^*(a)$ follows the same principle, with $Q_e^W = Q_e + Q_e^*$.[6] Energy is either consumed directly or used as an intermediate input by the goods sector.

Any good j can be produced in quantity q_j by combining energy E_j and labor L_j in a standard production function with constant-returns, so that:

$$q_j = \frac{1}{a_j} f(k_j) L_j, \tag{2}$$

where a_j (a_j^* in Foreign) is the unit input requirement, $k_j = E_j/L_j$ is the energy intensity, and $f(k)$ is the intensive form of the production function, increasing and concave. While k_j may vary, we treat f as common across goods and regions.

Services, in quantitites Q_s and Q_s^*, are provided in both regions at a unit labor requirement of 1.[7]

[5] Our assumption of quasi-linear preferences greatly simplifies the analysis of trade policy, as in Grossman and Helpman (1994). To ensure that the marginal utility of income is 1 we assume $C_s > 0$ and $C_s^* > 0$, a condition that is easily checked.

[6] We equate energy with a homogeneous fossil fuel measured by its carbon content. All energy that is extracted is combusted, so that global carbon emissions equal global extraction. (The exception is in Section 7 where we introduce multiple energy sources, including renewables.) Since energy is costly to extract, we set $E(0) = E^*(0) = 0$. For analytical tractability we assume that $E(a)$ is differentiable, with $E'(a) > 0$ and $E^{*\prime}(a) > 0$.

[7] We assume that the Foreign labor endowment is large enough to guarantee an active services sector, $Q_s^* > 0$. The competitive-equilibrium wage in Foreign is thus $w^* = 1$.

2.3 International Trade

Home and Foreign can trade services for energy at price p_e (quantity of services per unit of energy). Trade in manufactured goods follows DFS. Goods are ordered by Home comparative advantage:

$$\frac{a_j^*}{a_j} = A(j), \tag{3}$$

with $A(j)$ continuous and strictly decreasing, $A(0)$ arbitrarily large, and $A(1) = 0$.[8] Goods trade incurs iceberg costs $\tau \geq 1$ and $\tau^* \geq 1$ so that the overall unit input requirement is τa_j if Home exports good j and $\tau^* a_j^*$ if Home imports it.

2.4 Labor and Energy Requirements

Energy intensity may depend on where a good is produced and on where it will be shipped. We therefore distinguish Home exports, x_j, Home production for domestic consumption, y_j, Home imports, m_j, and Foreign production for domestic consumption, y_j^*, delivering $c_j = y_j + m_j$ and $c_j^* = y_j^* + x_j$. The corresponding energy intensities are k_j^i, for $i \in \{x, y, m, y^*\}$.[9]

Given an energy intensity k to produce good j, we can invert the production function (2) to get labor and energy requirements per unit of output of good j in Home:

$$l_j(k) = a_j/f(k); \qquad e_j(k) = kl_j(k) = \frac{a_j k}{f(k)}. \tag{4}$$

Replacing a_j with a_j^* gives unit input requirements $l_j^*(k)$ and $e_j^*(k)$ in Foreign.[10] These unit labor and energy requirements scale with trade costs for goods shipped abroad.

2.5 Carbon Accounting

Energy embodying carbon is extracted in one or both regions and possibly traded. Carbon is released when the energy is directly consumed (say, heating a house) or

[8] These assumptions on $A(j)$ simplify the analysis of goods trade. To simplify aggregation across goods, we assume that a_j and a_j^* are also continuous functions.

[9] Because Foreign can set k_j^{y*} independently from how it sets k_j^m, we do not include a so-called Brussels effect, as suggested by Bradford (2020).

[10] Since emissions are proportional to energy use, our *unit energy requirement* is sometimes called *emissions intensity* in the environmental economics literature, e.g. Shapiro and Walker (2018). We instead use the terms *energy intensity* or *carbon intensity* for energy per worker, k, by analogy to *capital intensity* for capital per worker in the macroeconomics literature.

used to produce a good. Goods embodying carbon are also traded before the carbon is consumed indirectly by households. We can trace carbon from where it's extracted, to where it's released into the atmosphere, and to where it's directly or indirectly consumed.

We define G_e as direct consumption of energy in Home plus intermediate demand for energy by Home goods producers. Home net exports of energy (Foreign net imports), which may be positive or negative, are $Q_e - G_e = G_e^* - Q_e^*$, accounting for the first level of trade in carbon.

The second level of trade in carbon is embodied in goods. The 2×2 matrix in the upper left of Table 1 depicts the implicit bilateral flows of carbon. Rows indicate the location of consumption and columns the location of production. For example, indirect consumption of carbon in Home is the sum of carbon released by producers in Home serving the local market, C_e^y, and carbon released by Foreign producers in supplying Home imports, C_e^m. The first row of the middle column constructs C_e^m by aggregating over goods the product of (i) the quantity imported by Home, (ii) the unit energy requirement (a function of energy intensity), and (iii) the trade cost. Adding direct consumption, C_e^d, to indirect consumption gives overall consumption, C_e, in Home, shown in the upper right. Just below is overall consumption of carbon in Foreign, C_e^*.

Table 1: Carbon Account

	Home	Foreign	Total
Home	$C_e^y = \int_0^1 e_j(k_j^y)y_j dj$	$C_e^m = \int_0^1 \tau^* e_j^*(k_j^m)m_j dj$	$C_e = C_e^y + C_e^m + C_e^d$
Foreign	$C_e^x = \int_0^1 \tau e_j(k_j^x)x_j dj$	$C_e^{y*} = \int_0^1 e_j^*(k_j^{y*})y_j^* dj$	$C_e^* = C_e^x + C_e^{y*} + C_e^{d*}$
Total	$G_e = C_e^y + C_e^x + C_e^d$	$G_e^* = C_e^m + C_e^{y*} + C_e^{d*}$	$G_e^W = C_e^W = Q_e^W$

2.6 Efficient Production

If p denotes either the global price of energy p_e or the planner's shadow value of energy λ_e (introduced below), the optimal energy intensity for good j is:

$$k(p) = \arg\min_k \{l_j(k) + pe_j(k)\} = \arg\min_k \left\{ \frac{1}{f(k)} + p\frac{k}{f(k)} \right\}, \tag{5}$$

with $k(p)$ decreasing in p. Given p, the same $k(p)$ applies to all goods j in Home, and in Foreign as well, since aside from a_j and a_j^*, the production function is everywhere the same.[11] Unit shadow costs of production are $a_j g(p)$ and $a_j^* g(p)$, where $g(p) = (1 + pk(p))/f(k(p))$. By Shepard's lemma, unit energy requirements are $e_j = a_j g'(p)$ and $e_j^* = a_j^* g'(p)$. Energy cost as a share of production costs is $\epsilon_g(p) = pg'(p)/g(p)$, the elasticity of $g(p)$.[12]

2.7 Business as Usual

The competitive equilibrium of the basic model with no carbon policy, *business as usual* or BAU, is a reference point for the planning problem that follows. In BAU, the relative wage is 1 and the global price of energy is $\bar{p}_e$ (our convention throughout is that $\bar{x}$ is the BAU value of x). Specialization in goods production mimics DFS. Good j is produced at cost $a_j g(\bar{p}_e)$ in Home and $a_j^* g(\bar{p}_e)$ in Foreign so that Foreign's relative cost is $A(j)$. Home consumers buy goods $j < \bar{j}_m$ from local producers and import goods $j > \bar{j}_m$, where $A(\bar{j}_m) = 1/\tau^*$. Foreign consumers buy goods $j > \bar{j}_x$ from local producers and import goods $j < \bar{j}_x$, where $A(\bar{j}_x) = \tau$. Analogs of these import and export thresholds, $\bar{j}_m$ and $\bar{j}_x$, play a key role in the planning problem.

3 Unilateral Planning Problem

A planner allocates the resources that it controls to maximize Home welfare. Outcomes in Foreign (such as how much energy it extracts) are dictated by market prices. The planner is subject to three constraints: (i) labor employed in the three sectors of the economy can't exceed global labor supply; (ii) energy directly consumed plus energy used to produce goods can't exceed global energy supply; and (iii) Foreign can't be made worse off than in BAU.[13] We consider market outcomes in Foreign and the planner's

[11] In their analysis of trade and the environment Copeland and Taylor (1994, 1995) also build on DFS, but make a restriction opposite to ours. Expressed in the notation here, they set $a_j = a$ and $a_j^* = a^*$ while allowing $f_j(k) = k^{\alpha(j)}$ to vary across goods. A hybrid of these two approaches (allowing both efficiency and energy intensity of production to vary across goods) could lead to additional insights but appears intractable.

[12] Given p, this share doesn't vary by good or by where a good is produced. Furthermore, for any good j that is traded, the fraction of energy used in transport is pinned down by iceberg costs only. For a good that Home exports, energy per unit is $\tau a_j g'(\lambda_e)$ with a fraction $(\tau - 1)/\tau$ due to transport. For a good that Home imports, energy per unit is $\tau^* a_j^* g'(p_e)$ with a fraction $(\tau^* - 1)/\tau^*$ due to transport.

[13] This constraint on Foreign welfare requires the planner to search for a Pareto improvement over BAU. We introduce it so that the optimal unilateral policy doesn't manipulate the terms of trade. To

constraints before stating the planning problem.

3.1 Foreign Outcomes

Energy extractors in Foreign hire labor at wage $w^* = 1$ and sell energy at price p_e. They therefore tap all energy deposits with a labor requirement below p_e:

$$Q_e^* = E^*(p_e). \tag{6}$$

Consumers in Foreign buy energy at price p_e, equating their marginal utility from direct consumption of energy to that price:

$$v^{*\prime}(C_e^{d*}) = p_e. \tag{7}$$

Goods producers in Foreign purchase energy at price p_e and hire labor at wage 1. They produce for the domestic market at energy intensity $k_j^{y*} = k(p_e)$, from (5). They can supply good j to consumers there at price $p_j^* = a_j^* g(p_e)$. This price sets an upper bound on the marginal utility of good j in Foreign:

$$u^{*\prime}(c_j^*) = u^{*\prime}(y_j^* + x_j) \leq a_j^* g(p_e) = p_j^*. \tag{8}$$

3.2 Constraints

The global labor constraint, given the endowment $L^W = L + L^*$, is:

$$L_e^W + L_g^W + L_s^W = L^W. \tag{9}$$

Expressions for sectoral employment are in Table 2.

The global energy constraint, given Q_e^* from (6), is:

$$G_e + G_e^* \leq Q_e + Q_e^* = Q_e^W. \tag{10}$$

Expressions for G_e and G_e^* are in Table 1. The Foreign welfare constraint, given BAU

satisfy this constraint, the planner can adjust transfers of services from Home to Foreign, subject to $C_s + C_s^* = Q_s + Q_s^*$. The planner is not constrained by trade balance.

Table 2: Labor Account

	Home	Foreign	Total
Energy	$L_e = \int_0^{a(Q_e)} aE'(a)da$	$L_e^* = \int_0^{p_e} aE^{*\prime}(a)da$	L_e^W
Goods	$L_g = \int_0^1 l_j(k_j^y)y_j + \tau l_j(k_j^x)x_j dj$	$L_g^* = \int_0^1 l_j^*(k_j^{y*})y_j^* + \tau^* l_j^*(k_j^m)m_j dj$	L_g^W
Services	$L_s = Q_s$	$L_s^* = Q_s^*$	L_s^W
Total	$L = L_e + L_g + L_s$	$L^* = L_e^* + L_g^* + L_s^*$	L^W

welfare of $\bar{U}^*$, is:

$$C_s^* + \int_0^1 u^*(c_j^*)dj + v^*(C_e^{d*}) - \varphi^* Q_e^W \geq \bar{U}^*. \tag{11}$$

3.3 Planner's Lagrangian

The planner seeks to maximize Home welfare subject to Foreign outcomes and the constraints (9), (10), and (11). In effect, it maximizes global welfare, $U^W = U + U^*$, transferring services between Home and Foreign to satisfy (11). We use (9) to substitute out $C_s^W = Q_s^W = L_s^W$, apply a Lagrange multiplier λ_e to the global energy constraint (10), and drop the constants L^W and $\bar{U}^*$ to write the planner's Lagrangian as:

$$\begin{aligned}\mathcal{L} = &\int_0^1 u(y_j + m_j)dj + \int_0^1 u^*(y_j^* + x_j)dj + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W \\ &- (L_e^W + L_g^W) - \lambda_e \left(C_e^y + C_e^m + C_e^x + C_e^{y*} + C_e^d + C_e^{d*} - Q_e^W\right).\end{aligned} \tag{12}$$

Elements of L_e^W and L_g^W are in Table 2 while elements of C_e^y, C_e^m, C_e^x, and C_e^{y*} are in Table 1. Foreign outcomes impose (6), (7), (8), and $k_j^{y*} = k(p_e)$.

For the *unilateral* optimum, the planner chooses the quantities of each good that Home consumes and each good that it exports, $\{y_j\}$, $\{m_j\}$, $\{x_j\}$, their energy intensities, $\{k_j^y\}$, $\{k_j^m\}$, $\{k_j^x\}$, energy extraction Q_e, direct consumption of energy C_e^d, and the price of energy, p_e. Choosing p_e amounts to choosing net exports of energy, $Q_e - G_e$, positive or negative. (A lower price p_e corresponds to higher net energy exports by Home.)

The Lagrangian also encompasses a number of other cases, which are determined by the resources that the planner is assumed to control. To derive the *global* optimum, we enlarge the planner's choices to include $\{y_j^*\}$, $\{k_j^{y*}\}$, Q_e^*, and C_e^{d*}.[14] Restricting the planner's choices allows us to derive simpler policies, which we explore in Section 5 and in our simulations.

We solve the unilateral optimum, starting with what Costinot, Donaldson, Vogel and Werning (2015) call the *inner problem*, involving optimality conditions for an individual good given values for Q_e, C_e^d, p_e, and λ_e. We then solve for Q_e, C_e^d, and p_e in what they call the *outer problem*. The Lagrange multiplier adjusts to satisfy the global energy constraint.

The solution can be characterized with various wedges (defined to be non-negative) between how the planner values things relative to the market in Foreign. For example, $\lambda_e - p_e$ is the *consumption wedge* in valuing energy, which is strictly positive in most cases since the planner wants energy to be used sparingly (we provide conditions below). If $\varphi^W = 0$ then wedges vanish and the planner's problem matches BAU.

3.4 Inner Problem

The inner problem, for any arbitrary good j, is to maximize:

$$\begin{aligned}\mathcal{L}_j = & \, u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ & - \left(l_j(k_j^y) + \lambda_e e_j(k_j^y)\right) y_j - \tau^* \left(l_j^*(k_j^m) + \lambda_e e_j^*(k_j^m)\right) m_j \\ & - \tau \left(l_j(k_j^x) + \lambda_e e_j(k_j^x)\right) x_j - \left(l_j^*(k(p_e)) + \lambda_e e_j^*(k(p_e))\right) y_j^*,\end{aligned}$$

subject to (8). Note that energy intensities, $k_j^i \quad i \in \{y, m, x\}$, enter the inner problem as in (5), replacing p with λ_e. It follows that the optimal values are:

$$k_j^i = k(\lambda_e), \quad i \in \{y, m, x\}.$$

The planner chooses the same energy intensity for production of any good consumed in Home (whether produced in Home or Foreign) and for production of any good in Home (whether serving consumers in Home or Foreign). Due to the consumption wedge, it's lower than the energy intensity $k(p_e)$ used by Foreign to serve its own consumers.

The associated unit shadow costs to produce and deliver y_j, m_j, and x_j are $a_j g(\lambda_e)$,

[14] Of course p_e drops out of the problem in this case. Appendix A provides a step-by-step solution.

$\tau^* a_j^* g(\lambda_e)$, and $\tau a_j g(\lambda_e)$, so that the inner problem simplifies to:

$$\begin{aligned}\mathcal{L}_j &= u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ &\quad - a_j g(\lambda_e) y_j - \tau^* a_j^* g(\lambda_e) m_j \\ &\quad - \tau a_j g(\lambda_e) x_j - a_j^* \left(g(p_e) + (\lambda_e - p_e) g'(p_e)\right) y_j^*,\end{aligned} \tag{13}$$

with $a_j^* g'(p_e)$ replacing $e_j^*(k(p_e))$. We maximize (13) to solve for y_j and m_j, and then x_j after imposing (8).

3.4.1 Goods for Home Consumers

The pair of first-order conditions for y_j and m_j are: (i) $u'(y_j + m_j) - a_j g(\lambda_e) \leq 0$, with equality if $y_j > 0$ and (ii) $u'(y_j + m_j) - \tau^* a_j^* g(\lambda_e) \leq 0$, with equality if $m_j > 0$. We define j_m as the good for which these two conditions hold with equality. Using (3), this import threshold satisfies:

$$A(j_m) = \frac{1}{\tau^*}. \tag{14}$$

The import threshold separates goods that Home produces for itself from those that it imports. If $j < j_m$, Home has a lower shadow cost of production, (ii) holds with a strict inequality (so that $m_j = 0$), and (i) holds with equality to determine $y_j > 0$. If $j > j_m$, Foreign has a lower shadow cost of production, (i) holds with a strict inequality (so that $y_j = 0$), and (ii) holds with equality to determine $m_j > 0$. Although Home imports are produced in Foreign, their shadow cost depends on λ_e, not p_e. Hence, j_m equals $\bar{j}_m$, the BAU import threshold.

3.4.2 Goods for Foreign Consumers

Unlike goods for Home consumers, in which the planner chooses both y_j and m_j, for Foreign consumers the planner chooses only x_j subject to the constraint (8).

If this constraint is not binding then $y_j^* = 0$ and x_j solves $u^{*\prime}(x_j) = \tau a_j g(\lambda_e)$. This solution applies to any $j < j_s$, where j_s is the good that Home and Foreign can supply at equal cost to Foreign consumers when energy used by Foreign producers is valued at its market price of p_e. Using (3), this threshold good satisfies:

$$A(j_s) = \tau \frac{g(\lambda_e)}{g(p_e)}. \tag{15}$$

It is apparent (by comparing to BAU) that $j_s \leq \bar{j}_x$, with a strict inequality if $\lambda_e > p_e$.

For any good $j \geq j_s$ the constraint (8) is binding so that Foreign consumption is set by $u^{*\prime}(c_j^*) = a_j^* g(p_e)$. The planner can choose $x_j = c_j^*$ thereby crowding out Foreign production until $y_j^* = 0$. This solution applies to any $j < j_x$, where j_x is the good that Home and Foreign producers can supply at equal cost to Foreign consumers if energy used by Foreign producers is valued at the planners shadow cost of λ_e. Using (3) and the formulation of cost in the last line of (13), this threshold good satisfies:

$$A(j_x) = \tau \frac{g(\lambda_e)}{g(p_e) + (\lambda_e - p_e) g'(p_e)}. \tag{16}$$

For goods $j > j_x$ the planner chooses $x_j = 0$ with $y_j^* = c_j^*$.[15]

As illustrated in Figure 1, the thresholds j_s and j_x divide the continuum of goods into three intervals. Interval 1 is goods $j < j_s$ where Home comparative advantage is strongest. Home exports these goods in quantities that equate Foreign marginal utility to Home's shadow cost of supply. Interval 2 is goods $j_s \leq j < j_x$ for which Home comparative advantage is moderate. Home exports these goods at the minimum quantities sufficient to deter Foreign from producing them for itself. Interval 3 is goods $j > j_x$ for which Home comparative advantage is weakest. Foreign produces these goods for itself.

Interval 2 implies a set of *export wedges*, $s_j = \tau a_j g(\lambda_e) - a_j^* g(p_e)$, between the shadow cost of Home producing and delivering these goods and Foreign's own cost, equal to its marginal utility of consuming them. The export wedge, relative to Foreign production cost, is strictly increasing in j over Interval 2, starting at zero for good j_s.[16]

[15] More formally, when (8) is binding, the derivative of (13) with respect to x_j becomes:

$$\partial \mathcal{L}_j / \partial x_j = -\tau a_j g(\lambda_e) + a_j^* g(p_e) + (\lambda_e - p_e) a_j^* g'(p_e),$$

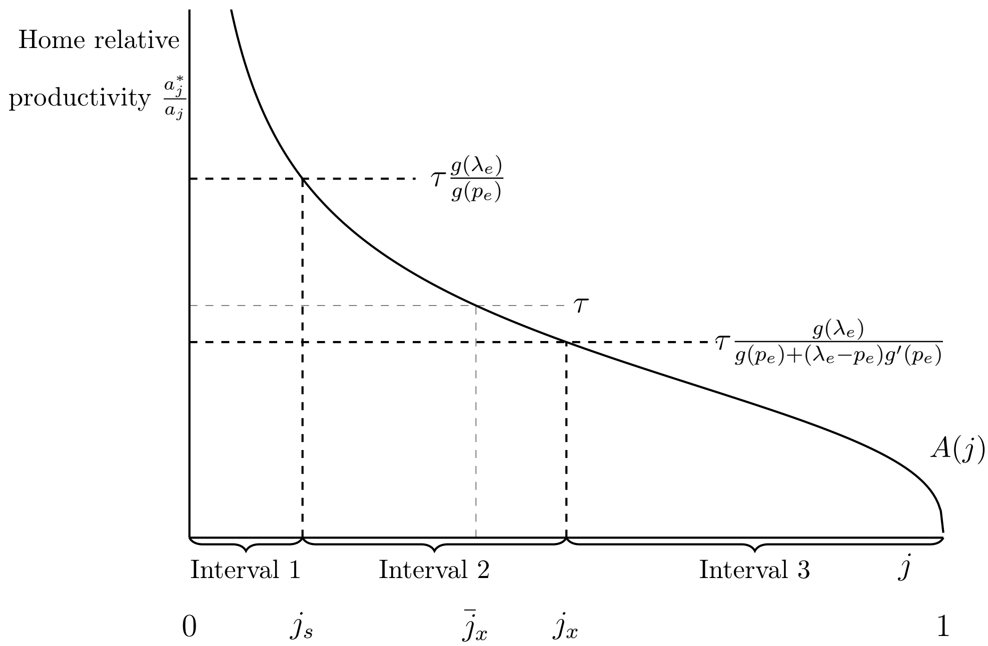
which is positive at $j = j_s$. This derivative is decreasing in j and hits zero when j reaches j_x. Therefore, the inner Lagrangian is maximized at the corner solutions $x_j = c_j^*$ (with $y_j^* = 0$) for $j \in [j_s, j_x)$ and $x_j = 0$ (with $y_j^* = c_j^*$) for $j > j_x$. If $\lambda_e = p_e$ then j_s and j_x both converge to the BAU export threshold of $\bar{j}_x$. Otherwise, for $\lambda_e > p_e$: (i) $(\lambda_e - p_e) g'(p_e) > 0$, which implies $A(j_x) < A(j_s)$ and hence $j_x > j_s$ and (ii) the denominator of (16) exceeds $g(\lambda_e)$, because $g(p)$ is concave, which implies $A(j_x) < \tau = A(\bar{j}_x)$ and hence $j_x > \bar{j}_x$.

[16] The export wedge relative to Foreign cost, for $j \in [j_s, j_x]$, is:

$$\frac{s_j}{a_j^* g(p_e)} = \frac{\tau g(\lambda_e)}{A(j) g(p_e)} - 1,$$

which is strictly increasing in j since $A(j)$ is strictly decreasing in j. Equation (15) implies that this ratio is 0 for good j_s. Equation (16) implies that its maximum value (for good j_x) is $(\lambda_e / p_e - 1)\epsilon_g(p_e)$, Interval 2 disappears if there is no consumption wedge. It emerges for $\lambda_e > p_e$ because the planner wants these goods to be produced with energy intensity $k(\lambda_e)$ rather than with the higher value $k(p_e)$ that Foreign producers would choose. Since the planner can't specify energy intensity in Foreign, it has Home export these goods instead. As a result Home expands its range of exports to j_x, which is beyond j_s and even beyond the BAU export threshold of $\bar{j}_x$.

