Physics Internal Assessment: Experiment Report

An Increasing Flow of Knowledge:

Investigating Torricelli's Law, or the Effect of Height on Flow Rate

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If you sit in a bathtub while it is draining, you may notice that the water level seems to drop quickly initially, but then slows down as the height of the water decreases. Although you could attribute this time distortion to being impatient and wanting the bathtub clean, it is also possible that there is a physics explanation for this. This paper will investigate the correlation between the height of water and the flow rate of the water. Our hypothesis is that as the height of the water increases, the flow rate of the water will increase linearly.

In order to perform this experiment, we will use a two liter bottle and drill a small hole in the side. We will fill the bottle with water and allow the water to drain from the bottle. The independent variable is the height of water above the hole in the two liter bottle, measured in meters. The dependent variable is the flow rate of water out of the bottle, measured in milliliters per second. However, these two variables are very difficult to measure directly with a high level of accuracy. Therefore, for practical purposes, we shall measure the amount of water poured into the two liter bottle as the independent variable, and the amount of water drained in the duration of 5.0 seconds as the dependent variable. We can easily convert these values to the units required for our lab.

One important control for this lab is that the walls of the two liter bottle are very similar to a perfect cylinder. Otherwise, we could not convert the volume of water in the bottle to the height of water above the drilled hole. Therefore, instead of drilling the hole at the bottom of the two liter bottle, where there are irregular shapes created by the "feet" of the two liter bottle, we will drill the hole roughly 5 cm above the bottom of the two liter bottle, at a region where the two liter bottle approximates a perfect cylinder.

Another control for this lab is the temperature of the water used. Because temperature has an effect on the density of water, a factor that could potentially affect the experiment, we want to keep this variable as constant as possible.

A very important control for this lab is the size of the hole drilled. For our experiment, we will only deal with a single hole of a constant diameter. Because we understand that a larger hole could potentially increase the flow rate, we choose to not change the diameter of the hole. In addition, having multiple holes could drain the water much faster than with a single hole, so we will only have one hole.

Maintaining a constant pressure on top of the water bottle is very important. If we keep the cap on the two liter bottle, this pressure above the water would change as the water drains. However, if we leave the cap off the two liter bottle, the pressure on top of the water would be a constant of 1 atmosphere, or roughly 101325 pascals. In addition, the pressure of the environment outside of the room should also be kept at a constant of roughly 101325 pascals.

The ranges of volumes of water we wish to fill the two liter bottle are chosen with care. The minimum value, of 500 mL, was chosen because below that amount, water does not flow properly from the hole. Rather than spewing out in a constant spray that could be collected by the beaker, the water would trickle down the side of the two liter bottle. This prevented us from collecting any data for volumes below 500 mL. The maximum amount of water we chose to fill the bottle with is 1700 mL because above that level of water, the two liter bottle's shape begins to change. The top of a two liter bottle is curved to form a small opening on the top. Therefore, using more than 1700 mL of water would distort the true purpose of the lab: to investigate the correlation of height on the flow rate.

We choose to measure 100 mL differences for water volume differences in order to get a full range of values between 500 and 1700 mL. In addition, we determined that at 1700 mL, the flow rate allows for the water to drain nearly 100 mL over the 5 seconds. If we used a value that was less than 100 mL as the difference between successive conditions, we would find that one trial would "overlap" into another trial. This is a fundamental error in the lab, but it is unavoidable provided the equipment that we were given. More on this error will be discussed later.

We choose use a period of 5.0 seconds for several reasons. Most importantly, this period allows us to collect a wide range of data for amount of drainage, between 27 and 96 mL. If we used a longer duration, such as 10.0 seconds, we would find that the amount of water drained would be roughly between 50 and 200 mL, which would overlap into the other trials far too much. The reason why we did not choose a value less than 5.0 seconds, such as 1.0 seconds, is because such a small value greatly amplifies the problem of human reaction time. Human reaction time is roughly 0.20 seconds, so using 1.0 seconds would result in a 20% error minimum. In addition, the values collected would fall between 5mL and 25 mL, which would be too small to be significant.

Finally, we chose to use 6 trials for each condition, primarily because we were concerned with the errors associated with liquids. As it is possible for a couple of droplets of water to spill here and there, disrupting the accuracy, we use more trials so that there is more ability to gather uncertainty data. Using 6 trials would exceed the 5 trial minimum required for statistically significant data.

Materials:

- Drill

- Drill bit with diameter of $\frac{3}{16}$ $inch$ $(4.8 \, mm)$

- Two Liter Bottle

- 100 ± 1 mL Graduated Cylinder

- Large bucket (Capacity greater than 2000 mL)

- 250 mL Beaker

- Water

- Timer App for iPhone 4 (Big Stopwatch by Yuri Yasoshima)

- String

- Scissors

- Ruler $(100 \pm 0.1 \, cm)$

Procedure:

1) Prepare the drill with the drill bit of diameter $\frac{3}{16}$ $inch$ $(4.8 \, mm)$

2) Drill a hole in the two liter bottle roughly 10 cm from the bottom of the bottle. There should be a thin line at the location, and the plastic should be relatively straight above the line.