Figure 1: Goods for Foreign Consumers



This figure illustrates the determination of export thresholds. The y-axis is Home's relative productivity. On the x-axis, interval 1 is the set of goods that Home exports at its cost. Interval 2 is the set of goods that Home exports at Foreign's cost. Interval 3 is the set of goods that Foreign supplies to itself.

Summary Table 3 displays the quantities of goods produced and traded, as determined by the inner problem, with rows indicating the location of consumption and columns indicating the location of production as in Table 1. The terms on the diagonal depend on preferences and costs as expected, but those for Home imports in the upper right and for Home exports in the lower left are unexpected: (i) while Home imports are produced in Foreign, with energy price p_e, the quantity is set as if producers there faced the same shadow cost of energy as producers in Home and (ii) while goods in Interval 2

the product of the ad-valorem consumption wedge and the share of energy in Foreign's production cost for its domestic market.

Table 3: Solution to Inner Problem (Goods-Sector Outcomes)

	Home		Foreign	
Home	$u'(y_j) = a_j g(\lambda_e)$	$j < j_m$	$u'(m_j) = \tau^* a_j^* g(\lambda_e)$	$j > j_m$
Foreign	$u^{*\prime}(x_j) = \begin{cases} \tau a_j g(\lambda_e) & j \leq j_s \\ a_j^* g(p_e) & j_s \leq j < j_x \end{cases}$		$u^{*\prime}(y_j^*) = a_j^* g(p_e)$	$j > j_x$

The thresholds, j_m, j_s, and j_x, are given by equations (14), (15), and (16), with $m_j = 0$ for $j < j_m$, $y_j = 0$ for $j > j_m$, $y_j^* = 0$ for $j < j_x$, and $x_j = 0$ for $j > j_x$. Good $j = j_m$ may be either imported by Home or produced domestically while good $j = j_x$ may be either exported by Home or produced by Foreign. Aggregate outcomes are invariant to this ambiguity.

are produced in Home, the quantities demanded by Foreign consumers depend on what it would cost to produce them in Foreign. In both cases the planner dictates the energy intensity to produce goods based on its shadow value of energy rather than letting them be produced with the higher energy intensity that Foreign producers would have chosen in the absence of the policy.[17]

3.5 Outer Problem

The outer problem is to maximize the Lagrangian (12) with respect to Q_e, C_e^d, and p_e, given the solution to the inner problem.

3.5.1 Extraction and Direct Consumption

The first-order condition for Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi^W - \frac{\partial L_e}{\partial Q_e} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor to extract a bit more energy, $\partial L_e / \partial Q_e$, is the labor requirement for Home's marginal energy deposit, $a(Q_e)$. Applying this result, the

[17]Treating $\lambda_e > p_e$, any good $j \leq j_x$ is produced by Home and exported to Foreign using less energy than if it were produced in Foreign. It's sufficient to demonstrate this result for good j_x. By the concavity of $g(p)$ we have $g(p_e) + g'(\lambda_e)(\lambda_e - p_e) < g(\lambda_e)$. Since $g'(\lambda_e) < g'(p_e)$ we have $g(p_e)g'(\lambda_e)/g'(p_e) + g'(\lambda_e)(\lambda_e - p_e) < g(\lambda_e)$. Hence $g'(\lambda_e)/g'(p_e) < g(\lambda_e)/(g(p_e) + g'(p_e)(\lambda_e - p_e))$. Multipling both sides by τ implies $\tau g'(\lambda_e)/g'(p_e) < A(j_x)$. Cross multiplication then gives the result that $a_{j_x} \tau g'(\lambda_e) < a_{j_x}^* g'(p_e)$.

first-order condition simplifies to:

$$Q_e = E\left(\lambda_e - \varphi^W\right), \tag{17}$$

for $\lambda_e - \varphi^W \geq 0$, with $Q_e = 0$ otherwise. This condition mimics (6), with $\lambda_e - \varphi^W$ in place of p_e. We refer to this gap, $p_e - (\lambda_e - \varphi^W)$, as the *extraction wedge*.

The first-order condition for C_e^d mimics (7), with λ_e replacing p_e:

$$v'(C_e^d) = \lambda_e. \tag{18}$$

3.5.2 Energy Price

The first-order condition for p_e is more intricate:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0,$$

where terms optimized in the inner problem dropped out due to the envelope condition. As shown in Appendix B the first-order condition simplifies to:

$$\left[p_e - (\lambda_e - \varphi^W)\right] \frac{\partial Q_e^*}{\partial p_e} + \left[\lambda_e - p_e\right] \frac{\partial C_e^{\tilde{y}*}}{\partial p_e} + \int_{j_s}^{j_x} \left[s_j\right] \frac{\partial x_j}{\partial p_e} dj = 0, \tag{19}$$

where $C_e^{\tilde{y}*} = C_e^{y*} + C_e^{d*}$.

Equation (19) balances different wedges introduced above (in square brackets), taking account of Foreign responses to a change in p_e of energy supply ($\partial Q_e^*/\partial p_e > 0$), energy demand ($\partial C_e^{\tilde{y}*}/\partial p_e < 0$), and export demand ($\partial x_j/\partial p_e < 0$). Wedges capture the welfare consequences of each Foreign response: (i) the extraction wedge between the value that the planner and Foreign place on extracting a unit of energy, (ii) the consumption wedge between the value that the planner and Foreign place on using a unit of energy, and (iii) the export wedges between the shadow cost to the planner and Foreign of supplying each good in Interval 2 to Foreign consumers. All three wedges are positive: (i) Foreign puts greater value on extraction since it ignores the global externality, (ii) once extracted, the planner values energy more highly to conserve on its use, and (iii) the planner tolerates a higher cost of supplying Foreign in order to crowd out Foreign production of goods in Interval 2.

If the planner raises the energy price (by reducing Home net exports of energy) the extraction wedge rises and the consumption and export wedges fall. Therefore, if the left side of (19) is negative, the planner increases p_e to rebalance it. If the left side of (19) is positive, the planner reduces p_e to rebalance it. When (19) is balanced, the energy price lies in the interval $(\lambda_e - \varphi^W, \lambda_e]$, with $p_e < \lambda_e$ if $\varphi^W \partial Q_e^* / \partial p_e > 0$.[18]

3.6 Properties of the Solution

Taking stock: (i) the inner problem gives C_e^y, C_e^m, C_e^x, and C_e^{y*} in terms of p_e and λ_e; (ii) equations (6) and (17) give Q_e^* and Q_e as functions of p_e and λ_e; (iii) equations (7) and (18) give C_e^{d*} and C_e^d as functions of p_e and λ_e; and (iv) equation (19) and (10) nail down p_e and λ_e. We now turn to the optimal wedges that characterize the solution.

Pigouvian Wedge Adding the extraction and consumption wedges yields the Pigouvian wedge, φ^W, i.e. the global social cost of carbon. It's the wedge between the planner's valuation of energy and its valuation for extracting energy. If the planner could control outcomes in Foreign, it would impose this Pigouvian wedge there too (see Appendix A). But, a unilateral policy can't achieve a globally harmonized wedge. Instead, the planner splits the Pigouvian wedge into extraction and consumption wedges based on Foreign demand and supply responses to the global energy price.

Balancing Extraction and Consumption Wedges The planner chooses a consumption wedge, $\lambda_e - p_e$, in the interval $[0, \varphi^W)$. It approaches φ^W if $\partial Q_e^* / \partial p_e$ is large relative to $\left|\partial C_e^{\tilde{y}*} / \partial p_e\right|$ resulting in a low value of p_e to limit Foreign extraction of energy. The extraction wedge, $p_e - (\lambda_e - \varphi^W) = \varphi^W - (\lambda_e - p_e)$, then approaches 0. At the other extreme, with perfectly inelastic Foreign supply, the planner chooses a high price to limit Foreign energy demand. The extraction wedge then equals the Pigouvian wedge and the consumption wedge is 0. In this case, the unilateral policy achieves the global optimum.[19]

[18] If $\varphi^W > 0$ then equation (19) can't be satisfied by $p_e \leq \lambda_e - \varphi^W$ as then the sum of the left-hand-side terms would be strictly negative. Likewise, if $p_e > \lambda_e$ then the inner problem would give $j_s = j_x < \bar{j}_x$, eliminating the integral in (19), so that all terms on the left side of (19) would be strictly positive. Hence, a solution to (19) implies $p_e \in (\lambda_e - \varphi^W, \lambda_e]$. If $\varphi^W \partial Q_e^* / \partial p_e = 0$ then $p_e = \lambda_e$ solves (19), with $j_s = j_x = \bar{j}_x$. If $\varphi^W \partial Q_e^* / \partial p_e > 0$, then balancing (19) requires at least one strictly negative term, hence $p_e < \lambda_e$.

[19] Following this logic, Harstad (2012) makes a case that the policy maker buy marginal energy deposits from Foreign to create a locally vertical Foreign supply curve. We have ruled out such an international market in Foreign energy deposits in our analysis here.

Export Wedges and Crosshauling The export threshold, j_x, is greater than in BAU when $\lambda_e > p_e$. With energy highly valued by the planner, global resources are saved by having goods in Interval 2 produced in Home rather than in Foreign. Export wedges emerge since these goods must be delivered to Foreign consumers at a price no higher than if they were produced in Foreign. Under the optimal policy there may be a set of goods that Home simultaneously imports and exports. Such crosshauling always occurs in the absence of trade costs as then $A(j_m) = 1/\tau^* = 1$ and $A(j_x) < 1$ implying $j_x > j_m$. With high enough trade costs, however, $A(j_x) > A(j_m)$ so that $j_x < j_m$. The inherent inefficiency of crosshauling then outweighs its advantage in crowding out Foreign production.

4 Optimal Taxes and Subsidies

We can now describe a set of taxes and subsidies that deliver the optimal unilateral policy as the outcome of a competitive equilibrium. In the resulting market economy, recall that services are the numéraire and the labor requirement for services pins the wage to 1 in both regions.

4.1 A Simple Implementation

We consider implementing the unilaterally optimal policy with three interventions, which correspond to different wedges:

1. Tax energy extraction in Home at *nominal* rate $t_e^N = \varphi^W$ (Pigouvian wedge).
2. Impose border adjustments on Home energy trade, and on energy embedded in Home imports of goods, at rate $t_b = \lambda_e - p_e$ (consumption wedge).
3. Subsidize Home exports of each good in Interval 2 at rate s_j (export wedges).

There are three *effective* carbon taxes implicit in this policy. The first is a tax on Home extraction, $t_e = t_e^N - t_b = \varphi^W - (\lambda_e - p_e)$ (extraction wedge). The second is a tax on carbon embodied in Home consumption, including imports, $t_c = t_b = \lambda_e - p_e$ (consumption wedge). The third is a tax on carbon used in Home production, including exports, $t_p = t_b = \lambda_e - p_e$ (consumption wedge). (Home production and consumption overlap when Home produces for itself, in which case only one tax is applied.) The per-unit subsidy to exports in Interval 2 is layered on top of these three carbon taxes.[20]

This set of taxes and subsidies is not the only way to implement the unilaterally optimal policy, but we find it the easiest to interpret.[21]

4.2 After-Tax Prices

Net prices faced by the various agents in the global economy are as follows:

1. The global price of energy, p_e, is paid by users of energy in Foreign and is received by energy extractors in Foreign.

2. If energy is imported by Home, it is subject to a border adjustment t_b, raising the price of energy for users in Home to $p_e + t_b$.

3. Energy extractors in Home sell energy domestically at price $p_e + t_b$, so after paying the extraction tax they net $p_e + t_b - t_e^N = p_e - t_e$.

4. Extractors in Home export energy at price p_e and get a rebate of t_b on the nominal extraction tax, so also net $p_e - t_e^N + t_b = p_e - t_e$.

5. Goods $j < j_m$ are produced in Home, using energy costing $p_e + t_b$, so that local consumers pay $p_j = a_j g(p_e + t_b)$.

6. Goods $j > j_m$ are imported by Home. Anticipating the border adjustment, Foreign produces these goods with energy intensity $k(p_e + t_b)$. The production cost, including the trade cost, is $\tau^* l_j^*(k(p_e + t_b)) + p_e \tau^* e_j^*(k(p_e + t_b))$. With the border adjustment, $t_b \tau^* e_j^*(k(p_e + t_b))$, consumers pay $p_j^m = \tau^* a_j^* g(p_e + t_b)$.

[20]This policy differs from that discussed in Staiger (2022) for three reasons. First, we assume that Foreign has no carbon policy. Second, we force Home's policy to generate a global Pareto improvement. Third, and more subtle, in our model carbon intensity is endogenous. As a consequence, border adjustments on goods must be imposed per unit of embodied carbon. Furthermore, this carbon tax is imposed on Home's exporters so that they choose the optimal carbon intensity. The per-unit subsidy for exporters doesn't undo this incentive, a result reminiscent of Fischer and Fox (2012).

[21]We can also implement the unilaterally optimal policy starting from a tax on emissions rather than a tax on extraction: (1) tax energy used for production of goods in Home (and on direct consumption of energy in Home) at *nominal* rate $t_p^N = \varphi^W$ (Pigouvian wedge); (2) impose border adjustments on energy embodied in Home imports of goods at rate $t_c = t_b = \lambda_e - p_e$ (consumption wedge); and (3) subsidize energy imports and tax energy exports at rate $t_e = t_p^N - t_b$ (extraction wedge). The export subsidy for goods in Interval 2 remains s_j (export wedge). This policy works because the subsidy to energy imports and tax on energy exports act as a tax on Home extractors. The resulting *effective* production tax is $t_p = t_p^N - t_e = t_b = \lambda_e - p_e$ (consumption wedge).

7. Goods $j < j_s$ in Interval 1 are produced in Home and exported. The producers use energy costing $p_e + t_b$, with no border adjustment for exports. The price in Foreign, including the trade cost, is $p_j^x = \tau a_j g(p_e + t_b)$.

8. Goods $j \in [j_s, j_x)$ in Interval 2 are also exported by Home. The producers use energy costing $p_e + t_b$, with no border adjustment for exports. They sell at price $p_j^* = a_j^* g(p_e)$ in Foreign. Producers get a subsidy from Home of s_j per unit, so they net $p_j^* + s_j = \tau a_j g(p_e + t_b)$, which covers their cost.

9. Goods $j > j_x$ in Interval 3 are produced in Foreign, using energy at price p_e. They are sold to local consumers at price $p_j^* = a_j^* g(p_e)$.

5 Constrained Optimal Policies

As shown above, the optimal unilateral policy can be implemented by combining carbon taxes on Home extraction, consumption, and production, together with subsidies for a range of goods (Interval 2) that Home exports. To assess this policy, we compare it to more conventional polices that involve some subset of these taxes with no subsidy for exports. We solve for the optimal version of each constrained policy following the approach we took for the optimal unilateral policy (there are seven constrained policies, one for each subset of the three types of taxes).[22]

For constrained policies involving an extraction tax, the tax rate on extraction equals the extraction wedge, $t_e = \varphi^W - (\lambda_e - p_e)$. For constrained policies involving a consumption tax, the tax rate on consumption equals the consumption wedge, $t_c = \lambda_e - p_e$. These results mimic the optimal unilateral policy. For constrained policies involving a production tax, the tax rate t_p is below the consumption wedge, unlike the optimal unilateral policy. With no export subsidy, a production tax induces *leakage* (increased energy use by Foreign) which reduces its efficacy.

Consider a constrained policy that includes a production tax but not a consumption tax. Let $v_e = p_e + t_p$ be the cost of energy to producers in Home. Raising v_e causes consumers in Home and in Foreign to substitute away, on both the extensive and

[22]We provide additional intuition for these conventional policies in Weisbach et al. (2023). There we solve the dual problem to get conditions for optimal tax rates directly (given the taxes allowed in each case). Here, to facilitate comparison to the optimal unilateral policy, we solve the primal problem to characterize the constrained-optimal allocation and then derive conditions for taxes that implement each allocation. Full solutions to the Lagrangian for each case are shown in Appendix C.

intensive margins, from goods produced in Home. The extensive-margin response is governed by the slope of the comparative advantage curve, $A(j)$, near the trade thresholds, with a flatter slope implying more leakage. Figure 2 illustrates the leftward shift in the two extensive margins. Relative to BAU, both j_m and j_x decline, in contrast to the optimal unilateral policy in which j_m doesn't change and j_x rises.

Leakage, on the margin, is (minus) the increase in emissions (here, energy use) in Foreign, relative to the decrease in Home, from a change in v_e:[23]

$$\Lambda = -\frac{\partial G_e^*/\partial v_e}{\partial G_e/\partial v_e}.$$

The planner optimizes, accounting for leakage, by setting the cost of energy in Home to $v_e = \Lambda p_e + (1 - \Lambda)\lambda_e$, closer to p_e if leakage is higher, and closer to λ_e if leakage is lower. The resulting production tax is therefore $t_p = (1 - \Lambda)(\lambda_e - p_e)$. The production tax rate, with no leakage, is equal to the consumption wedge. But, it declines linearly with leakage, and with 100% production leakage the optimal production tax is 0. These results continue to apply if the production tax is combined with an extraction tax.[24]

If the constrained policy includes a consumption tax then v_e applies only to Home production for export. Leakage takes place only via Foreign consumers substituting (on both the extensive and intensive margins) away from goods produced in Home. The expression for leakage becomes:

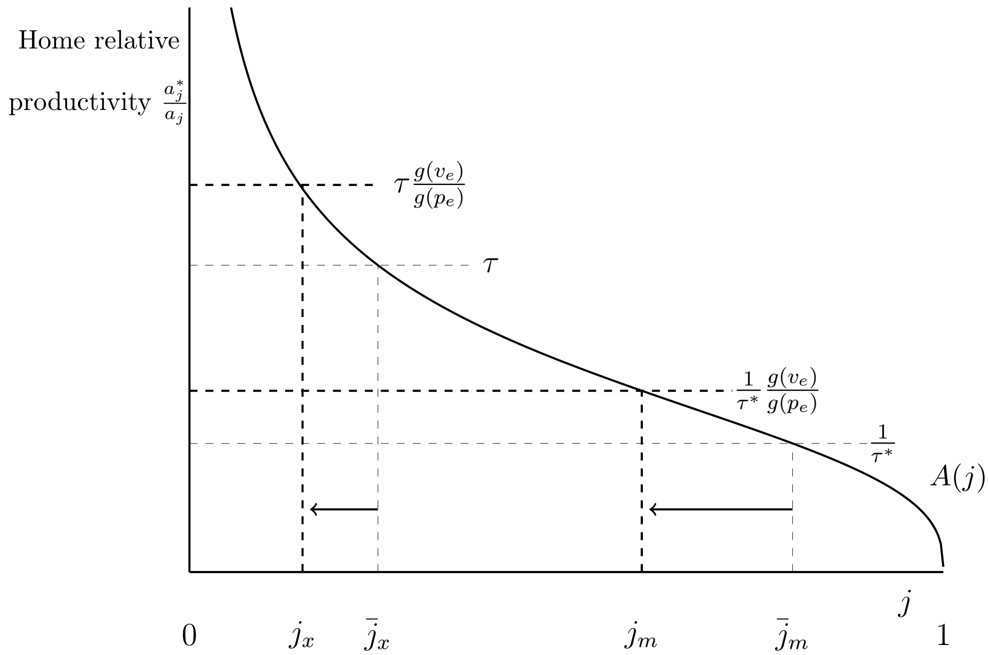
$$\Lambda^* = -\frac{\partial C_e^{y*}/\partial v_e}{\partial C_e^x/\partial v_e},$$

which we call "foreign leakage" to distinguish from conventional leakage. Unlike conventional leakage, foreign leakage does not include substitution by Home consumers away from Home production (since the cost of energy to supply Home, whether by

[23]Note that many analyses (e.g., Böhringer, Lange, and Rutherford (2014)) consider two channels of leakage. The first, referred to as the fuel price effect, is the change in Foreign emissions due to the change in p_e. The second, referred to as the competitiveness channel, refers to the change in the location of production due to the change in the relative price of energy in Foreign and Home. Our definition of leakage is limited to the latter. All of the policies we consider, including the unilaterally optimal policy, contend with the fuel-price effect.

[24]While an extraction/production hybrid tax must contend with production leakage, it has an offsetting advantage over other policies: it can be implemented simply and accurately. To implement this tax, Home could impose a nominal extraction tax of $t_e^N = t_e + t_p = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p$ and border adjustments on energy (but not on goods) at rate $t_b = t_p$. By avoiding border adjustments on goods, the tax avoids the need to measure the marginal emissions from the production of goods in foreign countries, which is the key problem in imposing border adjustments, see Kortum and Weisbach (2016).

Figure 2: Import/Export Thresholds under Pure Production Tax



This figure illustrates the determination of import and export thresholds with a production tax. The axes are as in Figure 1. For a policy that includes a production tax but not a consumption tax, both the import and export thresholds decrease, as shown. If a consumption tax is also part of the policy, the import threshold remains unchanged at $j_m = \bar{j}_m$.

domestic producers or imports, is $\lambda_e = p_e + t_c$). In this scenario, Figure 2 would show $j_m = \bar{j}_m$ since $(1/\tau^*)g(v_e)/g(p_e)$ would be replaced by $(1/\tau^*)g(\lambda_e)/g(\lambda_e) = 1/\tau^*$ to determine the import threshold. The planner optimizes, taking account of foreign leakage, by setting the cost of energy for Home exports to $v_e = \Lambda^* p_e + (1 - \Lambda^*)\lambda_e$. The resulting production tax, now applying only to energy used in Home exports, becomes: $t_p = (1 - \Lambda^*)(\lambda_e - p_e)$.

We have explained how each tax rate is set for each of the seven constrained policies, given the extraction wedge, the consumption wedge, and the appropriate leakage rate. The consumption wedge itself is based on the first-order condition for p_e, analogous to equation (19), but without export wedges since they are absent in all the constrained policies. The consumption wedge adjusts based on the supply and demand responses to the energy price. Supply and demand responses vary with the constrained policy being considered, hence so do the associated consumption wedges. Appendix C provides all derivations, with results summarized in Table 6.

6 Quantitative Illustration

We now calibrate the model, compute the optimal unilateral policy, explore its quantitative implications, and compare it to constrained optimal policies. Our computational strategy exploits convenient functional forms and conditions on observed global carbon flows in 2018, which we interpret as BAU outcomes.

Preferences We assume constant-elasticity demand, based on:

$$u(c_j) = \eta^{1/\sigma} \frac{c_j^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where η governs demand for goods and σ is the elasticity of substitution between goods. In aggregate, $\int_0^1 u(c_j)dj = u(C_g)$, where $C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}$ is total goods consumption. Foreign demand takes the same form, with parameters η^* and $\sigma^* = \sigma$.[25]

Demand for direct consumption of energy, based on $v(C_e^d)$, takes the same form, with parameters η_e and σ_e (in Foreign η_e^* and $\sigma_e^* = \sigma_e$).

Energy Supply We assume constant extraction elasticities, ϵ_S and ϵ_S^*, so that:

$$E(a) = Ea^{\epsilon_S}; \qquad E^*(a) = E^* a^{\epsilon_S^*},$$

where the E's govern the mass of energy deposits in each region.

Goods Production We assume a CES production function:

$$f(k) = (1 - \alpha + \alpha k^{(\rho-1)/\rho})^{\rho/(\rho-1)},$$

with elasticity of substitution ρ and energy shift parameter α.[26]

[25]If $\sigma \to 1$ we get $u(C_g) = \eta \ln C_g = \eta \int_0^1 \ln c_j dj$ (and likewise in Foreign).

[26]From (5), energy intensity is $k(p) = \tilde{\alpha} p^{-\rho}$ and $g(p) = (1-\alpha)^{\rho/(1-\rho)} \left(1 + \tilde{\alpha} p^{1-\rho}\right)^{1/(1-\rho)}$, where $\tilde{\alpha} = (\alpha/(1-\alpha))^{\rho}$. The energy share is thus $\epsilon_g(p) = \tilde{\alpha} p^{1-\rho} / \left(1 + \tilde{\alpha} p^{1-\rho}\right)$. In the Cobb-Douglas limit ($\rho \to 1$): $k(p) = (\alpha/(1-\alpha))p^{-1}$, $g(p) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} p^{\alpha}$, and $\epsilon_g(p) = \alpha$.

Comparative Advantage We parameterize input requirements for goods production in each region by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}; \qquad a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta},$$

where the A's govern absolute advantage and θ comparative advantage. Hence:

$$A(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta}.$$

This formulation has the convenient property that in BAU, the share of spending on imported goods is proportional to the measure of goods imported.

6.1 Counterfactual Policies

We capture the key counterfactual outcomes under the optimal unilateral policy in the vector $\boldsymbol{z}$. Its elements include carbon flows (energy extraction and the elements of Table 1), energy cost shares (which depend on the carbon price), and carbon-price variables (p_e and t_b). The structure of the model delivers these counterfactual outcomes succinctly in terms of their BAU values, $\bar{\boldsymbol{z}}$, so that:

$$\boldsymbol{z} = F(\bar{\boldsymbol{z}}; \boldsymbol{\Theta}),$$

where $\boldsymbol{\Theta}$ is a vector of parameters.[27] We describe the mapping F in what follows, with some details relegated to Appendix D.1.

Energy extraction is particularly simple to compute given carbon-price variables:

$$Q_e = \left(\frac{p_e + t_b - \varphi^W}{\bar{p}_e}\right)^{\epsilon_S} \bar{Q}_e; \qquad Q_e^* = \left(\frac{p_e}{\bar{p}_e}\right)^{\epsilon_S^*} \bar{Q}_e^*,$$

where $Q_e = 0$ if $p_e + t_b - \varphi^W \leq 0$. The E's disappear.

Given p (either p_e or $p_e + t_b$) we can use results in footnote 26 to compute the energy

[27] The strategy follows Dekle, Eaton, and Kortum (2007). For any individual outcome (or function of outcomes) x we let $\bar{x}$ be its value in BAU, x its value under a counterfactual carbon policy, $\hat{x} = x/\bar{x}$, and $\Delta x = x - \bar{x}$. In this application, by matching the BAU value of key outcomes to data, we ignore any existing carbon policies. Such policies were quite limited as of 2018, our base year for the calibration.

cost share in terms of its BAU value, $\bar{\epsilon}_g = \epsilon_g(\bar{p}_e)$:

$$\epsilon_g(p) = \frac{(p/\bar{p}_e)^{1-\rho}\bar{\epsilon}_g}{(p/\bar{p}_e)^{1-\rho}\bar{\epsilon}_g + (1-\bar{\epsilon}_g)}. \tag{20}$$

The parameter α disappears.

The optimal export threshold can also be expressed in terms of its BAU value:

$$j_x = \frac{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x}{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x + \hat{g}(p_e)^{-\theta}\left(1+(t_b/p_e)\epsilon_g(p_e)\right)^{-\theta}(1-\bar{j}_x)},$$

where $\hat{g}(p) = g(p)/g(\bar{p}_e) = \left(\bar{\epsilon}_g(p/\bar{p}_e)^{1-\rho} + 1 - \bar{\epsilon}_g\right)^{1/(1-\rho)}$. In BAU every good is produced with the same energy share $\bar{\epsilon}_g$ so that, exploiting the special property of $A(j)$, the BAU export threshold is simply $\bar{j}_x = \bar{C}_e^x/(\bar{C}_e^x + \bar{C}_e^{y*})$. Similarly, the BAU import threshold satisfies $1-\bar{j}_m = \bar{C}_e^m/(\bar{C}_e^y + \bar{C}_e^m)$, with $j_m = \bar{j}_m$.[28] The A's and τ's disappear.

Consumption of energy in Foreign from Foreign production is given by:

$$C_e^{y*} = \hat{D}(p_e)\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1+(1-\sigma)/\theta}\bar{C}_e^{y*},$$

where $\hat{D}(p) = D(p)/D(\bar{p}_e) = \hat{g}(p)^{\rho-\sigma}(p/\bar{p}_e)^{-\rho}$. The term $D(p)$ relates producers' demand for energy to its cost with an elasticity, $\epsilon_D(p) = \epsilon_g(p)\sigma + (1-\epsilon_g(p))\rho$, governed by the elasticity of demand for goods and the elasticity of substitution in production.[29] Direct consumption of energy in Foreign is $C_e^{d*} = (p_e/\bar{p}_e)^{-\sigma_e}\bar{C}_e^{d*}$, with elasticity σ_e. In these expressions for energy demand (a complete list is in Appendix Table 10) the η's and η_e's disappear.

Equation (19) delivers an expression for the border adjustment (consumption wedge).

[28]The mapping from BAU trade thresholds to observed carbon flows would be more intricate if we introduced intermediate goods. Suppose composite intermediates for Home and Foreign production, constructed like C_g, entered goods production with a Cobb-Douglas share α_n. We would then have:

$$(1-\bar{j}_m) = [1-\alpha_n(\bar{j}_m - \bar{j}_x)]\frac{\bar{C}_e^m}{\bar{C}_e^y + \bar{C}_e^m}; \quad \bar{j}_x = [1-\alpha_n(\bar{j}_m - \bar{j}_x)]\frac{\bar{C}_e^x}{\bar{C}_e^x + \bar{C}_e^{y*}},$$

where $\alpha_n(\bar{j}_m - \bar{j}_x)$ is the share of endogenously non-traded intermediates embodied in the production of Home's imports and exports. Since the data in Table 4 account for carbon embodied in intermediates, generalizing the model along these lines is a priority for future work.

[29]Here $D(p) = g'(p)g(p)^{-\sigma}$ and $\epsilon_D(p) = -pD'(p)/D(p)$. By the envelope condition $\epsilon_D(p_e)$ is the price elasticity of C_e^{y*}, since j_x is fixed with respect to deviations of p_e from its optimum. In the Cobb-Douglas limit ($\rho \to 1$) this elasticity is constant, $\epsilon_D = \sigma\bar{\epsilon}_g + 1 - \bar{\epsilon}_g$. This limit also implies $\hat{g}(p) = (p/\bar{p}_e)^{\bar{\epsilon}_g}$.

Observing that constant-elasticity preferences imply $\partial x_j / \partial p_e = -\sigma x_j \epsilon_g(p_e)/p_e$, defining $S = \int_{j_s}^{j_x} s_j x_j dj$, and employing elasticities introduced above, it simplies to:

$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^* - \sigma \epsilon_g(p_e) S}{\epsilon_S^* Q_e^* + \epsilon_D(p_e) C_e^{y*} + \sigma_e C_e^{d*}}. \tag{21}$$

The optimal border adjustment along with the energy price jointly satisfy (21) and clear the global energy market, $C_e^W = Q_e^W$.

All the expressions for computation shown above are designed for the unilaterally optimal policy. The minor modifications required to compute constrained optimal policies are described in Appendix D.2.

Home's welfare gain under an optimal policy (either the unilaterally optimal policy or a constrained optimal policy) can be measured as equivalent variation. Starting from BAU, raising income in Home by $EV = \Delta U = U - \bar{U}$ (which would be spent on consumption of services) delivers the same welfare gain as implementing the optimal policy. Since Foreign welfare is fixed, we can compute EV as the increase in global welfare, as shown in Appendix D.1, from elements of $\boldsymbol{z}$.

For a high-level perspective on the counterfactual results in Section 6.3, we will illustrate the trade-off between global emissions reductions ($-\Delta Q_e^W$) and reductions in global consumption, measuring the change in global consumption from an optimal policy as $\Delta C^W = EV + \varphi^W \Delta Q_e^W$ (where $\Delta C^W < 0$ if $\varphi^W > 0$). For a given policy, and fixed φ^W, we can plot $-\Delta Q_e^W$ on the vertical axis and ΔC^W on the horizontal. Varying φ^W traces out the frontier of possibilities for global emissions reductions and associated declines in global consumption for a given policy. The slope of this frontier is $-1/\varphi^W$ at the optimum. The frontier for a constrained optimal policy will, by definition, lie below the frontier for the optimal unilateral policy.

6.2 Calibration

We calibrate $\bar{\boldsymbol{z}}$ to the data shown in Table 4, a measure of the BAU energy price, and a measure of energy as a share of production costs in BAU. The carbon flow data for 2018 come from two sources. Energy extraction is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of fuels of different types to gigatons of CO_2. Indirect and direct consumption is from the Trade

Table 4: Carbon Flows: Home as the OECD

	Home	Foreign	Direct	Total
Home	$\bar{C}_e^y = 8.7$	$\bar{C}_e^m = 2.5$	$\bar{C}_e^d = 2.5$	$\bar{C}_e = 13.7$
Foreign	$\bar{C}_e^x = 1.0$	$\bar{C}_e^{y*} = 16.7$	$\bar{C}_e^{d*} = 2.2$	$\bar{C}_e^* = 19.9$
Direct	$\bar{C}_e^d = 2.5$	$\bar{C}_e^{d*} = 2.2$		
Total	$\bar{G}_e = 12.2$	$\bar{G}_e^* = 21.4$		$\bar{C}_e^W = 33.6$
Extraction	$\bar{Q}_e = 9.3$	$\bar{Q}_e^* = 24.3$		$\bar{Q}_e^W = 33.6$

The table shows carbon flows in 2018 from OECD countries (Home) to all other countries (Foreign), measured as gigatons of CO_2. Data construction is described in more detail in Appendix E.

Embodied in CO_2 ($TECO_2$) database, made available by the OECD.[30] For most of our results, members of the OECD form the taxing region, or Home, and non-OECD countries are Foreign, as in Table 4. By this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

We take the BAU energy price to be $\bar{p}_e = \$167$ per ton of CO_2.[31] The definition of BAU implies $\bar{t}_b = 0$. We compute the BAU energy share in cost to be $\bar{\epsilon}_g = 0.22$.[32] It follows that the global social cost of carbon φ^W is measured as dollars per ton of CO_2 (hence billions of dollars per gigaton) in 2018.

In addition to $\bar{\boldsymbol{z}}$ we need values for $\boldsymbol{\Theta} = \left\{\varphi^W, \theta, \sigma_e, \sigma, \rho, \epsilon_S, \epsilon_S^*\right\}$ in order to compute model outcomes $\boldsymbol{z}$. We calibrate these seven parameters from a variety of sources.

Under the optimal unilateral policy the sum of the extraction tax and the border adjustment equals the global social cost of carbon φ^W. We consider a range of values from 0 to 400 dollars per ton of CO_2. The EPA currently applies a value of \$190 for

[30]The values that we take from $TECO_2$ are broadly consistent with those available from the Global Carbon Project. Further details on the data construction are described in Appendix E.

[31]The Federal Reserve Economic Data (FRED) reports the 2018 global price per barrel of Brent Crude as \$71.6. The EPA Greenhouse Gases Equivalencies Calculator reports 0.43 metric tons of CO_2 per barrel of crude oil. Hence, $\bar{p}_e = 71.6/0.43 = 166.5$.

[32]The United Nations National Accounts reports a world total value added in agriculture plus manufacturing (what we consider the goods sector) of $\$17,364$ billion in 2018. From Table 4, total energy used in production is 28.9 gigatons of CO_2. Pricing it at \$167 per ton gives a value of \$4812 billion. Hence, $\bar{\epsilon}_g = 4812/(17,364 + 4812) = 0.217$.

CO_2 emissions in 2020.

A higher θ eases substitution between Home and Foreign production, raising the potential for carbon leakage. We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). We also consider a much higher value of $\theta = 16$ to see how export subsidies under the optimal unilateral policy respond to higher potential leakage.

The parameter σ_e is the elasticity of demand for direct consumption of energy. Most studies find this elasticity to be quite small. Addressing the endogeneity of energy prices, that bias estimates toward zero, Davis and Kilian (2011) use state-level variation in gas taxes to estimate $\sigma_e = 0.7$. They acknowledge that the long-run elasticity is likely larger, so we choose a slightly higher value of $\sigma_e = 1$.

The parameter σ is both the elasticity of substitution across goods and the elasticity of demand for aggregate goods consumption, C_g. Using both individual and aggregate data while taking account of non-homothetic demand for goods relative to services, estimates by Boppart (2014) imply $\sigma = 0.6$. This value is unusually small for the elasticity of substitution between goods, but that implication will hardly matter because, with our specification of $A(j)$, trade substitution is driven primarily by θ.

The elasticity of demand for energy by producers, $\epsilon_D(p)$, increases with ρ as well as with σ and the energy share. The magnitude of ρ (relative to 1) determines whether $\epsilon_g(p)$ is increasing or decreasing in p (see footnote 26). Antoszewski (2019) estimates industry-level elasticities of substitution between energy and value added using panel data across countries. The average over agriculture and manufacturing industries is 0.45, which we approximate as $\rho = 0.5$. Since long-run substitution possibilities could be much larger, making energy demand more elastic, we also consider $\rho = 2$.

Prior studies, such as Elliott et al. (2010), show that the foreign elasticity of energy supply, ϵ_s^*, is the key parameter affecting leakage and the effectiveness of a production tax. We calibrate $\epsilon_S = \epsilon_S^* = 0.5$ using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $E(a)$ and $E^*(a)$ among oil deposits with costs above the median (see Appendix E). Based on a literature review, Kotchen (2021) uses much higher values for the United States, with a point estimate for coal, 1.9, for natural gas, 1.6, and for gasoline, 2.0. Hence, we also consider a Foreign supply elasticity of $\epsilon_s^* = 2.0$.

In summary, our baseline parameter values are $\theta = 4$, $\sigma_e = 1$, $\sigma = 0.6$, $\rho = 0.5$, $\epsilon_S = 0.5$, $\epsilon_S^* = 0.5$, and a range for φ^W from 0 to 400. Alternative parameter values, considered one at a time, are $\theta = 16$, $\rho = 2$, and $\epsilon_S^* = 2$. Appendix E provides additional details on our data and on our calibration of the supply elasticities. For a given φ^W, our program uses a built-in solver to compute $F(\bar{\boldsymbol{z}}; \boldsymbol{\Theta})$.[33]

6.3 Results

We begin by computing the optimal policy (Home is OECD) as it varies with the global social cost of carbon, φ^W. Figure 3 shows, from left to right (top to bottom): (i) global emissions reductions, (ii) Home equivalent variation, (iii) the energy price, (iv) optimal taxes, (v) Home's export thresholds, and (vi) the maximum export subsidy normalized by the cost of the good in Foreign.[34]

Starting in the upper left, global emissions go down by up to ¼ as φ^W rises to \$400, a substantial reduction given that emissions in the OECD are only about ⅓ of global emissions (as reflected in the value of $\bar{G}_e$ in Table 4). Note that the substantial reduction from the OECD policy does not mean that the OECD's emissions are near zero. Some of the reductions arise in other parts of the world because of how the optimal policy expands the carbon price to trading partners. Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not, delivering gains (equivalent variation) of nearly \$1.5 trillion as φ^W hits \$400.

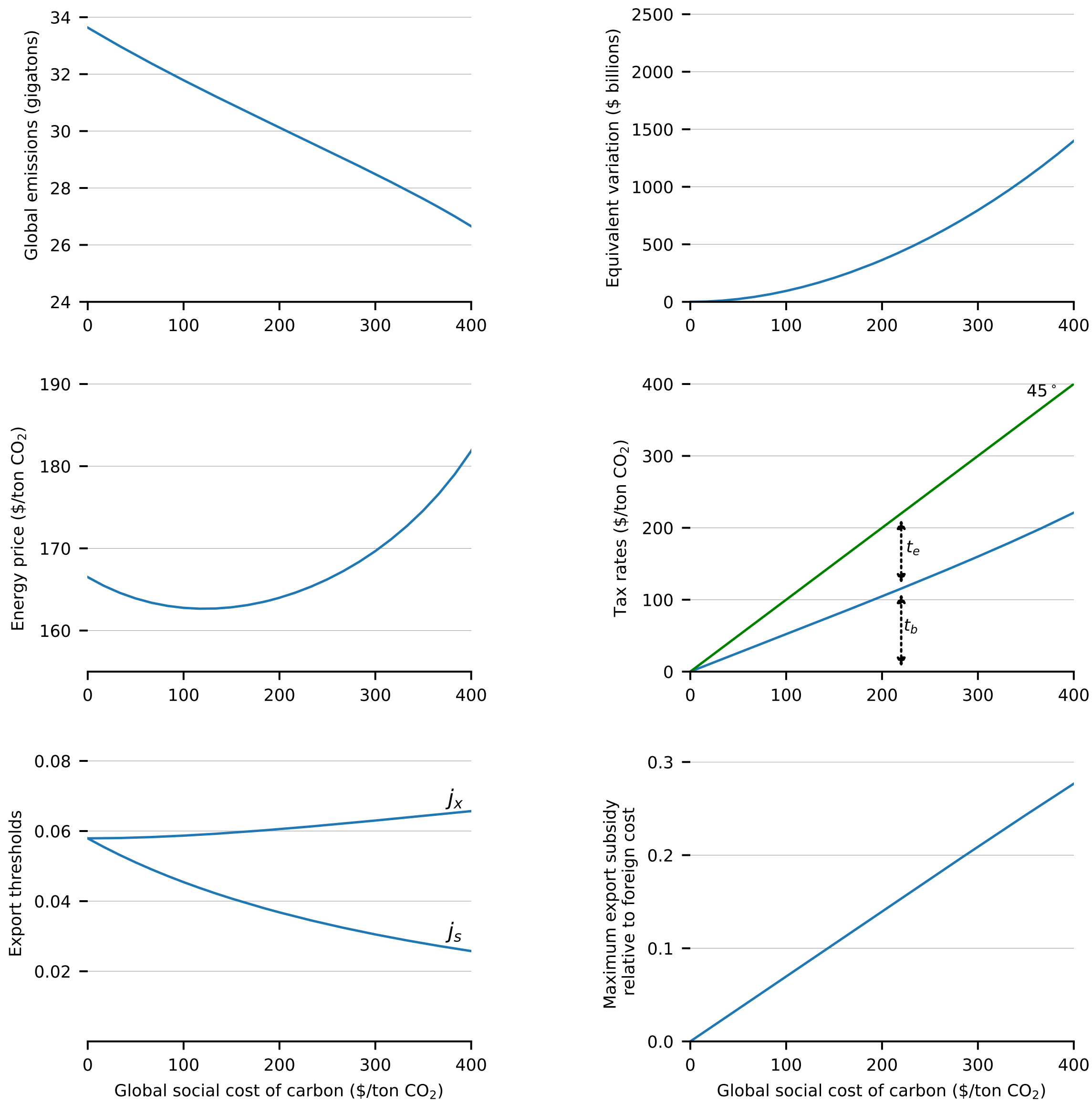
The OECD policy relies about evenly on an effective extraction tax (taxing supply) and a border adjustment (taxing demand), as shown in the middle right panel. The net effect is to push the global energy price below its BAU level for φ^W below \$100. For higher φ^W the energy price begins to rise. The extraction tax rises even faster, nearly equaling the energy price at $\varphi^W = \$400$, so that energy extractors in Home receive an after-tax price approaching zero.

Examining the lower left panel of Figure 3, we can see that Home expands the range of goods it exports as φ^W increases. By increasing its export threshold, Home is able to broaden the application of its carbon policy, which becomes more beneficial at a high global social cost of carbon. The vertical distance between the lower line and the upper line is the range of exported goods that Home subsidizes. These subsidies can

[33] As a check on our solution, which is based on first order conditions, we also directly maximize the planner's Lagrangian numerically. For the parameters discussed in the following subsection, we obtain nearly identical results. Our script is in Python and uses the built-in solver fsolve from Scipy. Our code is available at `https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy`.

[34] Using a result from footnote 16, this normalized maximum export subsidy can be expressed as $\frac{s_{jx}}{a^*_{jx} g(p_e)} = \frac{t_b}{p_e} \epsilon_g(p_e)$, the product of the ad-valorem border adjustment and the energy share in Foreign's production for its domestic market.

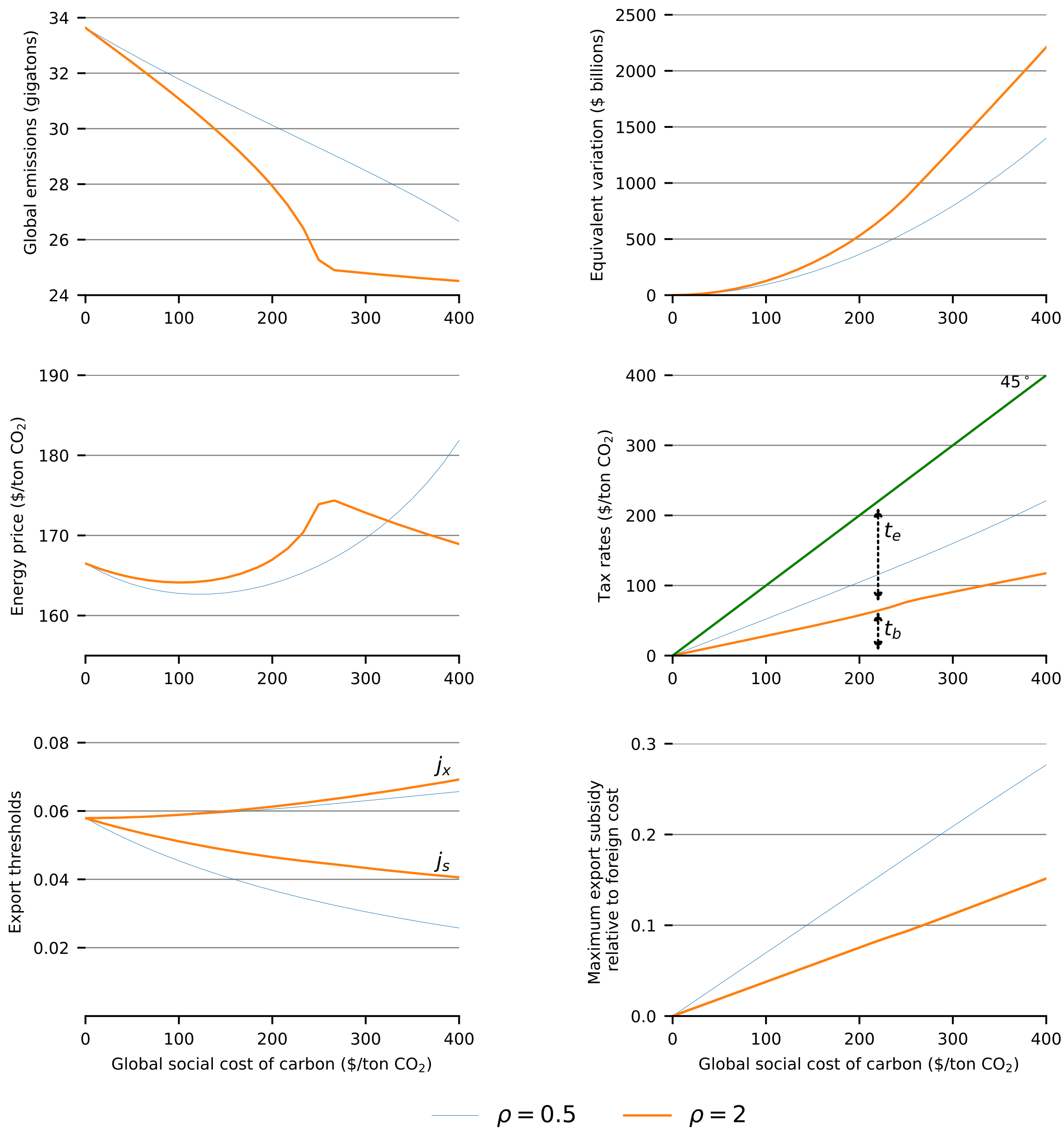
Figure 3: Optimal Policy in the OECD



become costly, reflected by the maximum subsidy (relative to Foreign's price) shown in the lower-right panel, which rises with φ^W.