3) Fill the two liter bottle with water such that the water level is right at the hole. Measure how much water is currently in the bottle by using the 100 mL graduated cylinder. Record this number.

4) Wrap string around the circumference of the two liter bottle. Use the scissors to cut off this piece of string.

5) Measure the length of the string with the ruler and record the length as the circumference of the two liter bottle.

6) Using the graduated cylinder, measure 500 mL water and pour into the large bucket.

7) Pour the water from the large bucket into the two liter bottle from the top by unscrewing the cap, making sure to seal the drilled hole with finger to prevent loss of water.

8) Unscrew the cap on the two liter bottle.

9) Drain the water from the two liter bottle into the 250 mL bottle. Use the timer app such that you unseal (remove finger from hole) the bottle at 0.00 seconds and reseal (place finger on hole) the bottle at 5.00 seconds.

10) Replace the cap on the two liter bottle and invert the bottle such that no water escapes.

11) Measure the amount of water drained by using the 100 mL graduated cylinder. Record value.

12) Unscrew the cap on the two liter bottle and replace water that was drained.

13) Repeat steps 8-12 five more times.

14) Repeat steps 6-13 thirteen more times, using increments of 100 mL for the amount of water poured by the 100 mL graduated cylinder.

Illustration:

Fig 1: Experimental Setup
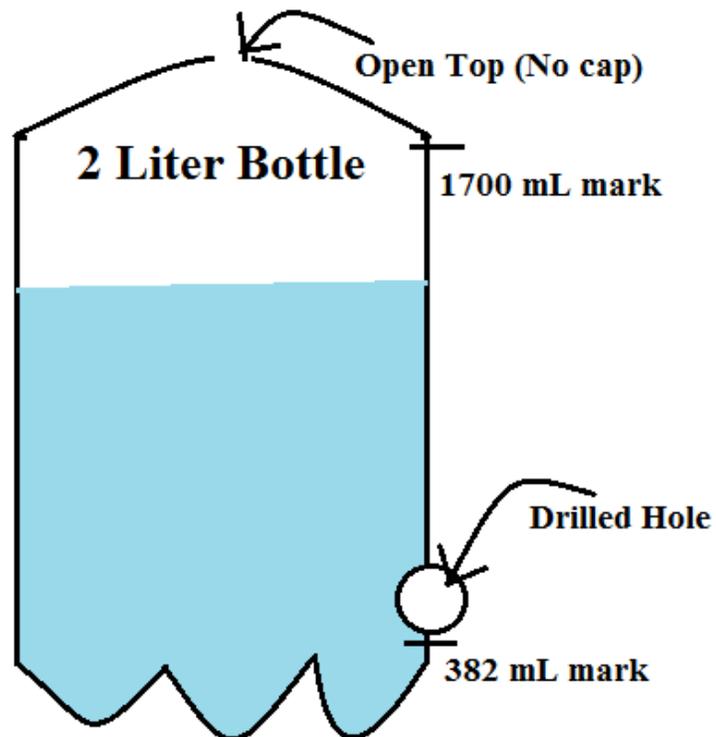
Fig 2: Schematic of Two Liter Bottle

Data Collection and Processing:

Volume of Water (mL)	Volume Lost over 5 seconds(± 1mL)						Error in Measuring Volume of Water (mL)
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	
500	25	26	27	27	27	27	5
600	36	37	36	35	37	38	6
700	45	45	44	47	43	43	7
800	52	53	51	50	54	50	8
900	56	58	61	59	57	57	9
1000	66	63	62	62	65	61	10
1100	69	67	68	67	66	67	11
1200	72	72	69	71	72	72	12
1300	74	73	75	73	72	77	13
1400	77	80	81	80	79	78	14
1500	86	84	84	83	80	83	15
1600	86	90	87	88	88	89	16
1700	89	90	90	91	89	93	17
1800	95	95	96	94	92	92	18

Volume of Water Below Hole (mL)	Error in Volume Below (mL)	Circumference of Two Liter Bottle (cm)	Error in Circumference of Bottle (cm)
382	4	34.4	0.4

An explanation for the large error in the circumference of the bottle: The string that was used to determine the circumference of the two liter bottle was found to be rather elastic. Therefore, the maximum that the string could be stretched was by an additional 0.4 cm. Therefore, the error in the circumference of the bottle was determined to be 0.4 cm, rather than the 0.1 cm that the measurement error of using the ruler would be.

The error in measuring the volume above is determined by the number of times the 100 mL graduated cylinder is used to pour water. We assume that because the graduated cylinder has a measurement error of $\pm 1\ mL$, each time we use this measurement, the error becomes compounded. Therefore, measuring 500 mL of water requires using the graduated cylinder five times, which would result in an error of $\pm 5\ mL$.