In Figure 4 we illustrate how the optimal unilateral policy would be effected if the elasticity of substitution ρ was 2 (shown with thick orange lines) in comparison to the baseline value of 0.5 as in the previous figure (shown in thin blue lines). We interpret high ρ as a world with technological innovations that reduce the cost of production at

Figure 4: Optimal Policy in the OECD - Effect of Higher ρ



low energy intensity. With a higher ρ, energy demand by Foreign producers is more elastic, tilting the optimal policy toward taxing supply rather than demand, and hence pushing up the energy price. This shift is clear in the middle right panel. For larger values of φ^W the net price received by energy extractors in the OECD, $p_e - t_e$, becomes negative and extraction in the OECD hits zero for φ^W approaching \$300. Extraction in Home remains at zero for higher values of φ^W, causing a kink in the energy price line as energy supply no longer declines with higher taxes.[35]

With a high elasticity of substitution global emissions fall more sharply and are reduced at lower cost, yielding greater policy gains as measured by equivalent variation. A more subtle change is that the greater reliance on the extraction tax leads to less distortions in the export market. The range of exported goods that are subsidized declines and the normalized maximum export subsidy is smaller.

6.3.1 Coalition Size

A key factor in global climate negotiations is the set of countries that will agree to emissions reductions. To examine the effects of coalition size, Figure 5 shows global emissions under optimal policies with five increasingly large coalitions, starting with just the EU and moving up to a global coalition.[36] Tables 16, 17 and 18 in the appendix provide the carbon flow data we calibrate to for the three new scenarios. Global emissions in BAU, the energy price and energy share in BAU, and the parameter vector $\boldsymbol{\Theta}$ remain the same across all these scenarios.

Figure 5, like a production possiblity frontier, shows the greater global emissions reductions (on the vertical axis) that can be achieved at the cost of ever greater declines in global consumption (on the horizontal axis), for a given scenario for Home. The x's for each scenario indicate the optimal emissions reduction when $\varphi^W = 190$.

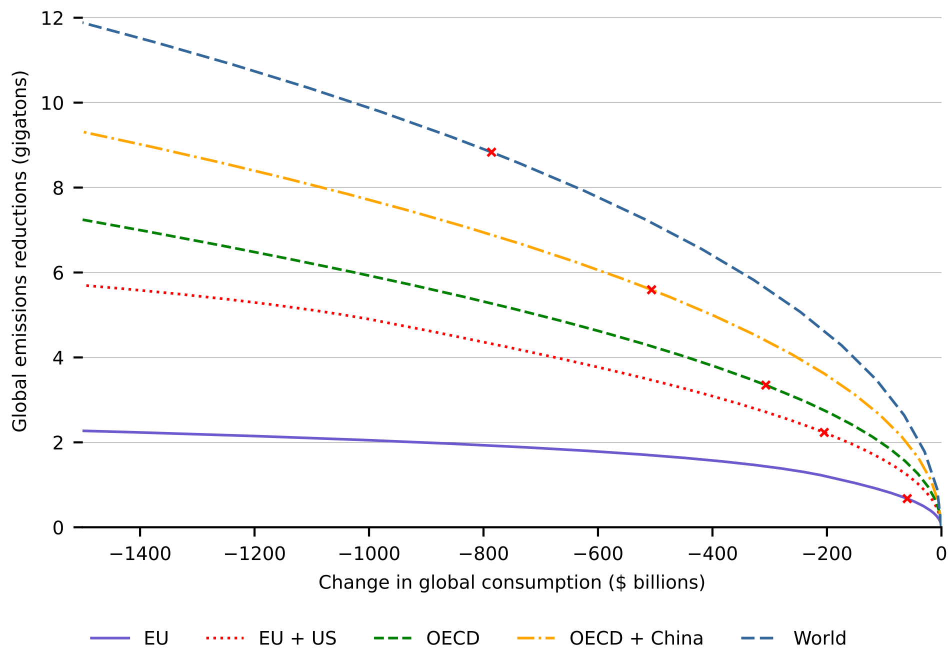
The consistent story is that there are substantial gains from expanding the taxing coalition. The EU alone has almost no power to reduce emissions. Adding the United States or the rest of the OECD countries helps significantly and increases the willingness of the coalition to incur costs to reduce emissions. Adding China to the taxing coalition leads to even greater emissions reductions for any given reduction in global consumption, and for $\varphi^W = \$190$ global emissions reductions are about ⅔ of what they would be for a global coalition.

Looking at the calibration tables in the appendix, we can see that the size of the extraction base is the key difference between the EU and the coalition of the EU and the United States. Energy used in production and consumption is roughly double,

[35]Once $t_e \geq p_e$, so that $Q_e = 0$, the exact value of the t_e becomes irrelevant (for convenience, however, we show t_e continuing to rise with φ^W even beyond that point).

[36]We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. For the EU-only case, we treat the EU as having 28 members as it had, prior to Brexit, in 2015.

Figure 5: Coalition Size



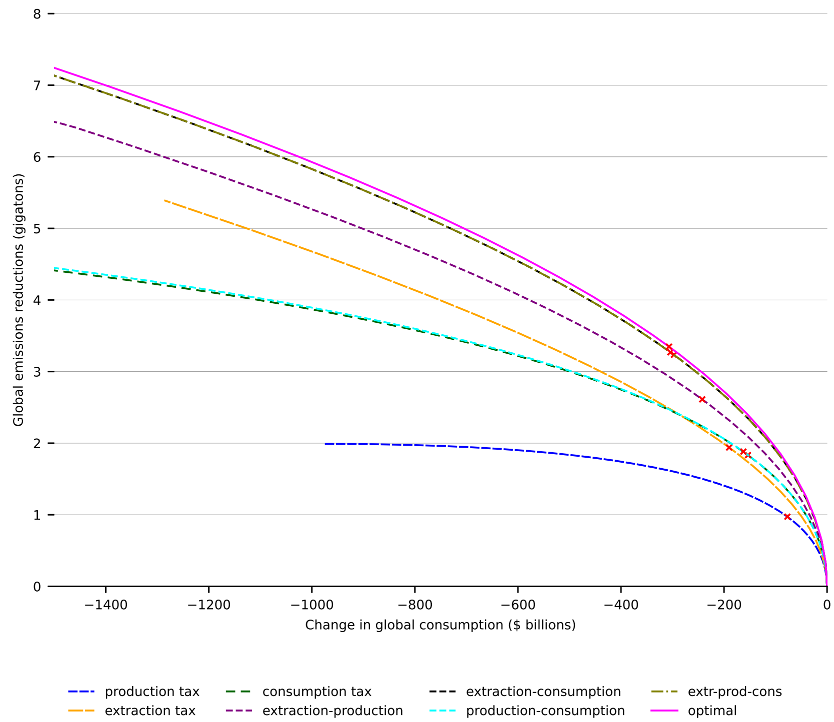
reflecting the relative size of the two economies, but extraction is higher by a factor of more than 5. With almost no extraction, the EU on its own gets little advantage from the extraction tax portion of the optimal policy, which means that acting alone, it is ineffective at reducing global emissions. Adding the United States expands the extraction base and makes the policy more effective.

6.3.2 Choice of Tax

Figure 6 compares the set of taxes and subsidies that implement the optimal policy to the seven constrained taxes (the three pure taxes and the hybrids of those taxes) with Home as the OECD.[37] A pure consumption tax significantly outperforms a pure production tax, confirming the widely-held view that adding border adjustments to

[37]The frontiers for a pure extraction tax and a pure production tax in Figure 6 stop short on the left. For the extraction tax, Home extraction Q_e hits zero when the net price hits zero (at $p_e = t_e = \$224$) for sufficiently high φ^W. A higher extraction tax becomes irrelevant beyond that point. For a production tax, eventually leakage hits 100% (at $t_p = \$327$ with $p_e = \$147$) even as $\varphi^W \to \infty$. An optimal production tax wouldn't go above the value at which $\Lambda = 1$ (100% leakage), as a higher tax with lower global consumption no longer buys lower global emissions. For any policy that involves a consumption tax, global emissions continue to decline (with t_c ever higher) as $\varphi^W \to \infty$, even at a higher cost in global consumption.

Figure 6: Choice of Tax

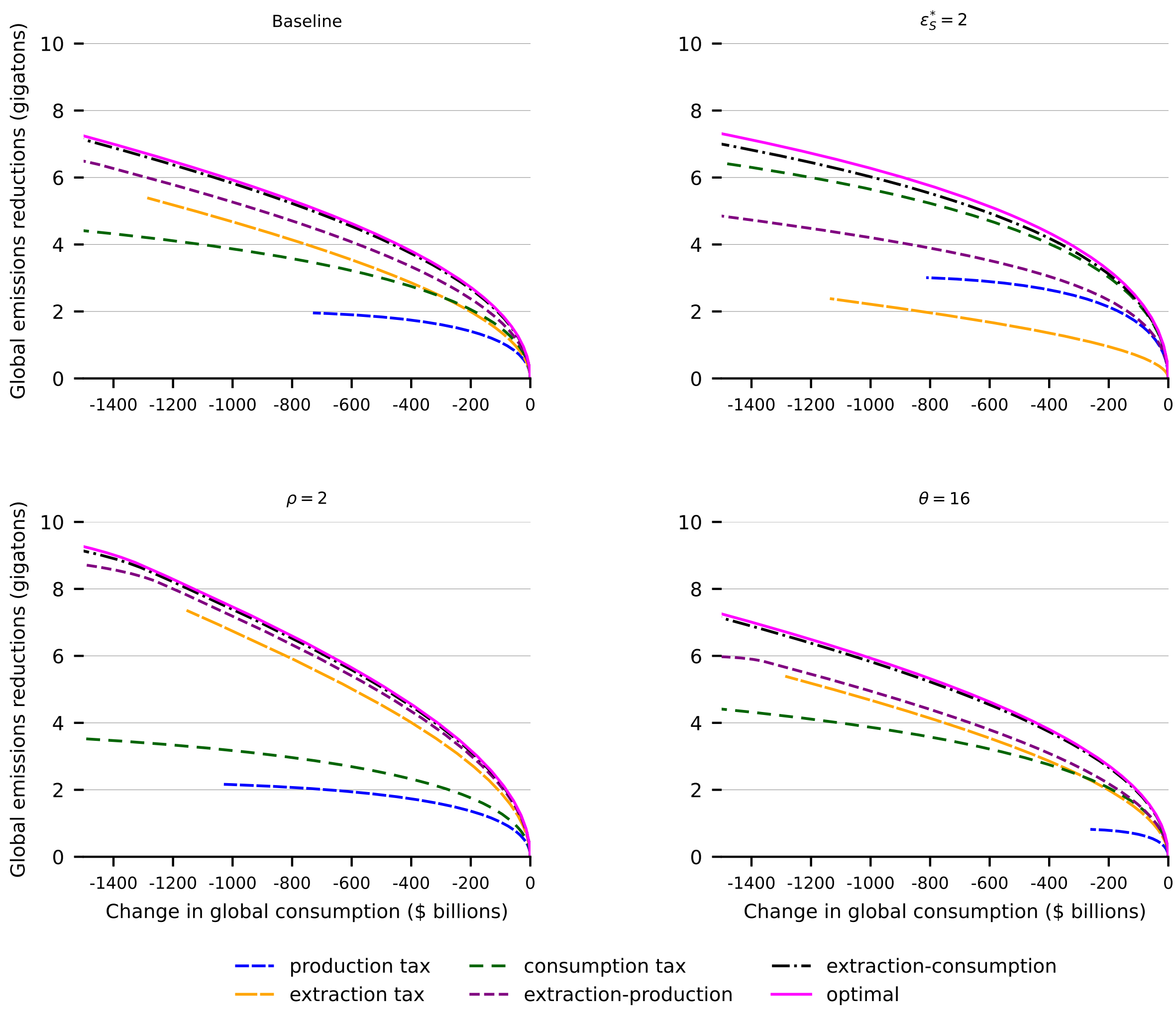


a production tax (which shift the tax downstream to consumption) may be desirable. Hybrid taxes that combine demand-side and supply-side taxes, however, provide an even greater benefit. For example, the extraction-production hybrid substantially outperforms a pure consumption tax, and the extraction-consumption hybrid and the hybrid of all three taxes (which are nearly indistinguishable in the figure) come close to the optimal.

In our static model, the EU's emissions trading system is a pure production tax, with a tax rate equal to the price of an emissions permit. Within the next few years, the EU CBAM will impose border taxes on imports (importers must purchase emissions permits) but will not rebate prior taxes paid (emissions permits purchased) on exports, which means that the EU CBAM is a production-consumption tax, with $t_p = t_c$. Comparing that tax to a pure consumption tax (so adding export rebates) shows that the two systems perform almost identically. Full export rebates do not hurt the system's performance very much, notwithstanding that exported goods no longer bear a tax.

Figure 7 explores how the constrained policies perform under different parameter values (for legibility, we do not show the production-consumption hybrid and the hybrid of all three taxes). When ϵ_S^* is high (top right), extraction taxes no longer perform

Figure 7: Effects of Different Taxes on Emissions



well. They induce a large response in Foreign extraction which offsets reductions in extraction in Home. Demand-side taxes, particularly a pure consumption tax, are correspondingly more effective because they lower p_e, causing a significant reduction in Foreign extraction. Because the extraction tax lacks power, combining it with a consumption tax does not significantly improve performance.

When ρ is high (bottom left), the reverse is true. Extraction taxes increase p_e which causes a large shift away from energy use in Foreign. Demand-side taxes lower p_e, and therefore, do not take advantage of the ability of Foreign to shift away from energy-intensive production. Extraction taxes in this scenario perform almost as well as the optimal policy, which itself relies heavily on extraction taxes (as seen in Figure

4, middle right panel).

Finally, when θ is high (bottom right), production taxes perform especially poorly. If θ is high, leakage will be more prevalent. Production taxes don't control leakage and, therefore, are ineffective in this scenario. Policies other than a production tax, however, perform similarly to our baseline case even when θ is high, indicating that concerns about leakage can readily be addressed through a number of different policies.

Figure 8: Energy Price and Location of Activities

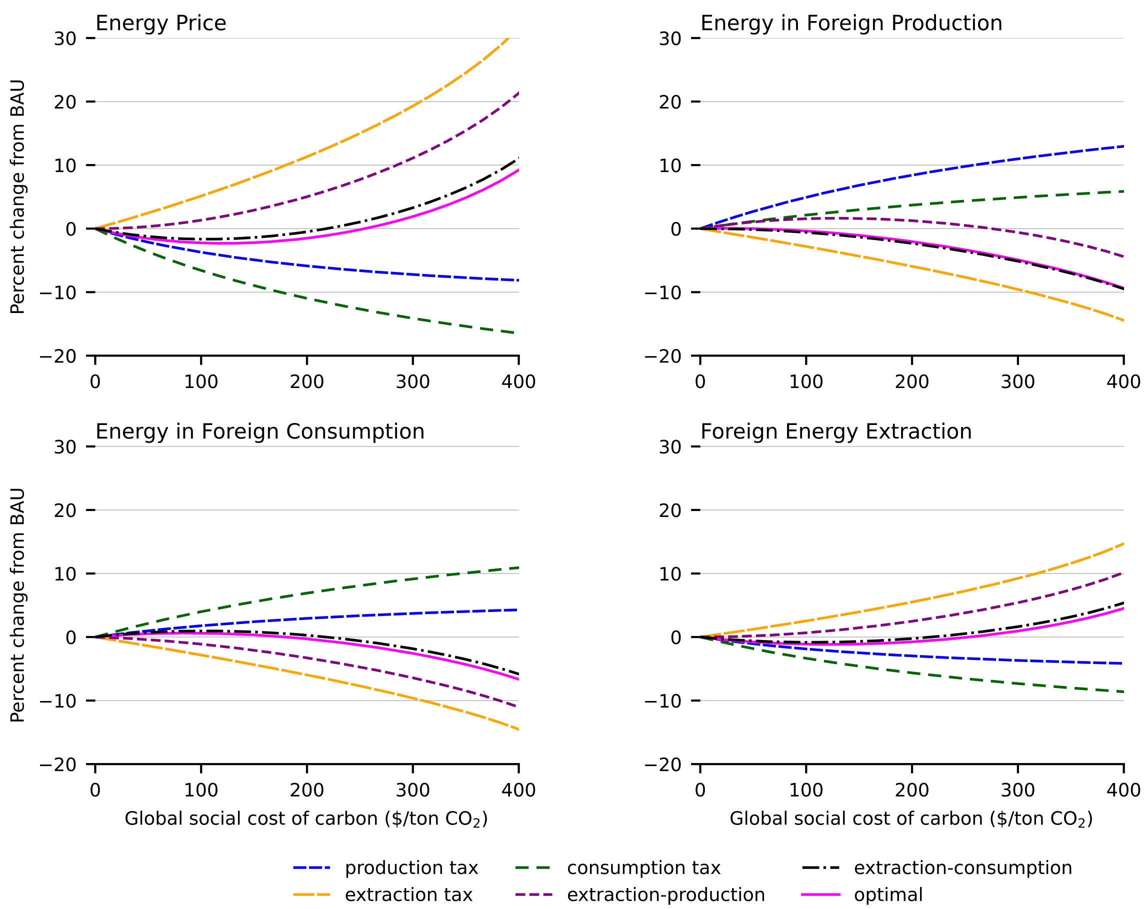


Figure 8 shows the effects of the same six policies (under our baseline calibration) on the price of energy and the location of extraction, production, and consumption, providing a visual sense of different types of leakage caused by various policies. Demand-side policies, such as a production tax or a consumption tax, suppress p_e, while supply-side policies increase p_e. For values of the social cost of carbon below \$300, the optimal policy keeps p_e roughly level, as does the extraction-consumption hybrid.

The effects on Foreign activities follow from the change in p_e. Because an extraction tax increases p_e, it causes Foreign to increase extraction and reduce production and consumption. The demand side policies, because they lower p_e, reduce Foreign extraction but increase Foreign production and consumption. The optimal policy minimizes changes in Foreign extraction and consumption, but causes a modest decline in Foreign production, possible due to the shift in $\bar{j}_x$.

Combined, Figures 6 through 8 show that the hybrid taxes generally perform better than the pure taxes and in some cases, come close to the optimal tax. The choice of policies depends on parameter values. If Foreign extraction or Foreign use of energy in production is highly elastic, extraction taxes or demand side taxes, respectively, become less attractive. Finally, all of the policies other than a pure production tax effectively deal with a high trade elasticity which means that there are a number of policies that can address leakage concerns.

7 Multiple Energy Sources

So far we have treated energy as a homogeneous fossil fuel measured in units of CO_2. Here we extend the analysis to accommodate $r = 1, 2, ..., R$ energy sources such as coal, natural gas, solar, and wind, which may differ in dirtiness, $d_r = CO_2$ per unit of energy. We take $r = 1$ to be oil, with $d_1 = 1$. If r is a renewable source then $d_r = 0$. Each source has a corresponding distribution of what we continue to call extraction costs, analogous to those for oil deposits, $E_r(a)$ in Home and $E_r^*(a)$ in Foreign.[38]

To keep this extension as simple as possible we take different sources of energy to be perfect substitutes for users. We continue to assume that oil is traded at price p_e, but again we simplify the analysis by treating other forms of energy as nontradeable.[39]

[38] We assume that each of these distributions satisfies the conditions described in footnote 6. This formulation is plausible for renewables as well, although the term *generation* is then more appropriate than *extraction*. Generating energy from solar, wind, and water power is constrained, like extracting fossil fuels, by scarce geographic factors.

[39] With this assumption we rule out interventions by Home that could shift Foreign extraction toward energy sources with lower CO_2 content. In contrast, Kotchen and Maggi (2024) consider trade in both "green energy" and fossil fuels in a simpler 2-country model with a climate externality. Their analysis can rationalize carbon policies that combine a subsidy to green energy with a tax on fossil-fuel extraction. The logic is related to why Home subsidizes exports of some goods in our paper, to crowd out dirty production in Foreign. While intriguing, a full analysis of such policies is beyond the scope of this paper.

The quantity of energy supplied by Home becomes $Q_e = \sum_{r=1}^{R} Q_{e,r}$, with CO_2 content $Q_c = \sum_{r=1}^{R} d_r Q_{e,r}$. Foreign supplies $Q_e^* = \sum_{r=1}^{R} Q_{e,r}^* = \sum_{r=1}^{R} E_r^*(p_e)$, with CO_2 content $Q_c^* = \sum_{r=1}^{R} d_r Q_{e,r}^*$. For $R = 1$ we return to our original analysis with no distinction between the quantity of energy and its carbon content.

7.1 Amendments to the Planning Problem

Incorporating multiple energy sources in this way requires only modest extensions to the unilaterally optimal solution presented in Section 3. Critically, $Q_c^W = Q_c + Q_c^*$ replaces Q_e^W as the source of the global externality, and hence the variable multiplying φ^W in the Lagrangian (12). The inner problem is unchanged since different sources of energy are perfect substitutes in production, with the Lagrange multiplier λ_e applying to energy without regard to its carbon content.

In the outer problem, the first-order condition for Home extraction from a given source r implies:

$$Q_{e,r} = E_r \left(\lambda_e - d_r \varphi^W \right), \tag{22}$$

for $\lambda_e - d_r \varphi^W \geq 0$ and $Q_{e,r} = 0$ otherwise. This condition is like (17) but with the dirtiness of energy source r weighting the global social cost of carbon. The planner's choice of direct consumption of energy satisfies (18).

The first-order condition for the energy price is like (19) but with a separate extraction wedge, $p_e - (\lambda_e - d_r \varphi^W)$, for each energy source. Aggregating over energy sources, the first-order condition becomes:

$$\varphi^W \frac{\partial Q_c^*}{\partial p_e} - [\lambda_e - p_e] \frac{\partial Q_e^*}{\partial p_e} + [\lambda_e - p_e] \frac{\partial C_e^{\tilde{y}*}}{\partial p_e} + \int_{j_s}^{j_x} [s_j] \frac{\partial x_j}{\partial p_e} dj = 0. \tag{23}$$

The response of the carbon content of Foreign extraction, $\partial Q_c^* / \partial p_e$, is a key determinate of the consumption wedge that balances (23). A higher Foreign response requires that Home increase net exports of energy, thereby lowering the global energy price and raising the consumption wedge. Foreign's *marginal* response is what matters. Even if most Foreign energy is from renewable sources, if coal is the dominant marginal source, the planner will maintain a larger consumption wedge to reduce Foreign extraction of this carbon-intensive fossil fuel.