Next, the total volume of water is converted into the amount of water that is above the hole. This is done by subtracting the amount of water that was poured into the bottle (recorded in the data table) from the amount of water that is below the hole (recorded in step 3 to be 382 mL). Therefore, as in the 500 mL case,

$$Volume\ Above\ (mL) = Total\ Volume\ (mL) - Volume\ Below\ (mL) = 500\ mL - 382\ mL$$

$$Volume\ Above\ (mL) = 184\ mL$$

The error in this reading is determined by the larger error between measuring the total volume and the volume below. Because the error in the volume below is set at $\pm 4mL$, the error for the total volume is always greater, and therefore, should be used.

In order to create useful data, we will first convert the volume loss into flow rate. This will be done by

$$Flow\ Rate = \frac{Volume\ Lost}{Time\ for\ Loss}$$

Therefore, the flow rate for Trial 1 of the 500 mL trial is

$$Flow\ Rate = \frac{25\ mL}{5.0\ seconds} = 5.0\ \frac{mL}{sec}$$

However, we also need to calculate the errors for this calculation. When dividing two numbers that both have measurement errors, we should add the percent errors and multiply by the final result. Our measurement error for volume lost is 1 mL due to the uncertainty in the

graduated cylinder. The measurement error for the time for loss is equivalent to human reaction time, or roughly 0.2 seconds. Therefore for the same case as above,

$$Error\ in\ Flow\ Rate = (Time\ \%Error + Volume\ \%Error) \cdot Flow\ Rate$$

$$= \left(\frac{0.2\ seconds}{5.0\ seconds} + \frac{(1\ mL)}{25\ mL}\right) \cdot 5.0\frac{mL}{second} = 0.40\frac{mL}{second}$$

Therefore, the following data tables are created:

Volume Above (mL)	Flow Rate (mL/sec)						Error in measuring Volume Above (mL)
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	
118	5.0	5.2	5.4	5.4	5.4	5.4	5
218	7.2	7.4	7.2	7.0	7.4	7.6	6
318	9.0	9.0	8.8	9.4	8.6	8.6	7
418	10.4	10.6	10.2	10.0	10.8	10.0	8
518	11.2	11.6	12.2	11.8	11.4	11.4	9
618	13.2	12.6	12.4	12.4	13.0	12.2	10
718	13.8	13.4	13.6	13.4	13.2	13.4	11
818	14.4	14.4	13.8	14.2	14.4	14.4	12
918	14.8	14.6	15.0	14.6	14.4	15.4	13
1018	15.4	16.0	16.2	16.0	15.8	15.6	14
1118	17.2	16.8	16.8	16.6	16.0	16.6	15
1218	17.2	18.0	17.4	17.6	17.6	17.8	16
1318	17.8	18.0	18.0	18.2	17.8	18.6	17
1418	19.0	19.0	19.2	18.8	18.4	18.4	18

Volume Above (mL)	Error in Flow Rate (mL/sec)					
	Error Trial 1	Error Trial 2	Error Trial 3	Error Trial 4	Error Trial 5	Error Trial 6
118	0.40	0.41	0.42	0.42	0.42	0.42
218	0.49	0.50	0.49	0.48	0.50	0.50
318	0.56	0.56	0.55	0.58	0.54	0.54
418	0.62	0.62	0.61	0.60	0.63	0.60
518	0.65	0.66	0.69	0.67	0.66	0.66
618	0.73	0.70	0.70	0.70	0.72	0.69
718	0.75	0.74	0.74	0.74	0.73	0.74
818	0.78	0.78	0.75	0.77	0.78	0.78
918	0.79	0.78	0.80	0.78	0.78	0.82
1018	0.82	0.84	0.85	0.84	0.83	0.82
1118	0.89	0.87	0.87	0.86	0.84	0.86
1218	0.89	0.92	0.90	0.90	0.90	0.91
1318	0.91	0.92	0.92	0.93	0.91	0.94
1418	0.96	0.96	0.97	0.95	0.94	0.94

As we study this data, we realize that the error from measurements is greater than the standard deviation of the measured flow rates. IB guidelines recommend for the maximum error in measurement to be used rather than using the standard deviation of the flow rate data.

Next, we want to process our data to find the average flow rate. Using the 118 mL data:

$$Flow\ Rate_{Avg} = \frac{T_1 + T_1 + T_3 + T_4 + T_5 + T_6}{6}$$

$$= \frac{5.0 + 5.2 + 5.4 + 5.4 + 5.4 + 5.4}{6} = 5.4\,\frac{mL}{seconds}$$

Our data tables are of the following:

Volume vs. Flow Rate			
Volume Above (mL)	Error in Volume Above (mL)	Average Flow Rate (mL / sec)	Max Error in Flow Rate (mL / sec)
118	5	5.3	0.42
218	6	7.3	0.50
318	7	8.9	0.58
418	8	10.3	0.63
518	9	11.6	0.69
618	10	12.6	0.73
718	11	13.5	0.75
818	12	14.3	0.78
918	13	14.8	0.82
1018	14	15.8	0.85
1118	15	16.7	0.89
1218	16	17.6	0.92
1318	17	18.1	0.94
1418	18	18.8	0.97

However, the goal of this report is to not determine the relationship between the volume of water above the hole with the flow rate; rather, we wish to determine the relationship between the height of the water with the fluid rate. Therefore, we shall convert the volume of water above the hole into height of the water.

Because we know the volume of water in milliliters, we are able to convert this value to cubic meters. Afterwards, we shall assume that our two liter bottle is a perfect cylinder. Therefore, the volume is equivalent to the cross sectional area multiplied by the height. Dividing the volume by the cross sectional area would therefore give the height that we desire.

We first convert milliliters to cubic meters by the following:

$$1L = 1\ dm^3 \therefore 1\ mL \cdot \frac{1\ L}{1000\ mL} \cdot \frac{1\ dm^3}{1\ L} \cdot \left(\frac{1\ m^3}{100\ dm^3}\right) = \frac{1\ meter^3}{100000\ mL}$$

Using this conversion factor, we calculate that

$$118 \; mL \cdot \frac{1 \; m^3}{100000 \; mL} = 0.000118 \; m^3$$

We determine the cross sectional area of the two liter bottle using the value for the

bottle's circumference that we determined in step 5 (34.4 cm). From this, we know that

$$Circle_{Area} = \pi r^2 = \pi \left(\frac{Area_{Circumference}}{2\pi} \right)^2 = \frac{Area^2_{Circumference}}{4\pi} = \frac{(34.4 \; cm)^2}{4\pi} = 94.2 \; cm^2$$

However, we also understand that $1 \; Pascal = \frac{1 \; Newton}{1 \; meter^2}$. Therefore, converting from

square centimeters to square meters:

$$Circle_{Area} = 94.17 \; cm^2 \cdot \frac{(1 \; m^2)}{10000 \; cm^2} = 0.0094 \; m^2$$

To determine the height of the water, we calculate for the 118 mL condition:

$$Height = \frac{0.000118 \; m^3}{0.0094 \; m^2} = 0.013 \; meters$$

To determine the error in this calculation, we would take the percent error for the area

and for the volume, add them together, and multiply by the final pressure.

However, the error for the area is a result of squaring the circumference value, which

changes the percent error. Therefore, this error is evaluated to be

$$Error = 2 \cdot \frac{\Delta length}{Circumference} \cdot Area$$

$$= 2 \cdot \frac{0.004 \; m}{.344 \; m} \cdot 0.0094 m^2 = 0.0002186 \; m^2$$

Therefore, the total error in height is calculated by:

$$Height_{Error} = \left(\frac{0.000219 \; m^2}{0.0094 \; m^2} + \frac{0.000005 m^3}{0.000118 \; m^3} \right) \cdot 0.013 \; meters = 0.00082 \; meters$$

This produces the following data table:

Height vs. Flow Rate			
Height (m)	Error in height (m)	Flow Rate (mL/sec)	Error in Flow Rate (mL/sec)
0.013	0.00082	5.30	0.42
0.023	0.00117	7.30	0.50
0.034	0.00153	8.90	0.58
0.044	0.00188	10.33	0.63
0.055	0.00223	11.60	0.69
0.066	0.00258	12.63	0.73
0.076	0.00294	13.47	0.75
0.087	0.00329	14.27	0.78
0.097	0.00364	14.80	0.82
0.108	0.00399	15.83	0.85
0.119	0.00435	16.67	0.89
0.129	0.00470	17.60	0.92
0.140	0.00505	18.07	0.94
0.151	0.00540	18.80	0.97

Therefore, the following graph is produced:

Flow Rate of Water vs. Height of Water

$y = 49.1x^{0.50}$
$R^2 = 0.999$
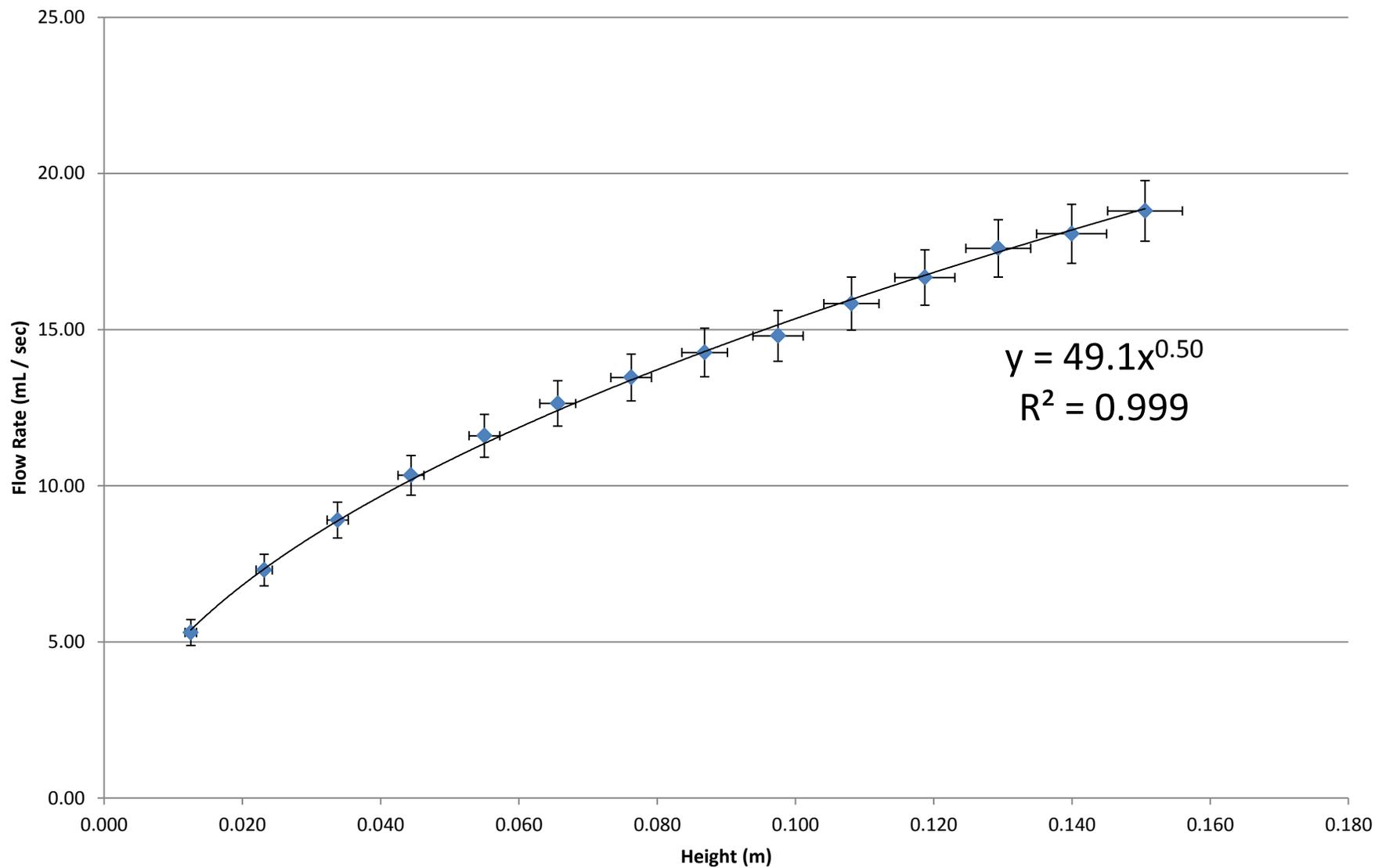
We can tell that our data seems to fit a square root curve very nicely. Our x axis is the height as measured in meters, while the y axis is the flow rate of the water out of the drilled hole as measured in milliliters per second. The x and y intercepts are at (0, 0), which implies that when there is no water above the hole, the flow rate will be zero milliliters per second. This makes a lot of sense, as if there is no water to drain out of the two liter bottle, the flow rate would be zero.

We also notice that the square root function has a domain of $\{x | x \in \mathbb{Z}^+\}$. This makes sense, as if the height of the water is below the height of the hole, the flow rate would always be zero. Although the "undefined" portion is questionable, it is not sufficient to ignore the trend.

The next step is to linearize the data. We will use the variable z to represent this linearized value. Our current regression indicates a square root relationship between the height of the water and the flow rate of the water. Therefore, we can linearize the data by taking the square root of the pressure (x-variable) as follows:

$$Linearized\ X = z = \sqrt{Height}$$

Using the 0.013 meter experiment, we have

$$z = \sqrt{0.013\ meters} = 0.11\ \sqrt{meter}$$

Evaluating the error of this linearized data is done by

$$z_{error} = \frac{1}{2} \cdot \frac{0.00082\ meters}{0.013\ meters} \cdot 0.11\ \sqrt{meters} = 0.0037\ m^{0.5}$$

Therefore, our data table is:

Linearized Height vs. Flow Rate			
Linearized Height $(\sqrt{m})$	Error in Height $(\sqrt{m})$	Flow Rate (mL/sec)	Error in Flow Rate (mL/sec)
0.112	0.0037	5.30	0.42
0.152	0.0039	7.30	0.50
0.184	0.0042	8.90	0.58
0.211	0.0045	10.33	0.63
0.235	0.0048	11.60	0.69
0.256	0.0050	12.63	0.73
0.276	0.0053	13.47	0.75
0.295	0.0056	14.27	0.78
0.312	0.0058	14.80	0.82
0.329	0.0061	15.83	0.85
0.345	0.0063	16.67	0.89
0.360	0.0065	17.60	0.92
0.374	0.0068	18.07	0.94
0.388	0.0070	18.80	0.97

Next, we determine the lines of maximum and minimum slope. The maximum slope is determined from the maximum uncertainty values of the highest and lowest points, so that the two points responsible for this calculation would be:

$(x_{min} + Uncert, \ y_{min} - Uncert)$ and $(x_{max} - Uncert, y_{max} + Uncert)$. Using our data, these two points would be (0.112+0.0037, 5.30-0.42) and (0.288-0.0070, 18.80+0.97), giving us the points (0.116, 4.88) and (0.381, 19.77). As the equation for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$, we can process by

$$m = \frac{19.77 - 4.88}{0.381 - 0.116}$$

$$m = \frac{14.89}{0.265}$$

$$m = 56.19 \frac{ml}{second \cdot \sqrt{meter}}$$

The minimum slope is similarly calculated, but with reversed signs in regards to adding or subtracting the uncertainty. Therefore, the points should be $(x_{min} - Uncert,\ y_{min} + Uncert)$ and $(x_{max} + Uncert, y_{max} - Uncert)$, or (0.112-0.0037, 5.30+0.42) and (0.388+0.0070, 18.80-0.97), giving us the points (0.108, 5.72) and (0.395, 17.83). Again, calculating for slope by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{17.83 - 5.72}{0.395 - 0.108}$$

$$m = \frac{12.11}{0.287}$$

$$m = 42.195 \frac{mL}{second \cdot \sqrt{meter}}$$

Putting all of this information together, the following graph is produced:

Flow Rate of Water vs. Linearized Height

$y = 48.4z + 0.0173$

$R^2 = 0.999$

Max Slope

$y = 56.1z - 1.60$

Min Slope

$y = 42.3z + 1.14$

Flow Rate (mL/sec)

Linearized Height (m^0.5)
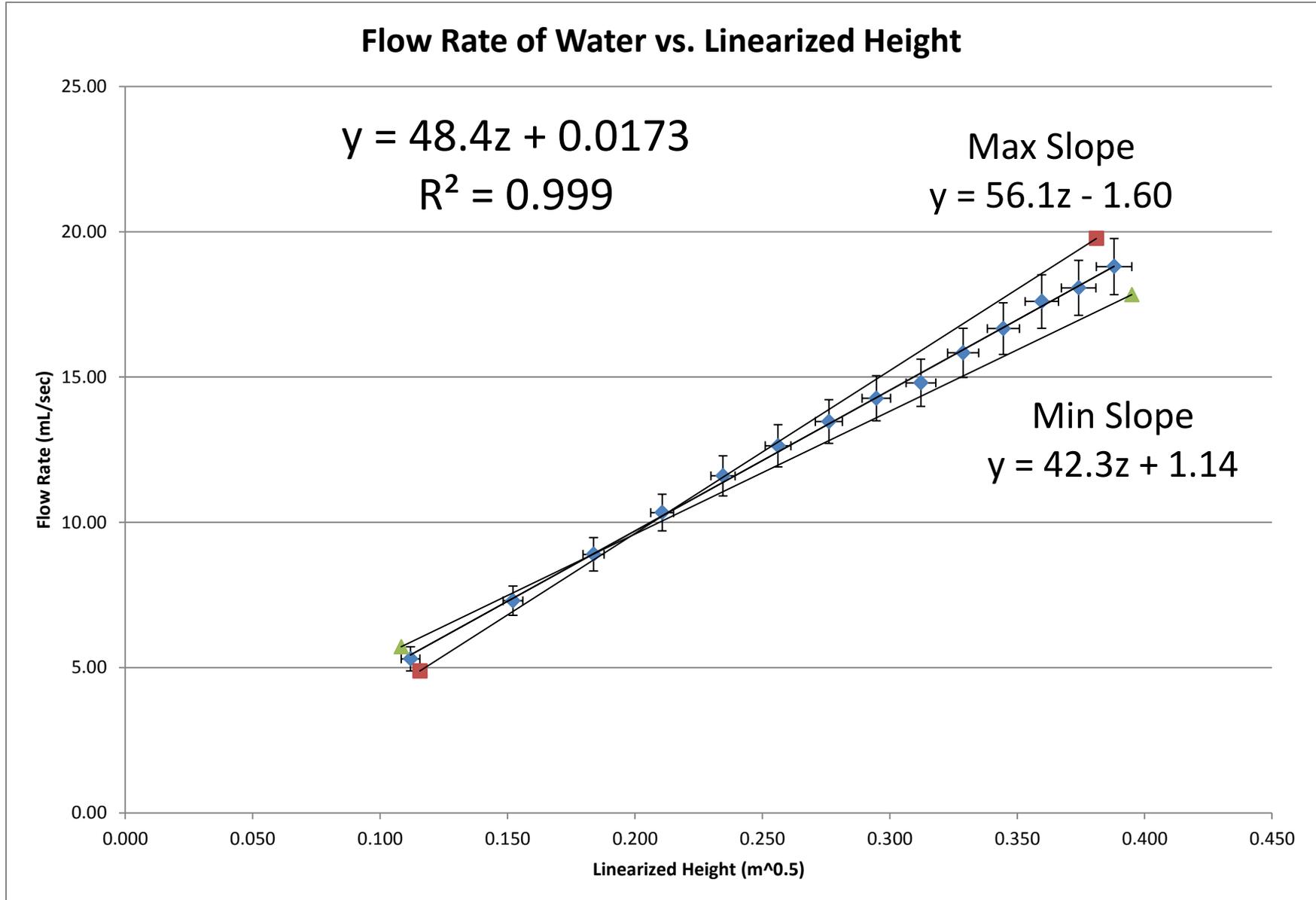
For this graph, we notice that the z variable is the linearized height, as shown in the SI units of $\sqrt{meters}$, while the y variable is the flow rate as measured in milliliters per second. We see that our linearization is a very close fit. The correlation factor is 0.999, but more importantly, the best fit line nearly passes through every single data point and the error regions associated with those data points. In addition, the lines of maximum and minimum slopes are well within the error regions. The large number of data points used for this experiment provides further confidence in the validity of our data. For this data, it is not possible that the error regions alone could have caused the trend that we see. Instead, the values clearly show a positive increase, from (0.112, 5.30) to (0.388, 18.80). There is a constant increase between every data point which supports the high correlation value of this graph. Finally, the minimum and maximum slopes are both positive and are very close to the slope of the best fit line.