7.2 Amendments to Optimal Taxes

The optimal policy can be implemented as in Section 4.1, with a nominal extraction tax per unit of CO_2 (Pigouvian wedge), a border adjustment per unit of energy (consumption wedge), and a subsidy per unit of goods (in Interval 2) exported by Home (export wedges). With multiple sources of energy, however, energy and CO_2 are not functionally the same. The nominal extraction tax, φ^W, applies to the carbon content of each source of energy, at a common rate per unit of CO_2. The tax rate per unit of energy depends on the dirtiness of each source, $t^N_{e,r} = d_r\varphi^W$. The border adjustment, $t_b = \lambda_e - p_e$, on the other hand, applies per unit of energy, whether on energy itself or on energy embodied in Home imports of goods.[40] The after-tax price of energy in Home, whether consumed directly or embodied in domestically produced or imported goods, is $p_e + t_b$ without regard to its source.

Combining the nominal extraction tax with the border adjustment, the effective tax per unit of energy supplied from source r is $t_{e,r} = t^N_{e,r} - t_b$, equal to the corresponding extraction wedge. Home extractors receive $p_e - t_{e,r}$ per unit of energy extracted from source r, and therefore extract a quantity satisfying (22). The effective extraction tax will be positive for dirty sources (with $d_r \geq 1$), for which $t_{e,r} \geq \varphi^W - (\lambda_e - p_e)$, but negative for renewables (with $d_r = 0$), for which $t_{e,r} = p_e - \lambda_e < 0$. Under the optimal policy, extraction from a high-carbon source will shrink or even cease, while renewables will expand due to the implicit subsidy per unit of renewable energy generated. The policy, as implemented here, provides a supply-side incentive to generate clean energy rather than a demand-side incentive to use clean energy.

7.3 Quantitative Illustration

To assess the implications of these amendments to the optimal unilateral policy, we modify the procedure described in Section 6, in a minimal way, by including just renewable energy ($r = 2$) along with oil ($r = 1$). We assume that renewables supply has the same functional form as oil deposits. We calibrate the level of renewables supply to BAU, with renewables accounting for 13% of energy supply (based on the same IEA source that we use for fossil fuel extraction). We consider the case of both a low

[40]This result formalizes the argument made in Kortum and Weisbach (2017) that the border adjustment should not be based on the carbon content of the energy source used to produce a particular good. Instead, what matters is the carbon content of the marginal energy source (with respect to a change in p_e) for the region exporting the good, which enters into equation (23).

renewables supply elasticity in Home and Foreign (of 0.5) and a high elasticity (of 2). All other parameters are set to their baseline values from Section 6.2.

Global carbon flows in BAU remain as in Table 4 (taking Home to be the OECD), but we need to include the additional energy from renewables when we calibrate energy demand. To deviate as little as possible from Section 6, we calibrate direct consumption of energy in Home and Foreign (implicitly η_e and η_e^*) to absorb this extra energy in BAU.[41]

Figure 9 shows what renewables contribute to the optimal unilateral policy. The left panel compares global emissions reductions, relative to BAU, without renewables and with renewables (under both a low and high renewables supply elasticity). At a social cost of carbon of \$300, global emissions are reduced by nearly an additional 2 gigatons due to renewables (in the low-elasticity case) and by about 4 gigatons (in the high-elasticity case). The right panel shows that the welfare gains for the policy are substantially higher in the presence of renewables, particularly in the high-elasticity case.

Figure 9: Optimal Outcomes with Renewables

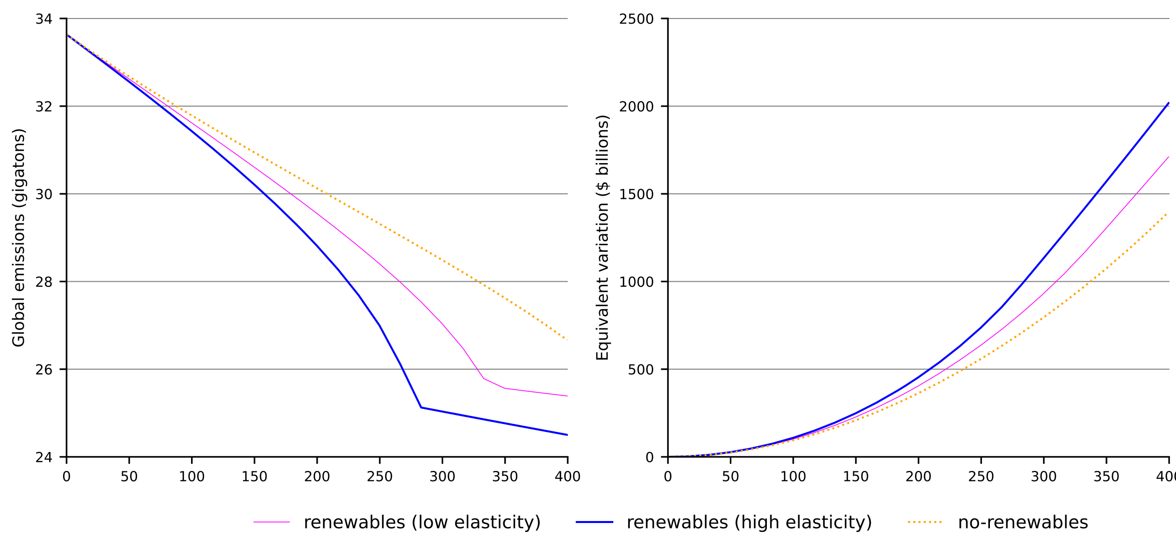
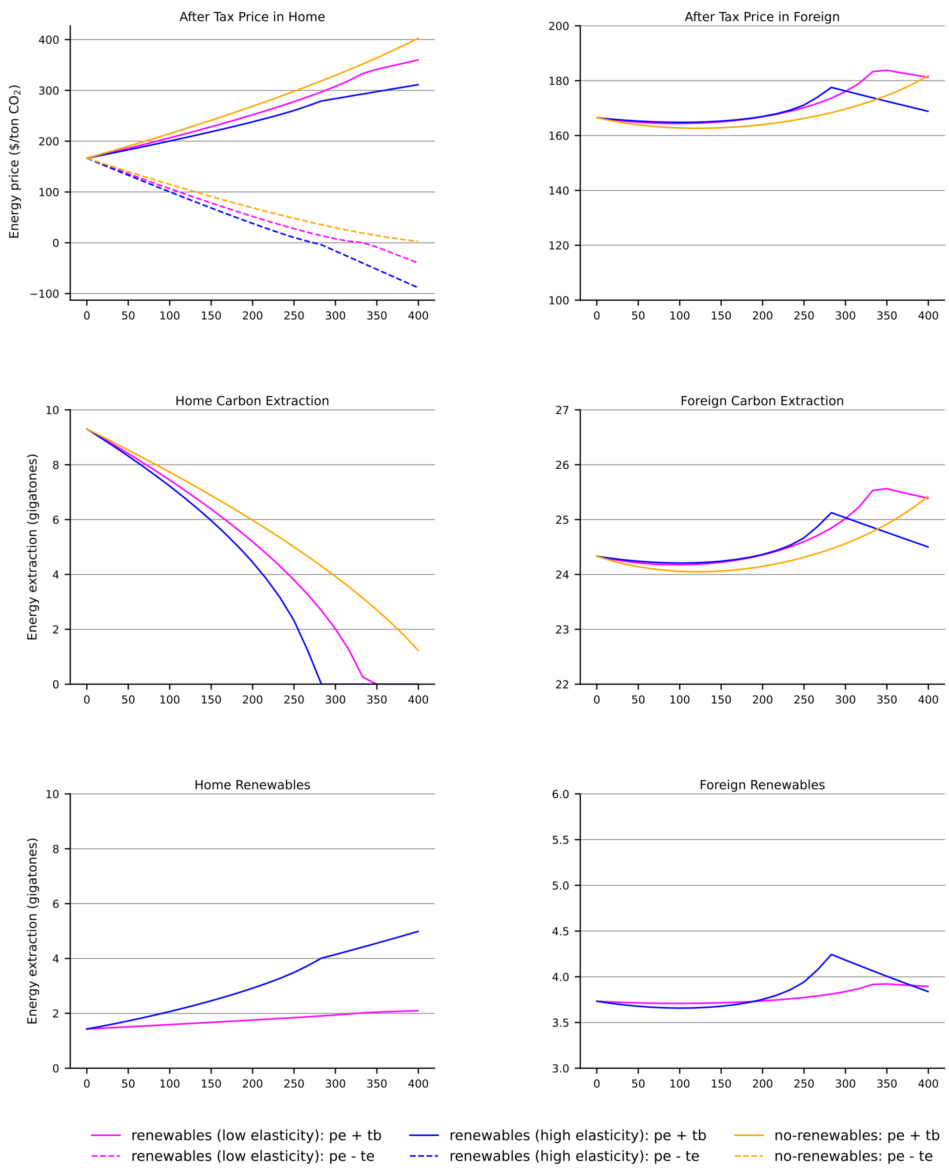


Figure 10 explores the mechanism behind these results. The upper left panel shows that in the presence of renewables, the optimal unilateral policy leads to a lower after-tax cost of energy for users in Home and a lower after-tax price for oil extractors in Home.

[41]This choice of calibration does not constrain where renewable energy is used under the optimal unilateral policy. For users of energy, oil and renewables are perfect substitutes.

The reason is that the policy tilts away from a border adjustment and toward a higher extraction tax when there is an alternative clean source of energy. This shift slightly raises the global energy price, relative to the scenario with no renewables, as shown in the upper right panel. Home extraction of fossil fuel declines quite dramatically, as shown in the middle left panel, and shuts down completely when the social cost of carbon exceeds \$350 (for the case of low renewables elasticity) or \$275 (for the high-elasticity case). Renewables supply in Home expands, as renewables do not pay the extraction tax.

Figure 10: Optimal Policy with Renewables



8 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. To go deeper requires pushing the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. Barresi (2022) shows how our unilateral carbon policy fits into the multi-country trade model of Eaton and Kortum (2002), with coalitions of countries representing Home and Foreign. Larch and Wanner (2024) provide a natural multi-country analysis of the energy sector. On the second extension, Golosov, Hassler, Krusell, and Tsyvinski (2014) and Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2021) are roadmaps for introducing dynamics.

Another important extension, in a multi-country world, is to consider endogenizing the region we call Home. Our current approach follows Markusen (1975) and CDVW in assuming that Foreign is intransigent. Home's optimal policy will likely be different if it can entice (or coerce) Foreign countries to join its coalition. Promising steps in this direction have been taken by Nordhaus (2015), Farrokhi and Lashkaripour (2024), and Barresi (2022).

References

Antoszewski, Michal (2019), "Wide-Rangee Estimation of Various Substitution Elasticities for CES Production Functions at the Sectoral Level," *Energy Economics,* 83, 272–289.

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2019), "(Mis) Allocation, Market Power, and Global Oil Extraction," *American Economic Review*, 109, 1568–1615.

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2018), "The Welfare Impact of Market Power: The OPEC Cartel," *Working Paper*, Duke University.

Barresi, Michael (2022), "Unilateral Carbon Policies and Multilateral Coalitions: An Analysis of Coalition Stability under the Optimal Unilateral Policy," Yale University Senior Essay in Economics.

Böhringer, Christoph, Edward J. Balistreri, and Thomas F. Rutherford (2012), "The Role of Border Carbon Adjustment in Unilateral Climate Policy: Overview of an Energy Modeling Forum Study (EMF 29)," *Energy Economics*, 34. The Role of Border Carbon Adjustment in Unilateral Climate Policy: Results from EMF 29, S97–110.

Böhringer, Christoph, Andreas Lange, and Thomas F. Rutherford (2014), "Optimal Emission Pricing in the Presence of International Spillovers: Decomposing Leakage and Terms-of-Trade Motives," *Journal of Public Economics*, 110, 101–111.

Boppart, Timo (2014), "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences," *Econometrica,* 82, 2167–2196.

Branger, Frédéric, and Philippe Quiron (2014) "Climate Policy and the 'Carbon Haven' Effect," *Wiley Interdisciplinary Reviews: Climate Change*, 5, 53–71.

Bradford, Anu (2020), *The Brussels Effect,* Oxford University Press, New York.

Copeland, Brian R. and M. Scott Taylor (1994), "North-South Trade and the Environment," *Quarterly Journal of Economics,* 109, 755–787.

Copeland, Brian R. and M. Scott Taylor (1995), "Trade and Transboundry Pollution," *American Economic Review,* 85, 755–787.

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Ivan Werning (2015), "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics,* 659–702.

Davis, Lucas and Lutz Kilian (2011), "Estimating the Effect of a Gasoline Tax on Carbon Emissions," *Journal of Applied Econometrics,* 26, 1187–1214.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum (2007), "Unbalanced Trade," *American Economic Review: Papers and Proceedings,* 97, 351–355.

Dornbusch, Rudiger, Stanley Fischer, and Paul A. Samuelson (1977), "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review,* 67, 823–839.

Eaton, Jonathan and Samuel Kortum (2002), "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach (2010), "Trade and Carbon Taxes," *American Economic Review: Papers and Proceedings,* 100, 465–469.

Elliott, Joshua, Ian Foster, Samuel Kortum, Gita K. Jush, Todd. Munson, and David Weisbach (2013), "Unilateral Carbon Taxes, Border Tax Adjustments and Carbon Leakage," *Theoretical Inquiries in Law,* 14.

EPA (2019): "Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-2017. Annex 2 (Methodology for estimating CO2 emissions from fossil fuel combustion), Table A-42 for C coefficient and Table A-52 for heat content," *U.S. Environmental Protection Agency, Washington, DC,* U.S. EPA #430-R-19-001 (PDF).

Farrokhi, Farid and Ahmad Lashkaripour (2024), "Can Trade Policy Mitigate Climate Change," unpublished, Purdue University.

Fischer, Carolyn, and Alan K. Fox (2012), "Comparing Policies to Combat Emissions Leakage: Border Carbon Adjustments versus Rebates," *Journal of Environmental Economics and Management*, 64, 199–216.

Fowlie, Meredith, Mar Reguant, and Stephen Ryan (2016), "Market-Based Emissions Regulation and Industry Dynamics, "*Journal of Political Economy*, 124, 249–302.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014), "Optimal Taxes on Fossil Fuels in General Equilibrium," *Econometrica*, 82, 41–88.

Grossman, Gene M. and Elhanan Helpman (1994), "Protection for Sale," *American Economic Review*, 84, 833–850.

Harstad, Bård (2012), "Buy Coal! A Case for Supply-Side Environmental Policy,"*Journal of Political Economy*, 120, 77–115.

Hoel, M. (1994), "Efficient Climate Policy in the Presence of Free Riders," *Journal of Environmental Economics and Management*, 27, 259-274

Hoel, M. (1996), "Should a Carbon Tax be Differentiated across sectors?," *Journal of Public Economics*, 59, 17–32.

Frictions," *Journal of Environmental Economics and Management*, 88, 950-113.

IEA (2020), "World Energy Balances," *IEA World Energy Statistics and Balances (database)*, https://doi.org/10.1787/data-00512-en.

IPCC (2008), "2006 IPCC Guidelines for National Greenhouse Gas Inventories – A primer," Prepared by the *National Greenhouse Gas Inventories Programme*, Eggleston H.S., Miwa K., Srivastava N. and Tanabe K. (eds). Published: IGES, Japan.

Rock and a Hard Place: A Trade-Theory Analysis of Leakage Under Production- and Consumption-Based Policies, *Environmental and Resource Economics*, 56, 47–72.

Keen, Michael, and Christos Kotsogiannis (2014), "Coordinating Climate and Trade Policies: Pareto Efficiency and the Role of Border Tax Adjustments," *Journal of International Economics*, 94, 119–128.

Kortum, Samuel and David Weisbach (2017), "The Design of Border Adjustments for Carbon Prices," *National Tax Journal*, 70, 421–446.

Kotchen, Matthew (2021), "The producer benefits of implicit fossil fuel subsidies in the United States," *Proceedings of the National Academy of Sciences* 118, e2011969118

Kotchen, Matthew and Giovanni Maggi (2024), "The Political Economy of Carbon Taxes and Green Subsidies in a World Economy," unpublished, Yale University.

Kotlikoff, Laurence, Felix Kubler, Andrey Polbin, Jeffrey Sachs, and Simon Scheidegger (2021), "Making Carbon Taxation a Generational Win Win," *International Economic Review*, 62, 3–46.

Kruse-Andersen, Peter Kjær, and Peter Birch Sørensen (2022), "Optimal Unilateral Climate Policy with Carbon Leakage at the Extensive and the Intensive Margin,"*CESifo Working Paper* 9185-2021.

Larch, Mario and Joschka Wanner (2024), "The Consequences of Non-Participation in the Paris Agreement,"*European Economic Review*, 163, 104699.

Markusen, James R. (1975), "International Externalities and Optimal Tax Structures," *Journal of International Economics*, 5, 15–29.

Metz et al (2007), Climate Change 2007—Mitigation of Climate Change, Contribution of Working Group III to the Fourth Assessment Report of the IPCC, Cambridge University Press.

Nordhaus, William (2015), "Climate Clubs: Overcoming Free-riding in International Climate Policy," *American Economic Review*, 105, 1339–1370.

OECD (2018), Input-Output Tables. For download at http://oe.cd/i-o. Organisation for Economic Co-operation and Development, Paris.

OECD (2019), "Carbon Dioxide Emissions Embodied in International Trade (TECO2) Database."

Shapiro, Joseph S. and Reed Walker (2018), "Why is Pollution from US Manufacturing Declining? The Roles of Environmental Regulation, Productivity, and Trade," *American Economic Review,* 108, 3814–3854.

Simonovska, Ina and Michael Waugh (2014), "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 92, 34–50.

Staiger, Robert W. (2022), *A World Trading System for the Twenty-First Century*, MIT Press.

Weisbach, David, Samuel S. Kortum, Michael Wang, and Yujia Yao (2023), "Trade, Leakage, and the Design of a Carbon Tax," *Environmental and Energy Policy and the Economy* 4, 43–90.

Weisbach, David and Sam Kortum (2023), "Climate Change in the International Context: Solving the Carbon Leakage Problem,"*N.Y.U. Environmental Law Journal* 31, 1-65

Zhang, Zhong Xiang (2012), "Competitiveness and Leakage Concerns and Border Carbon Adjustments," *International Review of Environmental and Resource Economics*, 6, 225–287.

A Global Planner's Problem

Suppose the planner controls all decisions in Foreign as well as in Home. We can solve for the optimal policy by maximizing the planner's Lagrangian from Section 3.3 while enlarging the set of choice variables to C_e^d, C_e^{d*}, Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{k_j^y\}$, $\{k_j^{y*}\}$, $\{k_j^x\}$, and $\{k_j^m\}$. (The energy price is no longer relevant.)

A.1 Solution

Following CDVW, we first solve the inner problem, involving conditions for an individual good j, given λ_e. We then turn to the outer problem, optimizing over C_e^d, C_e^{d*}, Q_e and Q_e^* while solving for λ_e.

A.1.1 Inner Problem

The inner problem is to choose y_j, y_j^*, x_j, m_j, k_j^y, k_j^{y*}, k_j^x, and k_j^m to maximize:

$$\begin{aligned}\mathcal{L}_j &= u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ &\quad - \left(l_j(k_j^y) + \lambda_e e_j(k_j^y)\right) y_j - \left(l_j^*(k_j^{y*}) + \lambda_e e_j^*(k_j^{y*})\right) y_j^* \\ &\quad - \tau\left(l_j(k_j^x) + \lambda_e e_j(k_j^x)\right) x_j - \tau^*\left(l_j^*(k_j^m) + \lambda_e e_j^*(k_j^m)\right) m_j.\end{aligned}$$

Energy intensities k_j^i, for $i \in \{y, y^*, x, m\}$, enter this objective as in (5), which implies $k_j^i = k(\lambda_e)$. We also get that the shadow cost of producing good j is $a_j g(\lambda_e)$ in Home and $a_j^* g(\lambda_e)$ in Foreign. The unit energy requirement is $e_j(k(\lambda_e)) = a_j g'(\lambda_e)$ in Home and $e_j^*(k(\lambda_e)) = a_j^* g'(\lambda_e)$ in Foreign.

The FOC for y_j implies $u'(y_j + m_j) \leq a_j g(\lambda_e)$, with equality if $y_j > 0$. The FOC for m_j implies $u'(y_j + m_j) \leq a_j^* \tau^* g(\lambda_e)$, with equality if $m_j > 0$. Both FOC's hold with equality for good $j = j_m$ satisfying $A(j_m) = 1/\tau^*$. For $j < j_m$ we have $y_j > 0$ and $m_j = 0$ while for $j > j_m$ we have $m_j > 0$ and $y_j = 0$.

The FOC for y_j^* implies $u^{*\prime}(y_j^* + x_j) \leq a_j^* g(\lambda_e)$, with equality if $y_j^* > 0$. The FOC for x_j implies $u^{*\prime}(y_j^* + x_j) \leq a_j \tau g(\lambda_e)$, with equality if $x_j > 0$. Both FOC's hold with equality for good $j = j_x$ satisfying $A(j_x) = \tau$. (Since $A(j)$ is monotonically decreasing, it follows that $j_x < j_m$.) For $j < j_x$ we have $x_j > 0$ and $y_j^* = 0$ while for $j > j_x$ we have $y_j^* > 0$ and $x_j = 0$.

Although consumption levels are different, the import and export thresholds are identical to BAU, $j_m = \bar{j}_m$ and $j_x = \bar{j}_x$.

Aggregating over goods, taking account of demand and unit energy requirements, implicit consumption of energy in Home is:

$$C_e^y(\lambda_e) + C_e^m(\lambda_e) = \int_0^{j_m} a_j g'(\lambda_e) u'^{-1}(a_j g(\lambda_e)) dj + \int_{j_m}^1 \tau^* a_j^* g'(\lambda_e) u'^{-1}(\tau^* a_j^* g(\lambda_e)) dj,$$

while in Foreign:

$$C_e^{y*}(\lambda_e) + C_e^x(\lambda_e) = \int_0^{j_x} \tau a_j g'(\lambda_e) u^{*\prime -1}(\tau a_j g(\lambda_e)) dj + \int_{j_x}^1 a_j^* g'(\lambda_e) u^{*\prime -1}(a_j^* g(\lambda_e)) dj.$$

Both are functions of the Lagrange multiplier λ_e.

A.1.2 Outer Problem

The outer problem is to choose C_e^d, C_e^{d*}, Q_e and Q_e^* while finding λ_e that clears the global energy market. The outer Lagrangian is unchanged. The first-order conditions for direct consumption give $v'(C_e^d) = v^{*\prime}(C_e^{d*}) = \lambda_e$. The first-order condition for Home energy extraction implies $Q_e = E(\lambda_e - \varphi^W)$, for $\lambda_e - \varphi^W \geq 0$, or else $Q_e = 0$. Likewise for Foreign extraction, $Q_e^* = E^*(\lambda_e - \varphi^W)$, for $\lambda_e - \varphi^W \geq 0$, or else $Q_e^* = 0$. The Lagrange multiplier solves:

$$C_e(\lambda_e) + C_e^*(\lambda_e) = E\left(\lambda_e - \varphi^W\right) + E^*\left(\lambda_e - \varphi^W\right),$$

with $C_e(\lambda_e) = C_e^y(\lambda_e) + C_e^m(\lambda_e) + C_e^d(\lambda_e)$ and $C_e^*(\lambda_e) = C_e^{y*}(\lambda_e) + C_e^x(\lambda_e) + C_e^{d*}(\lambda_e)$.