The slope in this graph is interpreted to mean how rapidly the flow rate increases as the square root of the height increases. If the linearized height increases by $0.300 \sqrt{meters}$, the increase in flow rate would be predicted to an increase of $14.52 \frac{mL}{second}$. These values correspond with our data, as the increase between the minimum and maximum points for linearized height, $0.276 \sqrt{meters}$, corresponds to the flow rate increasing by $13.5 \frac{mL}{second}$.

Our best fit regression line has the equation of $y = 48.4 \, z + 0.0173$. The reason for the apparent mismatch of precision is because in our scenario, the number of significant figures is more important that the actual precision of the number. All of these numbers are calculated values, not directly measured values. Therefore, it is valid to use a constant three significant figures in all of these equations.

In this graph, the z intercept is at (0, 0.0173) while the y intercept is at (-.000357, 0). The expected z and y intercept would be at (0, 0), because when the pressure is zero, we expect the flow rate to also be 0. We will attribute this to small measurement errors in this experiment.

However, the relationship of $Height \propto \sqrt{Flow\ Rate}$ is confirmed to be true with the linearized graph. Using the data from both the linearized graph as well as the data from the regular graph, we realize that the correlation is very high and indicates a very accurate experiment.

Conclusion:

Before we dive into the results of our lab, let us return to an old equation: The law of conservation of energy[1]. This states that

$$W = \Delta U + \Delta K$$

However, we can also express work done by a fluid by using the definition of pressure and work to determine that

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

For a liquid, we understand that $A_1 \Delta x_1 = V_1$, so that

$$W_1 = P_1 V$$

If we calculate for the work done by the liquid at some other point with different pressures and areas, we find that done by the liquid at some other point with different pressures and areas, we find that

$$W_2 = -P_2 V$$

Solving for the net work would give:

$$W_{Net} = W_1 + W_2 = (P_1 - P_2)V$$

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[1] Serway, Raymond A., and John W. Jewett. Physics for Scientists and Engineers. 5th ed. Pacific Grove, CA: Brooks/Cole, 2000. Print.

If we relate this to the kinetic and potential energies, which can be defined as

$$\Delta K = \frac{1}{2}m(v_2)^2 - \frac{1}{2}m(v_1)^2$$

$$\Delta U = mgy_1 - mgy_2$$

Therefore, combining all of these terms gives:

$$(P_1 - P_2)V = \frac{1}{2}m(v_2)^2 - \frac{1}{2}m(v_1)^2 + mgy_1 - mgy_2$$

Dividing by V and recalling that $\rho = \frac{m}{V}$, we find that

$$P + \frac{1}{2}\rho(v_1)^2 + \rho gy_1 = P + \frac{1}{2}\rho(v_2)^2 + \rho gy_2$$

In physics, this is a very useful equation for fluid dynamics known as Bernoulli's equation. Essentially, it is the same as the law of conservation of energy. However, it allows us to solve for velocities for liquids with different heights or pressures.

In our situation, we have the following diagram:
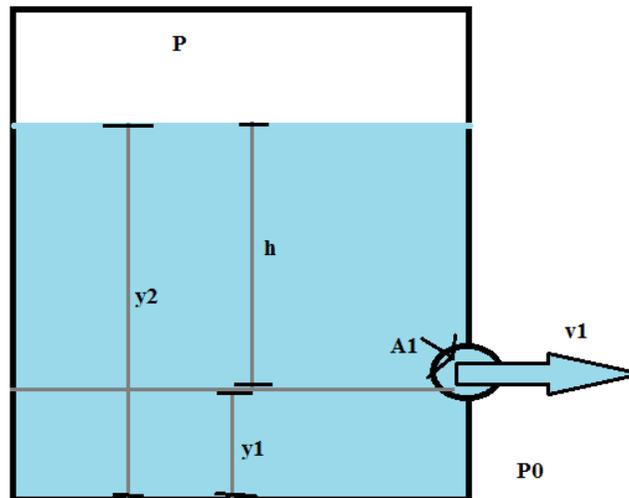
There are several things to observe here. Primarily, $y_2 - y_1 = h$ for the height of the water, $v_2 = 0$ for large openings, ρ is constant for the water, and that $P \approx P_0$. Using these conditions, we simplify Bernoulli's equation to be