A.2 Decentralized Global Optimum

We can interpret the planner's solution in terms of a decentralized economy with a price of energy $p_e = \lambda_e$. An extraction tax in Home and Foreign, equal to global marginal damages from emissions, $t_e = t_e^* = \varphi^W$, solves the global externality. Energy extractors in Home and Foreign receive an after-tax price of $p_e - \varphi^W$.[42]

[42]Inspection of the global market clearing condition for energy shows that extraction and consumption of energy remain the same if we instead set $p_e = \lambda_e + \varphi^W$. This change corresponds to adding full border adjustments, $t_b = t_b^* = \varphi^W$, to a nominal extraction tax, $t_e^N = t_e^{N*} = \varphi^W$, turning it into a consumption tax. Any differences in the distribution of services consumption between these two policies (a global extraction tax versus a global consumption tax) can be undone with transfers.

Table 5: Business as Usual (Goods-Sector Outcomes)

	Home		Foreign	
Home	$u'(y_j) = a_j g(p_e)$	$j < j_m$	$u'(m_j) = \tau^* a_j^* g(p_e)$	$j > j_m$
Foreign	$u^{*\prime}(x_j) = \tau a_j g(p_e)$	$j < j_x$	$u^{*\prime}(y_j^*) = a_j^* g(p_e)$	$j > j_x$

Thresholds: $A(j_m) = 1/\tau^*$ and $A(j_x) = \tau$.

A.3 Business as Usual

A decentralized global optimum with $\varphi^W = 0$ mimics business as usual. Alternatively, we can think of BAU as the competitive equilibrium with no carbon policy. For later reference, we list the outcomes in BAU for any good j in Table 5.

B Unilateral Planner's Outer Problem

Here, we fill in the missing derivation of equation (19) starting from the first-order condition for p_e, in which some terms are absent due to the envelope condition:[43]

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

We distinguish energy C_e^x and labor L_g^x used by Home to supply goods to Foreign consumers from energy C_e^{y*} and labor L_g^{y*} used by Foreign to supply goods to Foreign consumers. (Energy $C_e^y + C_e^m$ and labor $L_g^y + L_g^m$ used to supply goods to Home consumers don't depend on p_e since consumption by Home consumers is determined by the inner problem in Section 3.4.1.) Substituting in the equilibrium conditions from

[43] In particular, C_g, C_e^d, Q_e, and L_e don't appear. Although the thresholds, j_s and j_x, depend on the energy price, since the inner problem implies $\partial \mathcal{L}/\partial j_x = \partial \mathcal{L}/\partial j_s = 0$, we get:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \left.\frac{\partial \mathcal{L}}{\partial p_e}\right|_{j_s, j_x} + \frac{\partial \mathcal{L}}{\partial j_x}\frac{\partial j_x}{\partial p_e} + \frac{\partial \mathcal{L}}{\partial j_s}\frac{\partial j_s}{\partial p_e} = \left.\frac{\partial \mathcal{L}}{\partial p_e}\right|_{j_s, j_x}.$$

Consider, for example, energy used by Foreign producers to supply local demand:

$$C_e^{y*} = \int_{j_x}^1 e_j^*(k(p_e)) y_j^* dj = \int_{j_x}^1 a_j^* g'(p_e) u^{*\prime -1}(a_j^* g(p_e)) dj.$$

In differentiating C_e^{y*} as it enters the first-order condition for p_e, we thus ignore $\partial j_x / \partial p_e$.

Foreign, $\partial v^*(C_e^{d*})/\partial p_e = p_e \partial C_e^{d*}/\partial p_e$ and $\partial L_e^*/\partial p_e = p_e \partial Q_e^*/\partial p_e$, we can rearrange the first-order condition as:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = -\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj - p_e \frac{\partial C_e^{d*}}{\partial p_e} + \lambda_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} + \lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e}.$$

We now derive an expression for the first term on the right-hand side of the FOC for p_e above:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj = \int_0^1 u^{*\prime}(c_j^*) \frac{\partial c_j^*}{\partial p_e} dj.$$

From the inner problem in Section 3.4.2, the c_j^* in Region 1 don't depend on the energy price, the c_j^* in Region 2 are exported by Home in quantities x_j that equate Foreign marginal utility to what it would marginally cost Foreign to produce them itself, and the c_j^* in Region 3 are produced and consumed by Foreign in quantities y_j^*. Hence:

$$\int_0^1 u^{*\prime}(c_j^*) \frac{\partial c_j^*}{\partial p_e} dj = \int_{j_s}^{j_x} a_j^* g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{j_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj. \tag{24}$$

Aggregating the implicit cost functions for supplying Foreign consumption:

$$\lambda_e C_e^x + L_g^x + p_e C_e^{y*} + L_g^{y*} = \int_0^{j_x} \tau a_j g(\lambda_e) x_j dj + \int_{j_x}^1 a_j^* g(p_e) y_j^* dj.$$

Differentiating both sides (noting that x_j depends on p_e only in Region 2) and canceling out C_e^{y*} yields:

$$\lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} = \int_{j_s}^{j_x} \tau a_j g(\lambda_e) \frac{\partial x_j}{\partial p_e} dj + \int_{j_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj.$$

Combined with (24) and the equation preceding it:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj = \lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} - \int_{j_s}^{j_x} \left(\tau a_j g(\lambda_e) - a_j^* g(p_e)\right) \frac{\partial x_j}{\partial p_e} dj. \tag{25}$$

Substituting (25) into the FOC for p_e above yields equation (19) from the paper.

C Constrained-Optimal Policies

We derive the formulas for the constrained-optimal policies that are described in Section 5. Each maximizes the planner's Lagrangian from Section 3.4, but with with different constraints on the planner's choice variables. The results are summarized in Table 6.

Table 6: Effective Taxes Under Constrained-Optimal Policies

Policy	Effective Tax	Consumption Wedge: $\lambda_e - p_e$
Extrac. tax	$t_e = \varphi^W - (\lambda_e - p_e)$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_e^W/\partial p_e}$
Consum. tax	$t_c = \lambda_e - p_e$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e}$
Produc. tax	$t_p = (1 - \Lambda)(\lambda_e - p_e)$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial G_e^*/\partial p_e - \Lambda \partial G_e/\partial p_e}$
Extr/Cons	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_e^*/\partial p_e}$
Extr/Prod	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1 - \Lambda)(\lambda_e - p_e) \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial G_e^*/\partial p_e - \Lambda \partial G_e/\partial p_e}$
Prod/Cons	$\begin{cases} t_p = (1 - \Lambda^*)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^x/\partial p_e}$
Extr/Prod/Cons	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1 - \Lambda^*)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^x/\partial p_e}$

Overall leakage: $\Lambda = -\frac{\partial G_e^*/\partial v_e}{\partial G_e/\partial v_e}$; foreign leakage: $\Lambda^* = -\frac{\partial C_e^{y*}/\partial v_e}{\partial C_e^x/\partial v_e}$.

The Lagrangian of the outer problem, which we repeat here for convenience, is common to all seven policies (although individual terms differ across policies):

$$\mathcal{L} = \int_0^1 u(c_j)dj + \int_0^1 u^*(c_j^*)dj + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e^W - Q_e^W \right).$$

As in the paper, we first consider policies in which the planner chooses extraction and/or domestic consumption before turning to policies in which the planner controls domestic production, or some combination of extraction, consumption, and production. The primary challenge is to derive, for each policy, the optimal energy price (or prices in the case of a production tax) to maximize the outer Lagrangian.

A key step is to use a simpler version of (25) from Appendix B, which we derive next. As in Appendix B, we distinguish labor used in each of the four lines of goods production, L_g^i for $i \in \{y, x, m, y^*\}$ with $L_g^y + L_g^x = L_g$ and $L_g^m + L_g^{y*} = L_g^*$. Assuming that producers serving Foreign consumers face an energy price p_e (we will amend this assumption for the case of a production tax), we get a variant of (24):

$$\int_0^1 u^{*\prime}(c_j^*) \frac{\partial c_j^*}{\partial p_e} dj = \int_0^{j_x} \tau a_j g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{j_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj. \tag{26}$$

Aggregating the implicit cost functions for supplying Foreign consumption, all evaluated at energy price p_e, gives:

$$p_e C_e^x + L_g^x + p_e C_e^{y*} + L_g^{y*} = \int_0^{j_x} \tau a_j g(p_e) x_j dj + \int_{j_x}^1 a_j^* g(p_e) y_j^* dj$$

Differentiating with respect to the energy price and canceling out C_e^x and C_e^{y*}:

$$p_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} = \int_0^{j_x} \tau a_j g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{j_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj.$$

Combining this expression with (26) delivers the variant of (25) relevant for constrained-optimal policies:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj = p_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e}. \tag{27}$$

We will refer back to (27) in the derivations that follow.

C.1 Extraction Policy

For an extraction policy, we constrain the planner to choose only Q_e and p_e. Energy intensities, quantities produced, and quantities consumed of each good j are as in BAU (shown in Table 5), given p_e. Hence we can skip the inner problem and go directly to the outer problem.

C.1.1 Outer Problem

We maximize the outer Lagrangian over Q_e and p_e. The first-order condition for Q_e is identical to that for the unilaterally optimal policy. For $\lambda_e - \varphi^W \geq 0$ we have $Q_e = E(\lambda_e - \varphi^W)$, and otherwise $Q_e = 0$.

The first-order condition for p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \int_0^1 \frac{\partial u(c_j)}{\partial p_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v(C_e^d)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e}$$
$$- \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^W}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

Since direct consumers of energy in both countries face an energy price p_e, the equilibrium conditions imply:

$$\frac{\partial v(C_e^d)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} = p_e \left(\frac{\partial C_e^d}{\partial p_e} + \frac{\partial C_e^{d*}}{\partial p_e} \right)$$

Since all goods producers face price p_e, we can employ equation (27) for both Home and Foreign, which, in combination with the expression above, gives:

$$\int_0^1 \frac{\partial u(c_j)}{\partial p_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v(C_e^d)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} = \frac{\partial L_g^W}{\partial p_e} + p_e \frac{\partial C_e^W}{\partial p_e}.$$

Using this result and $\partial L_e^* / \partial p_e = p_e \partial Q_e^* / \partial p_e$, the first-order condition for p_e collapses to:

$$(\lambda_e - p_e) \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e} \right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e}.$$

C.1.2 Decentralization

In a market economy we can impose an extraction tax of $t_e = \varphi^W - (\lambda_e - p_e)$ so that the after-tax price, $p_e - t_e = \lambda_e - \varphi^W$, induces the optimal level of extraction in Home. The extraction tax rate is thus:

$$t_e = \varphi^W \frac{-C_e^W / \partial p_e}{\partial Q_e^* / \partial p_e - \partial C_e^W / \partial p_e}.$$

C.2 Consumption Policy

For a consumption policy we constrain the planner to choose only: $\{k_j^y\}$, $\{k_j^m\}$, $\{y_j\}$, $\{m_j\}$, C_e^d, and p_e. These choices involve both the inner problem and the outer problem.

Table 7: Consumption Policy (Goods-Sector Outcomes)

	Home		Foreign	
Home	$u'(y_j) = a_j g(\lambda_e)$	$j < j_m$	$u'(m_j) = \tau^* a_j^* g(\lambda_e)$	$j > j_m$
Foreign	$u^{*\prime}(x_j) = \tau a_j g(p_e)$	$j < j_x$	$u^{*\prime}(y_j^*) = a_j^* g(p_e)$	$j > j_x$

Thresholds: $A(j_m) = 1/\tau^*$ and $A(j_x) = \tau$.

C.2.1 Inner Problem

We first consider the inner problem (conditions for an individual good j given values for p_e and λ_e). The terms involving Foreign consumption drop out of the inner problem, as they are determined by p_e, leaving:

$$\mathcal{L}_j = u(y_j + m_j) - \left(l_j(k_j^y) + \lambda_e e_j(k_j^y)\right) y_j - \tau^* \left(l_j^*(k_j^m) + \lambda_e e_j^*(k_j^m)\right) m_j.$$

The first-order conditions for k_j^y, k_j^m, y_j, and y_j^m will clearly be identical to those for the unilaterally optimal policy. Results from the inner problem, together with market-determined outcomes, are summarized in Table 7.

All producers serving consumers in Home, whether domestic or foreign, use the same energy intensity, but Home uses a different energy intensity for serving Foreign consumers (unlike in the unilaterally optimal case). The import and export thresholds are the same as in BAU.

C.2.2 Outer Problem

We now maximize the outer Lagrangian over C_e^d and p_e. The first-order condition for C_e^d is $v'(C_e^d) = \lambda_e$, so that C_e^d is determined directly by λ_e.

The inner problem determines goods consumption in Home (and hence the labor and energy to produce these goods), while Q_e (like Q_e^*) is left to depend on the energy price. The first-order condition for p_e is therefore:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \frac{\partial L_e^W}{\partial p_e} - \frac{\partial L_g^x}{\partial p_e} - \frac{\partial L_g^{y*}}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

Since all goods producers serving consumers in Foreign face price p_e for energy, we can exploit (27). Together with $\partial v^*(C_e^{d*})/\partial p_e = p_e \partial C_e^{d*}/\partial p_e$ and $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$, the first-order condition collapses to:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \varphi^W \frac{\partial Q_e^W}{\partial p_e}.$$

C.2.3 Decentralization

In a market economy we can impose a consumption tax of $t_c = \lambda_e - p_e$ so that the after-tax price of energy embodied in goods consumed in Home, $p_e + t_c = \lambda_e$, induces the optimal level of demand. The consumption tax rate is thus:

$$t_c = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e}.$$

C.3 Extraction-Consumption Hybrid Policy

We now augment the consumption policy by allowing the planner to choose the amount of energy extraction in Home. To solve this problem we need only tweak the consumption case by replacing the competitively determined Q_e with the optimally chosen value. The inner problem is unchanged.

C.3.1 Outer Problem

We maximize the outer Lagrangian over C_e^d, Q_e and p_e. The first-order condition for Q_e is identical to that for the extraction policy. The first-order conditions for C_e^d and p_e are identical to those for the consumption policy except that $\partial Q_e^*/\partial p_e$ replaces $\partial Q_e^W/\partial p_e$ in the latter, so that:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e}.$$

C.3.2 Decentralization

In a market economy, the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_e^*/\partial p_e}.$$

Table 8: Production Policy (Goods-Sector Outcomes)

	Home		Foreign	
Home	$u'(y_j) = a_j g(v_e)$	$j < j_m$	$u'(m_j) = \tau^* a_j^* g(p_e)$	$j > j_m$
Foreign	$u^{*\prime}(x_j) = \tau a_j g(v_e)$	$j < j_x$	$u^{*\prime}(y_j^*) = a_j^* g(p_e)$	$j > j_x$

Thresholds: $A(j_m) = (1/\tau^*)g(v_e)/g(p_e)$ and $A(j_x) = \tau g(v_e)/g(p_e)$.

Since the optimal nominal extraction tax is $t_e^N = \varphi^W$ (as in the unilaterally optimal policy) the corresponding effective extraction tax is $t_e = \varphi^W - (\lambda_e - p_e)$.

C.4 Production Policy

For a production policy, we constrain the planner to choose only p_e together with the cost of using energy in Home, v_e. We don't constrain v_e to equal the shadow value of energy, λ_e.

C.4.1 Inner Problem

The inner problem reduces to competitive behavior conditional on energy costs, p_e and v_e. Goods prices are $p_j^* = a_j^* g(p_e)$ and $p_j = a_j g(v_e)$ with $p_j^m = \tau^* p_j^*$ and $p_j^x = \tau p_j$.

We get the export threshold by equating p_j^x with p_j^* at $j = j_x$, yielding $A(j_x) = \tau g(v_e)/g(p_e)$. For any good $j < j_x$ the quantity of Home exports demanded by Foreign satisfies $u^{*\prime}(x_j) = \tau a_j g(v_e)$, with $y_j^* = 0$. For any good $j > j_x$ the quantity demanded by Foreign from its local producers satisfies $u^{*\prime}(y_j^*) = a_j^* g(p_e)$, with $x_j = 0$.

We get the import threshold by equating p_j^m with p_j at $j = j_m$, yielding $A(j_m) = g(v_e)/(\tau^* g(p_e))$. For any good $j > j_m$, the quantity Home imports satisfies $u'(m_j) = \tau^* a_j^* g(p_e)$, with $y_j = 0$. For any good $j < j_m$ the quantity Home purchases from its local producers satisfies $u'(y_j) = a_j g(v_e)$, with $m_j = 0$.

Table 8 summarizes these results. The intensive margin of demand for goods produced in Home depends on v_e, the intensive margin for goods produced in Foreign depends on p_e, and the extensive margins of trade depend separately on v_e and p_e.

C.4.2 Outer Problem

We maximize the outer Lagrangian over C_e^d, p_e, and v_e. The first-order condition for C_e^d is $v'(C_e^d) = v_e$. We consider the first-order conditions for p_e and v_e together in what follows.

Two key results, analogs of (27), simplify these first-order conditions. To derive these results, start with global utility from goods consumption:

$$\int_0^1 u(c_j)dj + \int_0^1 u^*(c_j^*)dj = \int_0^{j_m} u(y_j)dj + \int_{j_m}^1 u(m_j)dj + \int_0^{j_x} u^*(x_j)dj + \int_{j_x}^1 u^*(y_j^*)dj,$$

and the global shadow cost of goods production:

$$\begin{aligned} & L_g^W + v_e(C_e^y + C_e^x) + p_e(C_e^m + C_e^{y*}) \\ &= \int_0^{j_m} a_j g(v_e) y_j dj + \int_{j_m}^1 \tau^* a_j^* g(p_e) m_j dj + \int_0^{j_x} \tau a_j g(v_e) x_j dj + \int_{j_x}^1 a_j^* g(p_e) y_j^* dj. \end{aligned}$$

Differentiating each (with respect to p_e or v_e), and then combining the two, we get:

$$\int_0^1 \frac{\partial u(c_j)}{\partial p_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj = \frac{\partial L_g^W}{\partial p_e} + v_e \frac{\partial (C_e^y + C_e^x)}{\partial p_e} + p_e \frac{\partial (C_e^m + C_e^{y*})}{\partial p_e} \tag{28}$$

and

$$\int_0^1 \frac{\partial u(c_j)}{\partial v_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial v_e} dj = \frac{\partial L_g^W}{\partial v_e} + v_e \frac{\partial (C_e^y + C_e^x)}{\partial v_e} + p_e \frac{\partial (C_e^m + C_e^{y*})}{\partial v_e}. \tag{29}$$

With these results in hand, we turn to the first-order conditions.

The FOC for p_e is:

$$\int_0^1 \frac{\partial u(c_j)}{\partial p_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \frac{\partial L_e^W}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^W}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

After substituting in $\partial v^*(C_e^{d*})/\partial p_e = p_e \partial C_e^{d*}/\partial p_e$ and $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$, it can be written as:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = -\frac{\partial u(C_g)}{\partial p_e} - \frac{\partial u^*(C_g^*)}{\partial p_e} - p_e \frac{\partial C_e^{d*}}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e} + \lambda_e \frac{\partial C_e^W}{\partial p_e}.$$

Substituting in (28) gives:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - v_e)\frac{\partial G_e}{\partial p_e} + (\lambda_e - p_e)\frac{\partial G_e^*}{\partial p_e}, \tag{30}$$

where $\partial G_e / \partial p_e = \partial (C_e^y + C_e^x)/\partial p_e$ since $\partial C_e^d / \partial p_e = 0$.

The FOC for v_e is:

$$\int_0^1 \frac{\partial u(c_j)}{\partial v_e} dj + \int_0^1 \frac{\partial u^*(c_j^*)}{\partial v_e} dj + \frac{\partial v(C_e^d)}{\partial v_e} - \frac{\partial L_g^W}{\partial v_e} - \lambda_e \frac{\partial C_e^W}{\partial v_e} = 0.$$

Substituting in $\partial v(C_e^d)/\partial v_e = v_e \partial C_e^d / \partial v_e$ and using (29) we have:

$$(\lambda_e - v_e)\frac{\partial G_e}{\partial v_e} = (p_e - \lambda_e)\frac{\partial G_e^*}{\partial v_e},$$

where $\partial G_e^* / \partial v_e = \partial (C_e^m + C_e^{y*})/\partial v_e$ since $\partial C_e^{d*}/\partial v_e = 0$.

The optimal v_e balances the two wedges, $\lambda_e - v_e$ and $\lambda_e - p_e$, based on the extent of production leakage:

$$\Lambda = \frac{-\partial G_e^* / \partial v_e}{\partial G_e / \partial v_e}.$$

Production leakage is the ratio of the increase in Foreign use of energy in goods production relative to the decline in Home use of energy, from a rise in v_e. In terms of production leakage, the first-order condition for v_e implies:

$$\frac{\lambda_e - v_e}{\lambda_e - p_e} = \Lambda.$$

Substituting it into (30), while noting that $\partial C_e^W / p_e = \partial G_e / p_e + \partial G_e^* / p_e$, yields:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^W / \partial p_e}{\partial Q_e^W / \partial p_e - (1 - \Lambda)\partial G_e^* / \partial p_e - \Lambda \partial C_e^W / \partial p_e}.$$

Since $\Lambda \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq v_e$ as well.

C.4.3 Decentralization

In a market economy we can impose a production tax of $t_p = v_e - p_e$ so that the after-tax price of energy used to produce goods in Home, $p_e + t_p = v_e$, induces the optimal energy intensity. The production tax rate is thus:

$$t_p \quad = \quad v_e - p_e = \varphi^W \frac{(1-\Lambda)\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - (1-\Lambda)\partial G_e^*/\partial p_e - \Lambda \partial C_e^W/\partial p_e}.$$

With 100% leakage, the production tax is zero. In the case of no trade in goods there is no leakage and the production tax becomes the same as the pure consumption tax.

C.5 Extraction-Production Hybrid Policy

We now allow the planner to also choose Q_e. The inner problem remains identical to the production policy.

C.5.1 Outer Problem

We maximize the outer Lagrangian over Q_e, C_e^d, p_e, and v_e. The first-order condition for Q_e is identical to that for the extraction policy. The first-order conditions for C_e^d and v_e are identical to the production policy.

The first-order conditions for p_e is identical to that for the consumption policy except that $\partial Q_e^*/\partial p_e$ replaces $\partial Q_e^W/\partial p_e$. Hence:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1-\Lambda)\partial G_e^*/\partial p_e - \Lambda \partial C_e^W/\partial p_e}.$$

C.5.2 Decentralization

In a market economy, the optimal production tax rate is:

$$t_p = v_e - p_e = \varphi^W \frac{(1-\Lambda)\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1-\Lambda)\partial G_e^*/\partial p_e - \Lambda \partial C_e^W/\partial p_e}.$$

From the first-order condition for Q_e we know that the after-tax price received by extractors must satisfy:

$$v_e - t_e^N = p_e - t_e = \lambda_e - \varphi^W.$$

The optimal nominal extraction tax is thus:

$$t_e^N = \varphi^W - (\lambda_e - v_e) = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p,$$

Table 9: Production-Consumption Hybrid Policy (Goods-Sector Outcomes)

	Home		Foreign	
Home	$u'(y_j) = a_j g(\lambda_e)$	$j < j_m$	$u'(m_j) = \tau^* a_j^* g(\lambda_e)$	$j > j_m$
Foreign	$u^{*\prime}(x_j) = \tau a_j g(v_e)$	$j < j_x$	$u^{*\prime}(y_j^*) = a_j^* g(p_e)$	$j > j_x$

Thresholds: $A(j_m) = 1/\tau^*$ and $A(j_x) = \tau g(v_e)/g(p_e)$.

while the corresponding effective extraction tax is:

$$t_e = t_e^N - t_p = \varphi^W - \frac{t_p}{1-\Lambda}.$$

C.6 Production-Consumption Hybrid Policy

We now augment the consumption policy by allowing the planner to choose v_e, the cost of energy for producing Home's exports. (The consumption policy determines the cost of producing in Home for Home consumers.) The choice of v_e therefore only alters the set of goods Home exports and the quantity demanded of those goods. We summarize the results of the inner problem in Table 9.