$$\frac{1}{2}(v_1)^2 + gy_1 = gy_2$$

Rearranging terms provides:

$$(v_1)^2 = 2gh$$

$$v_1 = \sqrt{2gh}$$

This statement is also called Torricelli's Law. Therefore, our data corresponds and verifies this law of fluid dynamics because we found that $flow\ rate \propto \sqrt{h}$. We understand that flow rate is directly correlated with the velocity of a liquid, so that Torricelli's law is satisfied.

We can verify this relationship by using some sample points. Because we can rearrange our relationship to be $\frac{flow\ rate}{\sqrt{height}} = Constant$, if we select several of our data points, they should result in the same constant.

Flow Rate (mL/sec)	$\sqrt{Height}$ ($\sqrt{meter}$)	$\dfrac{Flow\ Rate}{\sqrt{height}}$
5.30	0.013	46.48
8.90	0.034	48.27
11.60	0.055	49.46
13.47	0.076	48.86
14.80	0.097	47.52
16.67	0.119	48.32
18.07	0.140	48.29

As we can see from these selected trials, by dividing the flow rate by the square root of the height, we get roughly a constant value of 48.The variation in the constant are normal and attributable to measurement errors.

One oddity about this equation is that Torricelli's equation clearly shows that the constant is determined by $c = \sqrt{2g}$. However, $\sqrt{2g} \approx 4.429$. What does this mean for our experiment?

For one, we measured the flow rate, not the velocity of the liquid. The reason that we did not choose to measure the velocity of the liquid is because of the uncertainty of the size of the hole.

In order to determine the velocity that the water exits the two liter bottle, we must use

$$Flow\ Rate\ \left(\frac{m^3}{second}\right) \cdot \frac{1}{Area}\left(\frac{1}{m^2}\right) = Velocity\ \left(\frac{m}{s}\right)$$

However, our method of creating the hole is by using a drill of minimal precision into a plastic bottle. We are not able to guarantee that the hole drilled has a diameter of exactly 0.0048 meters, as reported above. In addition, we are not even able to claim that the "hole" is perfectly circular. There were small jagged edges around the edge of the opening that were a result of the non-perfect drill. Therefore, the velocity of the liquid is too difficult to determine accurately for this experiment, and therefore this aspect of Torricelli's equation should be ignored.

There is not too much to evaluate for error in this experiment. Given the precision and accuracy of our data, there is not too much to improve upon this lab. However, there are minor details that could assist in creating even more accurate data.

The largest error in this lab is that we wished to find the instantaneous flow rate of the liquid given a height. However, our procedure only permits for an average flow rate to be found. Therefore, an average flow rate should also require using the average height, or $height = height_{initial} - height_{final}$. This could potentially provide more accurate data, especially as the height increases. In addition, because the height drop for the 1700 mL trial was nearly 100 mL, we cannot be absolutely certain about the values collected there. This error causes a systematic increasing of error, skewed downwards, for the flow rate as the height increased.

The most convenient way to fix this error would be to decrease the amount of time that the water drains. However, as discussed earlier in this paper, we are unable to decrease this value, as it would begin interfering with the human reaction time, a random human error. Therefore, we

cannot decrease time too far. This becomes a balancing problem between human reaction time errors with average height errors. For this experiment, we determined that using a time period of 5.0 seconds would be the best average between the two errors.

Another potential source of error was that every time the hole was sealed or unsealed, a drop or two of water would be electrostatically attracted to the finger that plugged the hole. Although this would most likely result in an increase in error of $\pm\,0.1\,mL$, which seems extremely insignificant when we are dealing with volumes of excess 100 mL at the very minimum, and that our reported uncertainty for the volume is already in excess of 5 mL.

One potential solution to this systematic error is to use some sort of non-charged material to block the hole, such as a small piece of wood or metal. However, there is not too much to be gained from this, as a disadvantage of using such materials may cause inadequate blocking of the small hole.

In general, most of the major errors have already been corrected for in the design of the lab. Potential errors included maintaining a perfect cylinder for the two liter bottle, which we achieved by only using the flat portions of the bottle and preserving the same pressure above the water, achieved by leaving the cap off. These potential errors have already been accounted for in the lab design, which contributed to the high precision of the data.

In conclusion, this lab has thoroughly explored fluid dynamics, designing and conducting an experiment around Torricelli's law for fluids. We have seen how water obeys the laws for flow rate changes through different heights. A greater understanding of fluid mechanics has been gained as a result of this lab. In general, this experiment is a great success, given the extremely high levels of accuracy that the data models, and the correct interpretation of the data.