C.6.1 Outer Problem

We maximize the outer Lagrangian over C_e^d, p_e, and v_e. For direct energy consumption, $v'(C_e^d) = \lambda_e$. The first-order condition for p_e is:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \frac{\partial L_e^W}{\partial p_e} - \frac{\partial L_g^x}{\partial p_e} - \frac{\partial L_g^{y*}}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

We can simplify it using an analog of equation (27):

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial p_e} dj + \frac{\partial v^*(C_e^{d*})}{\partial p_e} = \frac{\partial L_g^x}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} + v_e \frac{\partial C_e^x}{\partial p_e} + p_e \frac{\partial C_e^{z*}}{\partial p_e},$$

where recall that $C_e^{z*} = C_e^{d*} + C_e^{y*}$. Substituting this result into the first-order condition, and using $\partial L_e^W / \partial p_e = p_e \partial Q_e^W / \partial p_e$, we get:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - v_e) \frac{\partial C_e^x}{\partial p_e} + (\lambda_e - p_e) \frac{\partial C_e^{z*}}{\partial p_e}.$$

The first-order condition for v_e is:

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial v_e} dj - \frac{\partial L_g^x}{\partial v_e} - \frac{\partial L_g^{y*}}{\partial v_e} - \lambda_e \frac{\partial (C_e^x + C_e^{y*})}{\partial v_e} = 0.$$

We can simplify it by substituting in the analog of equation (29):

$$\int_0^1 \frac{\partial u^*(c_j^*)}{\partial v_e} dj = \frac{\partial L_g^x}{\partial v_e} + \frac{\partial L_g^{y*}}{\partial v_e} + v_e \frac{\partial C_e^x}{\partial v_e} + p_e \frac{\partial C_e^{y*}}{\partial v_e}.$$

The result is:

$$(\lambda_e - v_e) \frac{\partial C_e^x}{\partial v_e} = (p_e - \lambda_e) \frac{\partial C_e^{y*}}{\partial v_e}.$$

The optimal v_e balances the two wedges, $\lambda_e - v_e$ and $\lambda_e - p_e$, based on what we call foreign leakage:

$$\Lambda^* = \frac{-\partial C_e^{y*}/\partial v_e}{\partial C_e^x/\partial v_e}.$$

Foreign leakage is the ratio of the increase in Foreign use of energy to serve its own customers relative to the decline in Home use of energy to serve Foreign customers, due to a rise in v_e. In terms of foreign leakage, the first-order condition for v_e becomes:

$$\frac{\lambda_e - v_e}{\lambda_e - p_e} = \Lambda^*.$$

Substituting into the first-order condition for p_e, noting that $\partial C_e^x/\partial p_e = \partial C_e^*/\partial p_e - \partial C_e^{\tilde{y}*}/\partial p_e$, yields:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - (1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

Since $\Lambda^* \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq v_e$ as well.

C.6.2 Decentralization

In a market economy the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - (1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

The optimal production tax on Home's exports is:

$$t_p = v_e - p_e = (1 - \Lambda^*)t_c.$$

C.7 Extraction-Production-Consumption Hybrid Policy

The final case augments the production-consumption policy to allow the planner to choose Q_e. Many of the results for the production-consumption case carry over, including those for individual goods shown in Table 9.

C.7.1 Outer Problem

We maximize the outer Lagrangian over Q_e, p_e, and v_e. The first-order condition for Q_e is identical to that for the extraction policy. The first-order condition for p_e is the same as for the production-consumption case, except with $\partial Q_e^*/\partial p_e$ in place of $\partial Q_e^W/\partial p_e$. The first-order condition for v_e is unchanged from the production-consumption case. Thus, we have:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

C.7.2 Decentralization

In a market economy the optimal nominal extraction tax is $t_e^N = \varphi^W$, while the effective rate is:

$$t_e = \varphi^W - (\lambda_e - p_e) = \varphi^W \frac{-(1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

The optimal pure consumption tax, applying to Home consumption of both domestically produced and imported goods is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1 - \Lambda^*)\partial C_e^{\tilde{y}*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

The optimal production tax on Home exports of goods is:

$$t_p = v_e - p_e = (1 - \Lambda^*)t_c.$$

D Solutions for Quantitative Illustration

Here we provide all the equations associated with the parameterized version of the model, that we use in Section 6 of the paper. For each outcome, we start with the BAU competitive equilibrium value that we calibrate the model to. We then show how to express the model outcomes, either for the optimal unilateral policy or one of the constained optimal policies, in terms of these BAU values.

To derive expressions for energy embodied in goods (C_e^y, C_e^m, C_e^x, and C_e^{y*}) we need to integrate of intervals of the unit continuum of goods. Here we provide general solutions to such integrals:

$$\int_{j_1}^{j_2} a_j^{1-\sigma} dj = (j_2^{1-\tilde{\sigma}} - j_1^{1-\tilde{\sigma}}) \frac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}};$$

$$\int_{j_1}^{j_2} (a_j^*)^{1-\sigma} dj = ((1-j_1)^{1-\tilde{\sigma}} - (1-j_2)^{1-\tilde{\sigma}}) \frac{(A^*)^{\tilde{\sigma}}}{1-\tilde{\sigma}}.$$

where $\tilde{\sigma} = (\sigma - 1)/\theta$. We apply these formulas with j_1 and j_2 replaced by an import threshold, an export threshold, 0, or 1, depending on the application.

D.1 Expressions to Compute the Optimal Unilateral Policy

We start with a typical example of the many equations that follow, showing how the integrals above deliver an expression for energy embodied in goods that Home produces for itself:

$$C_e^y = \int_0^{j_m} e_j(k) y_j dj = \eta g(p_e + t_b)^{-\sigma} g'(p_e + t_b) \int_0^{j_m} a_j^{1-\sigma} dj = \eta D(p_e + t_b)(\bar{j}_m)^{1-\tilde{\sigma}} \frac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}},$$

where recall that $D(p) = g'(p)g(p)^{-\sigma} = \epsilon_g(p)g(p)^{1-\sigma}/p$ and $j_m = \bar{j}_m$. In BAU we have:

$$\bar{C}_e^y = \eta D(\bar{p}_e)(\bar{j}_m)^{1-\tilde{\sigma}} \frac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}}.$$

In terms of BAU, the value under the optimal unilateral policy is:

$$C_e^y = \hat{D}(p_e + t_b)\bar{C}_e^y.$$

Using results from Section 6.1, for p equal to p_e or $p_e + t_b$, we have:

$$\hat{D}(p) = \frac{D(p)}{D(\bar{p}_e)} = \hat{g}(p)^{1-\sigma}(\epsilon_g(p)/\bar{\epsilon}_g)/(p/\bar{p}_e) = \hat{g}(p)^{\rho-\sigma}(p/\bar{p}_e)^{-\rho},$$

where:

$$\hat{g}(p) = \frac{g(p)}{g(\bar{p}_e)} = \left(\bar{\epsilon}_g(p/\bar{p}_e)^{1-\rho} + 1 - \bar{\epsilon}_g\right)^{1/(1-\rho)}.$$

Expressions for all the relevant variables under the optimal unilateral policy are in Table 10. They can be evaluated in conjunction with the global energy market-clearing condition and equation (21) to solve for p_e and t_b. Note that to evaluate (21) we need an expression for $S = \int_{j_s}^{j_x} s_j x_j dj$, which appears on the last row of Table 10.

Table 10: Expressions for Variables Under Optimal Unilateral Policy

BAU	Unilaterally Optimal in Terms of BAU
$\bar{Q}_e = E\bar{p}_e^{\epsilon_S}$	$Q_e = \max\left\{\left(\frac{p_e + t_b - \varphi^W}{\bar{p}_e}\right)^{\epsilon_S}, 0\right\}\bar{Q}_e$
$\bar{Q}_e^* = E^*\bar{p}_e^{\epsilon_S^*}$	$Q_e^* = \left(\frac{p_e}{\bar{p}_e}\right)^{\epsilon_S^*}\bar{Q}_e^*$
$\bar{j}_m = \frac{\bar{C}_e^y}{\bar{C}_e^y + \bar{C}_e^m}$	$j_m = \bar{j}_m$
$\bar{j}_x = \frac{\bar{C}_e^x}{\bar{C}_e^x + \bar{C}_e^{y*}}$	$j_x = \frac{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x}{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x + (1+\frac{t_b}{p_e}\epsilon_g(p_e))^{-\theta}\hat{g}(p_e)^{-\theta}(1-\bar{j}_x)}$
$\bar{j}_s = \bar{j}_x$	$j_s = \frac{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x}{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x + \hat{g}(p_e)^{-\theta}(1-\bar{j}_x)}$
$\bar{C}_e^d = \eta_e\bar{p}_e^{-\sigma_e}$	$C_e^d = \left(\frac{p_e+t_b}{\bar{p}_e}\right)^{-\sigma_e}\bar{C}_e^d$
$\bar{C}_e^{d*} = \eta_e^*\bar{p}_e^{-\sigma_e}$	$C_e^{d*} = \left(\frac{p_e}{\bar{p}_e}\right)^{-\sigma_e}\bar{C}_e^{d*}$
$\bar{C}_e^y = \eta D(\bar{p}_e)(\bar{j}_m)^{1-\tilde{\sigma}}\frac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C_e^y = \hat{D}(p_e+t_b)\bar{C}_e^y$
$\bar{C}_e^m = \eta(\tau^*)^{1-\sigma}D(\bar{p}_e)(1-\bar{j}_m)^{1-\tilde{\sigma}}\frac{(A^*)^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C_e^m = \hat{D}(p_e+t_b)\bar{C}_e^m$
$\bar{C}_e^{y*} = \eta^* D(\bar{p}_e)(1-\bar{j}_x)^{1-\tilde{\sigma}}\frac{(A^*)^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C_e^{y*} = \hat{D}(p_e)\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\bar{C}_e^{y*}$
$\bar{C}_e^x = \eta^*\tau^{1-\sigma}D(\bar{p}_e)(\bar{j}_x)^{1-\tilde{\sigma}}\frac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C_e^x = C_e^{x,1} + C_e^{x,2}$
$\bar{C}_e^{x,1} = \bar{C}_e^x$	$C_e^{x,1} = \hat{D}(p_e+t_b)\left(\frac{j_s}{\bar{j}_x}\right)^{1-\tilde{\sigma}}\bar{C}_e^x$
$\bar{C}_e^{x,2} = 0$	$C_e^{x,2} = B_1\bar{C}_e^x$
$\bar{S} = 0$	$S = \left(\frac{p_e+t_b}{\epsilon_g(p_e+t_b)}B_1 - B_2\right)\bar{C}_e^x$

$\tilde{\sigma} = \frac{\sigma-1}{\theta}$; $C_e^{x,1}$ covers Interval 1 and $C_e^{x,2}$ covers Interval 2 as defined in Section 3.4.2.

$$B_1 = (1-\tilde{\sigma})\left(\frac{1-\bar{j}_x}{\bar{j}_x}\right)^{\sigma/\theta}\left(\frac{\hat{g}(p_e)}{\hat{g}(p_e+t_b)}\right)^{-\sigma}\hat{D}(p_e+t_b)\frac{B\left(j_x, \frac{1+\theta}{\theta}, \frac{\theta-\sigma}{\theta}\right) - B\left(j_s, \frac{1+\theta}{\theta}, \frac{\theta-\sigma}{\theta}\right)}{\bar{j}_x^{1-\tilde{\sigma}}}.$$

$B(x,a,b) = \int_0^x i^{a-1}(1-i)^{b-1}di$ (incomplete beta function); $B_2 = \hat{g}(p_e)^{1-\sigma}\frac{\left((1-j_s)^{1-\tilde{\sigma}} - (1-j_x)^{1-\tilde{\sigma}}\right)}{\bar{j}_x(1-\bar{j}_x)^{-\tilde{\sigma}}}\frac{\bar{p}_e}{\bar{\epsilon}_g}$.

Having solved for $\boldsymbol{z}$ (carbon flows, the energy share, the border adjustment, and the corresponding energy price), we turn to equivalent variation, $EV = \Delta C^W - \varphi^W \Delta Q_e^W$. The term ΔC^W entering that expression is what we call the change in global consumption:

$$\Delta C^W = \Delta C_s^W + \Delta u(C_g) + \Delta u^*(C_g^*) + \Delta v(C_e^d) + \Delta v^*(C_e^{d*}).$$

To evaluate the change in global consumption, we define the value of spending on goods as $V_g = u'(C_g)C_g$. It follows that $\Delta u(C_g) = \frac{\sigma}{\sigma-1}\Delta V_g$ (and likewise for Foreign). We also have that $\Delta C_s^W = \Delta L^W - \Delta L_g^W - \Delta L_e^W$, with $\Delta L^W = 0$. The change in global consumption can then be expressed as:

$$\Delta C^W = \frac{\sigma}{\sigma-1}\Delta V_g + \frac{\sigma}{\sigma-1}\Delta V_g^* + \Delta v(C_e^d) + \Delta v^*(C_e^{d*}) - \Delta L_g^W - \Delta L_e^W. \tag{31}$$

Table 11 summarizes expressions to evaluate the right-hand side of (31), which involves only elements of $\boldsymbol{z}$, $\bar{\boldsymbol{z}}$, and $\boldsymbol{\Theta}$. Since V_g and V_g^* have just been introduced, we derive the expressions for these terms in Table 11 and also solve for their limiting cases.[44]

The value of Home spending on goods can be computed, using results in Table 10, by dividing spending on energy by the energy share:

$$V_g = \frac{(p_e + t_b)\left(C_e^y + C_e^m\right)}{\epsilon_g(p_e + t_b)} = \frac{1}{\bar{\epsilon}_g}\hat{g}(p_e + t_b)^{1-\sigma}\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^m\right),$$

which under BAU reduces to $\bar{V}_g = (1/\bar{\epsilon}_g)\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^m\right)$. For $\sigma \to 1$ we apply L'Hopital's rule to $\frac{\sigma}{\sigma-1}\Delta V_g$, which gives:

$$\lim_{\sigma\to 1}\frac{\left(\hat{g}(p_e + t_b)^{1-\sigma} - 1\right)\bar{V}_g}{1 - 1/\sigma} = -\ln\hat{g}(p_e + t_b)\frac{\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^m\right)}{\bar{\epsilon}_g}.$$

The value of Foreign spending on goods is more intricate, requiring a separate treatment of goods in each of the three intervals defined in Section 3.4.2. Foreign

[44] For $\sigma_e \to 1$ we can apply L'Hopital's rule to get:

$$\Delta v(C_e^d) = \ln\left(\frac{C_e^d}{\bar{C}_e^d}\right)\bar{p}_e\bar{C}_e^d; \qquad \Delta v^*(C_e^{d*}) = \ln\left(\frac{C_e^{d*}}{\bar{C}_e^{d*}}\right)\bar{p}_e\bar{C}_e^{d*}.$$

Table 11: Expressions for Computing Equivalent Variation

Change from BAU
$\Delta L_e = \frac{\epsilon_S}{\epsilon_S+1}\left(\left(\frac{Q_e}{\bar{Q}_e}\right)^{(\epsilon_S+1)/\epsilon_S} - 1\right)\bar{p}_e\bar{Q}_e$
$\Delta L_e^* = \frac{\epsilon_S^*}{\epsilon_S^*+1}\left(\left(\frac{Q_e^*}{\bar{Q}_e^*}\right)^{(\epsilon_S^*+1)/\epsilon_S^*} - 1\right)\bar{p}_e\bar{Q}_e^*$
$\Delta L_g = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e+t_b}{\bar{p}_e}\right)^{\rho}\frac{C_e^y+C_e^x}{\bar{C}_e^y+\bar{C}_e^x} - 1\right)\bar{p}_e\left(\bar{C}_e^y+\bar{C}_e^x\right)$
$\Delta L_g^* = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e+t_b}{\bar{p}_e}\right)^{\rho}\frac{C_e^m}{\bar{C}_e^m+\bar{C}_e^{y*}} + \left(\frac{p_e}{\bar{p}_e}\right)^{\rho}\frac{C_e^{y*}}{\bar{C}_e^m+\bar{C}_e^{y*}} - 1\right)\bar{p}_e\left(\bar{C}_e^m+\bar{C}_e^{y*}\right)$
$\Delta v(C_e^d) = \frac{\sigma_e}{\sigma_e-1}\left(\left(\frac{C_e^d}{\bar{C}_e^d}\right)^{(\sigma_e-1)/\sigma_e} - 1\right)\bar{p}_e\bar{C}_e^d$
$\Delta v^*(C_e^{d*}) = \frac{\sigma_e}{\sigma_e-1}\left(\left(\frac{C_e^{d*}}{\bar{C}_e^{d*}}\right)^{(\sigma_e-1)/\sigma_e} - 1\right)\bar{p}_e\bar{C}_e^{d*}$
$\Delta V_g = \left(\hat{g}(p_e+t_b)^{1-\sigma} - 1\right)\frac{\bar{p}_e\left(\bar{C}_e^y+\bar{C}_e^m\right)}{\bar{\epsilon}_g}$
$\Delta V_g^* = \left(\hat{g}(p_e)^{1-\sigma}\frac{(1-j_s)^{1-\tilde{\sigma}}}{(1-\bar{j}_x)^{-\tilde{\sigma}}} + \hat{g}(p_e+t_b)^{1-\sigma}\frac{(j_s)^{1-\tilde{\sigma}}}{(\bar{j}_x)^{-\tilde{\sigma}}} - 1\right)\frac{\bar{p}_e\left(\bar{C}_e^x+\bar{C}_e^{y*}\right)}{\bar{\epsilon}_g}$

spending on goods in Interval 1, which Home exports at a price that covers its cost, is:

$$V_g^{*,1} = \frac{(p_e+t_b)C_e^{x,1}}{\epsilon_g(p_e+t_b)} = \frac{1}{\bar{\epsilon}_g}\hat{g}(p_e+t_b)^{1-\sigma}\left(\frac{j_s}{\bar{j}_x}\right)^{1-\tilde{\sigma}}\bar{p}_e\bar{C}_e^x.$$

Foreign spending on goods in Interval 3, which Foreign produces for itself, is:

$$V_g^{*,3} = V_g^{y*} = \frac{p_e C_e^{y*}}{\epsilon_g(p_e)} = \frac{1}{\bar{\epsilon}_g}\hat{g}(p_e)^{1-\sigma}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\bar{p}_e\bar{C}_e^{y*}$$

Goods in Interval 2 are exported by Home at the price that Foreign could have produced them for itself. Foreign spending on these goods is therefore the same as if it produced them itself. Simply replacing j_x with j_s in the expression above gives overall Foreign spending on goods in Intervals 2 and 3. Combining these results we get total Foreign spending on goods, $V_g^* = V_g^{*,1} + V_g^{*,2} + V_g^{*,3}$. Since $\bar{j}_x = \bar{C}_e^x/(\bar{C}_e^x + \bar{C}_e^{y*})$ and

$1-\bar{j}_x = \bar{C}_e^{y*}/(\bar{C}_e^x + \bar{C}_e^{y*})$, we can write it as:

$$V_g^* = \left[\bar{j}_x \hat{g}(p_e+t_b)^{1-\sigma}\left(\frac{j_s}{\bar{j}_x}\right)^{1-\tilde{\sigma}} + (1-\bar{j}_x)\hat{g}(p_e)^{1-\sigma}\left(\frac{1-j_s}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\right]\frac{\bar{p}_e\left(\bar{C}_e^x+\bar{C}_e^{y*}\right)}{\bar{\epsilon}_g}.$$

In BAU it collapses to $\bar{V}_g^* = \bar{p}_e\left(\bar{C}_e^x + \bar{C}_e^{y*}\right)/\bar{\epsilon}_g$. If $\sigma = 1$ we apply L'Hopital's rule to $(\sigma/(\sigma-1))\Delta V_g^*$, which gives:

$$\begin{aligned}
&\lim_{\sigma\to 1}\frac{\left(j_s\hat{g}(p_e+t_b)^{1-\sigma}\left(\frac{j_s}{\bar{j}_x}\right)^{-\tilde{\sigma}} + (1-j_s)\hat{g}(p_e)^{1-\sigma}\left(\frac{1-j_s}{1-\bar{j}_x}\right)^{-\tilde{\sigma}} - 1\right)}{1-1/\sigma}\bar{V}_g^* \\
&= -j_s\ln\left(\hat{g}(p_e+t_b)\frac{(j_s)^{1/\theta}}{(\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* - (1-j_s)\ln\left(\hat{g}(p_e)\frac{(1-j_s)^{1/\theta}}{(1-\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* \\
&= -\ln\left(\hat{g}(p_e)\frac{(1-j_s)^{1/\theta}}{(1-\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* - j_s\ln\left(\frac{\hat{g}(p_e+t_b)(1-\bar{j}_x)^{1/\theta}(j_s)^{1/\theta}}{\hat{g}(p_e)(\bar{j}_x)^{1/\theta}(1-j_s)^{1/\theta}}\right)\bar{V}_g^* \\
&= -\left[\ln\hat{g}(p_e) + \frac{1}{\theta}\ln\left(\frac{1-j_s}{1-\bar{j}_x}\right)\right]\frac{\bar{p}_e\left(\bar{C}_e^x+\bar{C}_e^{y*}\right)}{\bar{\epsilon}_g},
\end{aligned}$$

where the second term on the third line disappears because $\frac{j_s}{1-j_s} = \frac{\hat{g}(p_e+t_b)^{-\theta}\bar{j}_x}{\hat{g}(p_e)^{-\theta}(1-\bar{j}_x)}$, as seen in Table 10.

D.2 Expressions to Compute Constrained-Optimal Policies

Many of the expressions needed for the constrained optimal policies are closely related to those for the unilateral optimal policy listed above. For policies involving a production tax, however, we need to incorporate the cost $v_e = p_e + t_p$ of energy in Home.

Table 12: Expressions for Variables under Constrained Policies (consumed in Home)

Expressions	Constrained Taxes
$p = p_e$	E
$p = p_e + t_p = v_e$	P, EP
$p = p_e + t_b = \lambda_e$	C, EC, PC, EPC
$j_m = \bar{j}_m$	E, C, EC, PC, EPC
$j_m = \dfrac{\hat{g}(p_e + t_p)^{-\theta}\bar{j}_m}{\hat{g}(p_e + t_p)^{-\theta}\bar{j}_m + \hat{g}(p_e)^{-\theta}(1 - \bar{j}_m)}$	P, EP
$C_e^d = \left(\dfrac{p}{\bar{p}_e}\right)^{-\sigma_e} \bar{C}_e^d$	
$C_e^y = \hat{D}(p)\left(\dfrac{j_m}{\bar{j}_m}\right)^{1-\tilde{\sigma}} \bar{C}_e^y$	
$C_e^m = \hat{D}(p_e)\left(\dfrac{1 - j_m}{1 - \bar{j}_m}\right)^{1-\tilde{\sigma}} \bar{C}_e^m$	E, P, EP
$C_e^m = \hat{D}(p_e + t_b)\left(\dfrac{1 - j_m}{1 - \bar{j}_m}\right)^{1-\tilde{\sigma}} \bar{C}_e^m$	C, EC, PC, EPC

E = Pure Extraction, P = Pure Production, C = Pure Consumption, EP = Extraction-Production, EC = Extraction-Consumption, PC = Production-Consumption, EPC = Extraction-Production-Consumption.

Table 13: Expressions for Variables under Constrained Policies (consumed in Foreign)

Expressions	Constrained Taxes
$p = \begin{cases} p_e \\ p_e + t_p = v_e \end{cases}$	E, C, EC P, EP, PC, EPC
$j_x = \begin{cases} \bar{j}_x \\ \dfrac{\hat{g}(p_e+t_p)^{-\theta}\bar{j}_x}{\hat{g}(p_e+t_p)^{-\theta}\bar{j}_x + \hat{g}(p_e)^{-\theta}(1-\bar{j}_x)} \end{cases}$	E, C, EC P, EP, PC, EPC
$C_e^{d*} = \left(\dfrac{p_e}{\bar{p}_e}\right)^{-\sigma_e} \bar{C}_e^{d*}$	
$C_e^{x} = \hat{D}(p)\left(\dfrac{j_x}{\bar{j}_x}\right)^{1-\tilde{\sigma}} \bar{C}_e^{x}$	
$C_e^{y*} = \hat{D}(p_e)\left(\dfrac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}} \bar{C}_e^{y*}$	

E = Pure Extraction, P = Pure Production, C = Pure Consumption, EP = Extraction-Production, EC = Extraction-Consumption, PC = Production-Consumption, EPC = Extraction-Production-Consumption.

Table 14: Expressions for Computing Equivalent Variation

Expressions	Constrained Taxes
$\Delta V_g = \left((\hat{g}(p_e))^{1-\sigma} - 1\right)\bar{V}_g$	E
$\Delta V_g = \left(\bar{j}_m\left(\hat{g}(p_e+t_p)\right)^{1-\sigma}\left(\frac{j_m}{\bar{j}_m}\right)^{1-\tilde{\sigma}} + (1-\bar{j}_m)\left(\hat{g}(p_e)\right)^{1-\sigma}\left(\frac{1-j_m}{1-\bar{j}_m}\right)^{1-\tilde{\sigma}} - 1\right)\bar{V}_g$	P, EP
$\Delta V_g = \left((\hat{g}(p_e+t_b))^{1-\sigma} - 1\right)\bar{V}_g$	C, EC, PC, EPC
$\Delta V_g^* = \left((\hat{g}(p_e))^{1-\sigma} - 1\right)\bar{V}_g^*$	E, C, EC
$\Delta V_g^* = \left(\bar{j}_x\left(\hat{g}(p_e+t_p)\right)^{1-\sigma}\left(\frac{j_x}{\bar{j}_x}\right)^{1-\tilde{\sigma}} + (1-\bar{j}_x)\left(\hat{g}(p_e)\right)^{1-\sigma}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}} - 1\right)\bar{V}_g^*$	P, EP, PC, EPC
$\Delta L_g = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e}{\bar{p}_e}\right)^{\rho}\frac{C_e^y + C_e^x}{\bar{C}_e^y + \bar{C}_e^x} - 1\right)\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^x\right)$	E
$\Delta L_g = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e+t_p}{\bar{p}_e}\right)^{\rho}\frac{C_e^y + C_e^x}{\bar{C}_e^y + \bar{C}_e^x} - 1\right)\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^x\right)$	P, EP
$\Delta L_g = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e+t_b}{\bar{p}_e}\right)^{\rho}\frac{C_e^y}{\bar{C}_e^y + \bar{C}_e^x} + \left(\frac{p_e}{\bar{p}_e}\right)^{\rho}\frac{C_e^x}{\bar{C}_e^y + \bar{C}_e^x} - 1\right)\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^x\right)$	C, EC
$\Delta L_g = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e+t_b}{\bar{p}_e}\right)^{\rho}\frac{C_e^y}{\bar{C}_e^y + \bar{C}_e^x} + \left(\frac{p_e+t_p}{\bar{p}_e}\right)^{\rho}\frac{C_e^x}{\bar{C}_e^y + \bar{C}_e^x} - 1\right)\bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^x\right)$	PC, EPC
$\Delta L_g^* = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e}{\bar{p}_e}\right)^{\rho}\frac{C_e^{y*} + C_e^m}{\bar{C}_e^{y*} + \bar{C}_e^m} - 1\right)\bar{p}_e\left(\bar{C}_e^{y*} + \bar{C}_e^m\right)$	E, P, EP
$\Delta L_g^* = \frac{1-\bar{\epsilon}_g}{\bar{\epsilon}_g}\left(\left(\frac{p_e}{\bar{p}_e}\right)^{\rho}\frac{C_e^{y*}}{\bar{C}_e^{y*} + \bar{C}_e^m} + \left(\frac{p_e+t_b}{\bar{p}_e}\right)^{\rho}\frac{C_e^m}{\bar{C}_e^{y*} + \bar{C}_e^m} - 1\right)\bar{p}_e\left(\bar{C}_e^{y*} + \bar{C}_e^m\right)$	C, EC, PC, EPC

E = Pure Extraction, P = Pure Production, C = Pure Consumption, EP = Extraction-Production, EC = Extraction-Consumption, PC = Production-Consumption, EPC = Extraction-Production-Consumption.

The expressions for V_g and V_g^* are modified under the constrained taxes. Using L'Hopital's rule, we solve for the limiting cases of $\frac{\sigma}{\sigma-1}V_g$ and $\frac{\sigma}{\sigma-1}V_g^*$.

Home spending takes three forms based on the inclusion of a production or consumption tax. The values for V_g can be computed following a procedure similar to the optimal case but with values from Table 12. Table 14 provides the respective ΔV_g expressions for each tax case. Irrespective of the tax case, home spending collapses to $\bar{V}_g = \bar{p}_e\left(\bar{C}_e^y + \bar{C}_e^m\right)$ in BAU.

Energy in home produced goods and imports face the same price p_e with the application of an extraction tax alone which results in

$$\lim_{\sigma\to 1}\frac{\left(\hat{g}(p_e)^{1-\sigma}-1\right)\bar{V}_g}{1-1/\sigma}=-\ln\hat{g}(p_e)\frac{\bar{p}_e\left(\bar{C}_e^y+\bar{C}_e^m\right)}{\bar{\epsilon}_g}.$$

The application of a production tax (pure production tax or a combination of extraction-production) changes the price Home faces. Home faces p_e+t_p while Foreign maintains p_e resulting in a two-part expression for V_g. In the limit as $\sigma\to 1$, $\frac{\sigma}{\sigma-1}\Delta V_g$ can be expressed as

$$\begin{aligned}
&\lim_{\sigma\to 1}\frac{\left(\bar{j}_m\left(\hat{g}(v_e)\right)^{1-\sigma}\left(\frac{j_m}{\bar{j}_m}\right)^{1-\tilde{\sigma}}+(1-\bar{j}_m)\left(\hat{g}(p_e)\right)^{1-\sigma}\left(\frac{1-j_m}{1-\bar{j}_m}\right)^{1-\tilde{\sigma}}-1\right)}{1-1/\sigma}\bar{V}_g\\
&=-\bar{j}_m\ln\left(\hat{g}(p_e+t_p)\frac{(j_m)^{1/\theta}}{(\bar{j}_m)^{1/\theta}}\right)\bar{V}_g-(1-\bar{j}_m)\ln\left(\hat{g}(p_e)\frac{(1-j_m)^{1/\theta}}{(1-\bar{j}_m)^{1/\theta}}\right)\bar{V}_g\\
&=-\ln\left(\hat{g}(p_e)\frac{(1-j_m)^{1/\theta}}{(1-\bar{j}_m)^{1/\theta}}\right)\bar{V}_g-\bar{j}_m\ln\left(\frac{\hat{g}(p_e+t_p)(1-\bar{j}_m)^{1/\theta}(j_m)^{1/\theta}}{\hat{g}(p_e)(\bar{j}_m)^{1/\theta}(1-j_m)^{1/\theta}}\right)\bar{V}_g\\
&=-\left[\ln\hat{g}(p_e)+\frac{1}{\theta}\ln\left(\frac{1-j_m}{1-\bar{j}_m}\right)\right]\frac{\bar{p}_e\left(\bar{C}_e^y+\bar{C}_e^m\right)}{\bar{\epsilon}_g},
\end{aligned}$$

where the second term on the third line disappears because $\frac{j_m}{1-j_m}=\frac{\hat{g}(p_e+t_p)^{-\theta}\bar{j}_m}{\hat{g}(p_e)^{-\theta}(1-\bar{j}_m)}$, as seen in Table 12.

Home produced goods and imports face the same price, p_e+t_b, with the inclusion of a consumption tax (pure consumption or a combination with extraction or production). When $\sigma=1$, $\frac{\sigma}{\sigma-1}\Delta V_g$ is

$$\lim_{\sigma\to 1}\frac{\left(\hat{g}(p_e+t_b)^{1-\sigma}-1\right)\bar{V}_g}{1-1/\sigma}=-\ln\hat{g}(p_e+t_b)\frac{\bar{p}_e\left(\bar{C}_e^y+\bar{C}_e^m\right)}{\bar{\epsilon}_g}.$$

In contrast, Foreign spending only takes on two forms given that consumption taxes do not affect foreign consumption. The values for V_g^* are computed based on expressions in Table 13. The ΔV_g^* computations themselves can be found in Table 14.

In the absence of a production tax (pure extraction, pure consumption or extraction-consumption), the limiting case of $\frac{\sigma}{\sigma-1}\Delta V_g^*$ can be expressed as

$$\lim_{\sigma\to 1}\frac{\left(\hat{g}(p_e)^{1-\sigma}-1\right)\bar{V}_g^*}{1-1/\sigma}=-\ln\hat{g}(p_e)\frac{\bar{p}_e\left(\bar{C}_e^{y*}+\bar{C}_e^x\right)}{\bar{\epsilon}_g}.$$

In tax scenarios involving a production tax (pure production or a combination with consumption or extraction), consumers face two different prices: p_e on Foreign produced goods and $p_e + t_p$ on exports. As a result, V_g^* becomes a two part expression. As $\sigma \to 1$, $\frac{\sigma}{\sigma-1}\Delta V_g^*$ gives

$$\begin{aligned}
&\lim_{\sigma\to 1} \frac{\left(\bar{j}_x \hat{g}(p_e+t_p)^{1-\sigma}\left(\frac{j_x}{\bar{j}_x}\right)^{1-\tilde{\sigma}} + (1-\bar{j}_x)\hat{g}(p_e)^{1-\sigma}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}} - 1\right)}{1-1/\sigma}\bar{V}_g^* \\
&= -\bar{j}_x \ln\left(\hat{g}(p_e+t_p)\frac{(j_x)^{1/\theta}}{(\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* - (1-\bar{j}_x)\ln\left(\hat{g}(p_e)\frac{(1-j_x)^{1/\theta}}{(1-\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* \\
&= -\ln\left(\hat{g}(p_e)\frac{(1-j_x)^{1/\theta}}{(1-\bar{j}_x)^{1/\theta}}\right)\bar{V}_g^* - \bar{j}_x \ln\left(\frac{\hat{g}(p_e+t_p)(1-\bar{j}_x)^{1/\theta}(j_x)^{1/\theta}}{\hat{g}(p_e)(\bar{j}_x)^{1/\theta}(1-j_x)^{1/\theta}}\right)\bar{V}_g^* \\
&= -\left[\ln \hat{g}(p_e) + \frac{1}{\theta}\ln\left(\frac{1-j_x}{1-\bar{j}_x}\right)\right]\frac{\bar{p}_e\left(\bar{C}_e^x + \bar{C}_e^{y*}\right)}{\bar{\epsilon}_g},
\end{aligned}$$

where the second term on the third line disappears because $\frac{j_x}{1-j_x} = \frac{\hat{g}(p_e+t_p)^{-\theta}\bar{j}_x}{\hat{g}(p_e)^{-\theta}(1-\bar{j}_x)}$, as seen in Table 13.

The derivatives with respect to v_e are used for leakage computations for policies with a production tax. The derivatives with respect to p_e appear in equilibrium conditions for all constrained policies. Note, however, that for policies without a production tax, the partial derivatives for import/export margins are 0. All formulas for key variables, welfare, and derivatives are shown in Tables **??**, 14, and 15, respectively.

We compute the partial derivative for C_e^{y*} as an example. Taking the derivative with respect to v_e yields:

$$\begin{aligned}
-\frac{\partial C_e^{y*}}{\partial v_e} &= -\frac{D(p_e)}{D(\bar{p}_e)}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\frac{1-\tilde{\sigma}}{1-j_x}\left(-\frac{\partial j_x}{\partial v_e}\right)\bar{C}_e^{y*} \\
&= \frac{1-\tilde{\sigma}}{(1-j_x)}\frac{\partial j_x}{\partial v_e}C_e^{y*}.
\end{aligned}$$

Taking the derivative with respect to p_e gives:

$$\begin{aligned}
\frac{\partial C_e^{y*}}{\partial p_e} &= \frac{D'(p_e)}{D(\bar{p}_e)}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\bar{C}_e^{y*} + \frac{1-\tilde{\sigma}}{1-j_x}\frac{D(p_e)}{D(\bar{p}_e)}\left(\frac{1-j_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma}}\left(-\frac{\partial j_x}{\partial p_e}\right)\bar{C}_e^{y*} \\
&= \frac{D'(p_e)}{D(p_e)}C_e^{y*} - \frac{1-\tilde{\sigma}}{1-j_x}\frac{\partial j_x}{\partial p_e}C_e^{y*} \\
&= -\frac{\epsilon_D(p_e)}{p_e}C_e^{y*} - \frac{1-\tilde{\sigma}}{1-j_x}\frac{\partial j_x}{\partial p_e}C_e^{y*}.
\end{aligned}$$

The partial derivative of the export margin is:

$$\begin{aligned}
\frac{\partial j_x}{\partial p_e} &= -\frac{g(v_e)^{-\theta}\bar{j}_x}{(g(v_e)^{-\theta}\bar{j}_x + g(p_e)^{-\theta}(1-\bar{j}_x))^2}(-\theta g(p_e)^{-\theta-1}g'(p_e)(1-\bar{j}_x)) \\
&= \frac{g(v_e)^{-\theta}\bar{j}_x}{g(v_e)^{-\theta}\bar{j}_x + g(p_e)^{-\theta}(1-\bar{j}_x)}\frac{g(p_e)^{-\theta}(1-\bar{j}_x)}{g(v_e)^{-\theta}\bar{j}_x + g(p_e)^{-\theta}(1-\bar{j}_x)}\left(\theta\frac{g'(p_e)}{g(p_e)}\right) \\
&= \theta\frac{g'(p_e)}{g(p_e)}j_x(1-j_x) \\
&= \theta\frac{\epsilon_g(p_e)}{p_e}j_x(1-j_x)
\end{aligned}$$

Table 15: Partial Derivatives for Policies Involving a Production Tax

Derivatives wrt v_e	Derivatives wrt p_e
$-\frac{\partial C_e^{y*}}{\partial v_e} = -[\theta(1-\tilde{\sigma})j_x\epsilon_g(v_e)]\frac{C_e^{y*}}{v_e}$	$\frac{\partial C_e^{y*}}{\partial p_e} = -\left[\epsilon_D(p_e) + \theta(1-\tilde{\sigma})j_x\epsilon_g(p_e)\right]\frac{C_e^{y*}}{p_e}$
$\frac{\partial C_e^{x}}{\partial v_e} = -\left[\epsilon_D(v_e) + \theta(1-\tilde{\sigma})(1-j_x)\epsilon_g(v_e)\right]\frac{C_e^{x}}{v_e}$	$\frac{\partial C_e^{x}}{\partial p_e} = [\theta(1-\tilde{\sigma})(1-j_x)\epsilon_g(p_e)]\frac{C_e^{x}}{p_e}$
$-\frac{\partial C_e^{m}}{\partial v_e} = -[\theta(1-\tilde{\sigma})j_m\epsilon_g(v_e)]\frac{C_e^{m}}{v_e}$	$\frac{\partial C_e^{m}}{\partial p_e} = -\left[\epsilon_D(p_e) + \theta(1-\tilde{\sigma})j_m\epsilon_g(p_e)\right]\frac{C_e^{m}}{p_e}$
$\frac{\partial C_e^{y}}{\partial v_e} = -\left[\epsilon_D(v_e) + \theta(1-\tilde{\sigma})(1-j_m)\epsilon_g(v_e)\right]\frac{C_e^{y}}{v_e}$	$\frac{\partial C_e^{y}}{\partial p_e} = \left[\theta(1-\tilde{\sigma})(1-j_m)\epsilon_g(p_e)\right]\frac{C_e^{y}}{p_e}$

$\epsilon_D(p) = -pD'(p)/D(p)$, $\epsilon_g(p) = pg'(p)/g(p)$ for $p \in \{p_e, v_e\}$

E Data

We combine data on fossil fuel extraction with data on where fossil fuels are explicitly consumed, used in goods production, and implicitly consumed (via goods consumption). Our common unit for energy is gigatons of CO_2 released by combustion.

We consider several scenarios for the regions that define Home and Foreign. In the first, members of the Organization for Economic Cooperation and Development (OECD37) are Home and all other countries are Foreign, as in Table 4. The alternative scenarios for Home (with all other countries Foreign) are: (i) the European Union prior to Brexit (EU28), (ii) the European Union plus the United States, and (iii) the OECD plus China. Tables 16, 17 and 18 show the data for these other scenarios.

Our data source for energy consumption in these tables is the Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2018. Carbon dioxide embodied in global consumption in 2018 is 33.63 gigatons. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and global totals. The data are provided in units of terajoules. In order to keep the units consistent with the energy consumption data (gigatons of carbon dioxide), we apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are: (i) coal and coal products, (ii) natural gas, (iii) peat and peat products, (iv) oil shale and oil sands, and (v) crude oil, NGL, and feedstocks. The emission factors are for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. Using this calculation, global extraction is 37.26 gigatons of carbon dioxide.

To explain the discrepancy between global consumption and global extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land.

Table 16: Carbon Flows: Home as the European Union

	Home	Foreign	Direct	Total
Home	$\bar{C}_e^y = 3.4$	$\bar{C}_e^m = 1.2$	$\bar{C}_e^d = 1.1$	$\bar{C}_e = 5.7$
Foreign	$\bar{C}_e^x = 0.5$	$\bar{C}_e^{y*} = 23.8$	$\bar{C}_e^{d*} = 3.6$	$\bar{C}_e^* = 27.9$
Direct	$\bar{C}_e^d = 1.1$	$\bar{C}_e^{d*} = 3.6$		
Total	$\bar{G}_e = 5.0$	$\bar{G}_e^* = 28.6$		$\bar{C}_e^W = 33.6$
Extraction	$\bar{Q}_e = 0.9$	$\bar{Q}_e^* = 32.7$		$\bar{Q}_e^W = 33.6$

The table shows carbon flows in 2018 from the European Union (Home) to all other countries (Foreign), measured as gigatons of CO_2.

Table 17: Carbon Flows: Home as the EU and United States

	Home	Foreign	Direct	Total
Home	$\bar{C}_e^y = 5.6$	$\bar{C}_e^m = 2.1$	$\bar{C}_e^d = 2.0$	$\bar{C}_e = 9.7$
Foreign	$\bar{C}_e^x = 0.8$	$\bar{C}_e^{y*} = 20.4$	$\bar{C}_e^{d*} = 2.7$	$\bar{C}_e^* = 23.9$
Direct	$\bar{C}_e^d = 2.0$	$\bar{C}_e^{d*} = 2.7$		
Total	$\bar{G}_e = 8.4$	$\bar{G}_e^* = 25.2$		$\bar{C}_e^W = 33.6$
Extraction	$\bar{Q}_e = 5.6$	$\bar{Q}_e^* = 28.0$		$\bar{Q}_e^W = 33.6$

The table shows carbon flows in 2018 from the European Union and United States (Home) to all other countries (Foreign), measured as gigatons of CO_2.

Given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis. According to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to global extraction, we get a number close to global consumption ($37.26 \times 0.92 = 34.28$, vs. 33.63). To make global extraction equal to global consumption, we divide the original extraction data by $1.019 = 34.28/33.63$. Tables 4, 16, 17, and 18 display the resulting data we use in our calibration for each regional scenario (defining Home and Foreign).

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard-Wexler, and De Loecker (2018, 2019) on the distribution across oil fields of extraction costs.

	Home	Foreign	Direct	Total
Home	$\bar{C}_e^y = 17.8$	$\bar{C}_e^m = 1.9$	$\bar{C}_e^d = 3.0$	$\bar{C}_e = 22.7$
Foreign	$\bar{C}_e^x = 1.4$	$\bar{C}_e^{y*} = 7.8$	$\bar{C}_e^{d*} = 1.7$	$\bar{C}_e^* = 10.9$
Direct	$\bar{C}_e^d = 3.0$	$\bar{C}_e^{d*} = 1.7$		
Total	$\bar{G}_e = 22.2$	$\bar{G}_e^* = 11.4$		$\bar{C}_e^W = 33.6$
Extraction	$\bar{Q}_e = 16.9$	$\bar{Q}_e^* = 16.7$		$\bar{Q}_e^W = 33.6$

Table 18: Carbon Flows: Home as the OECD and China

The table shows carbon flows in 2018 from the OECD and China (Home) to all other countries (Foreign), measured as gigatons of CO_2.

(We take the distribution for oil extraction to be representative of all fossil fuels.) The data come in the form of quantiles ($q = 0.05, 0.10, ..., 0.95$), separately for the EU, the US, OPEC, and ROW (i.e. q% of oil in a region is extracted at a cost below \$$a$ per barrel). We aggregate the EU and US to form Home and we aggregate OPEC and ROW to form Foreign. To aggregate the quantiles for two regions, we combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 11 and 12, to reveal the supply elasticity as the slope.

The most costly oil fields in either region would be the first to be abandoned under a carbon policy that lowered the after-tax energy price. Thus, the upper end of the cost distribution is most relevant for calibrating supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median.

Figure 11: Calibration of the Extraction Supply Elasticity in Home

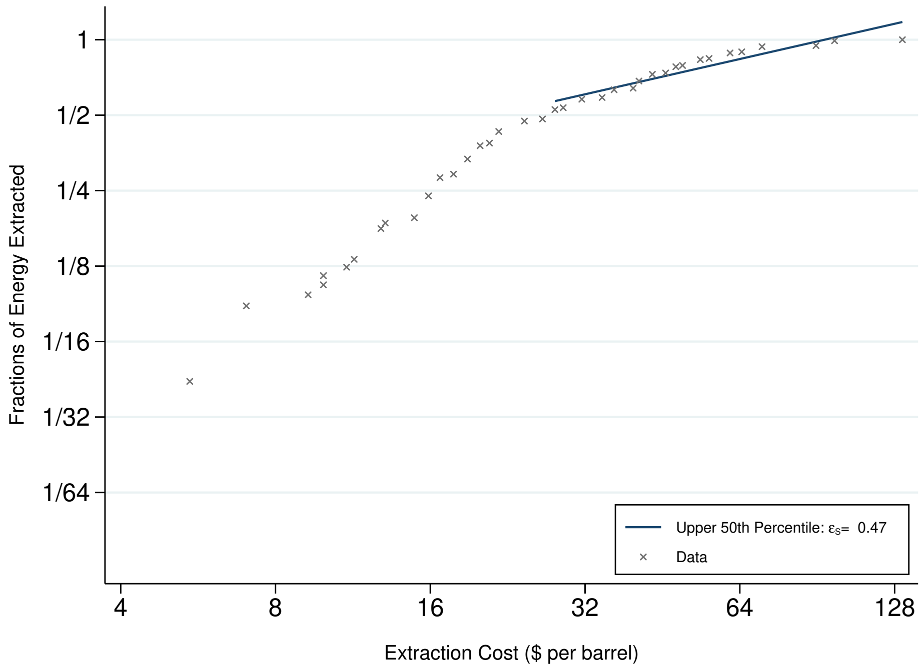


Figure 12: Calibration of the Extraction Supply Elasticity in Foreign