Electronic Counter-Countermeasure Capabilities of Chaos Based FM Radar

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Abstract

Four dissipative chaotic systems Lorenz, Rossler, Moore-Spiegel, and ACT, were considered in a monostatic radar setting for analysis of high range resolution capabilities. To simulate performance in a radar communication system, an FM waveform for each system acted as a simulated transmission signal. The autocorrelation, ambiguity function, signature analysis, and entropy were calculated on each of the system's signals. While all the systems had high resolution imaging capability, the Rossler system had the lowest sidelobes in autocorrelation, one of the least amount of noise in the ambiguity function, the highest resolution in the signature analysis, and the least amount of entropy indicating that it would most likely be the best system to implement in a monostatic radar.

Background

A monostatic radar is a configuration in which a waveform is both transmitted and received from the same antenna. This is beneficial as the receiver knows the transmitted signal and does not have to be synchronized. One issue with this configuration, however, is that these systems are more susceptible to jamming. It is hypothesized that the implementation of a chaotic system in radar could help avoid this problem. Previous research has found the use of a random Gaussian waveform efficient in avoiding a jammer [1]. Because a chaotic system has a similar random property, it is hypothesized that a chaotic system could avoid jamming as well. Additionally, research has found that the implementation of a chaotic system in a bistatic radar to have high range resolution imaging capabilities [2]. This is because of a chaotic waveform's resemblance to noise and that a chaotic system has a high bandwidth. This allows for a lower range resolution as the higher a signal's bandwidth is, the less distance is required between two objects for a radar to distinguish between both objects. In this paper, multiple chaotic systems, including the previously analyzed Lorenz system, were examined to see if they have high range resolution capabilities in a monostatic radar system. The systems that were tested were Moore-Spiegel, Lorenz, ACT, and Rossler.

The Attractors

The four attractors are each represented by three state variables and nonlinear differential equations. The Moore-Spiegel system is calculated by:

$$\frac{dx}{dt} = Cy$$
$$\frac{dy}{dt} = Cz$$
$$\frac{dz}{dt} = C(-z - (T - R + Rx^2)y - Tx)$$

Where T = 6, R = 20. The Lorenz system is calculated by:

$$\frac{dx}{dt} = C\sigma(y - x)$$
$$\frac{dy}{dt} = C(x(\rho - z) - y)$$
$$\frac{dz}{dt} = C(xy - \beta z)$$

Where the control parameters, σ, ρ, and □ are equal to 10, 28 and 8/3 respectively.

The ACT system is calculated by:

$$\frac{dx}{dt} = C\alpha(x - y)$$
$$\frac{dy}{dt} = C(-4\alpha y + xz + \mu x^3)$$
$$\frac{dz}{dt} = C(-\delta\alpha z + xy + \beta z^2)$$

Where α = 1.8, □ = -0.07, δ = 1.5, and μ = 0.05. The Rossler system takes the following form:

$$\frac{dx}{dt} = C(-y - z)$$
$$\frac{dy}{dt} = C(x + ay)$$
$$\frac{dz}{dt} = C(b + z(x - c))$$

With the control parameters a = 0.2, b=0.5, and c=5.7.

With these waveforms, it was essential for them to all have a similar oscillation and bandwidth so they are all evenly compared. Thus, the compression factor C was modified for each individual waveform to ensure similar oscillation and bandwidth. For Moore-Spiegel, C = 1.5 x $10^{6,}$, for Lorenz C = 5 x 10^{5}, for Rossler C = 3 x 10^{6}, and for ACT C = 1.8 x 10^{6}. Another system, the Chen System, was also tested but the only way to ensure a similar bandwidth as the other systems was by decreasing the compression factor to a point where the oscillation of Chen was much lower than the other systems. Thus, the Chen system was discarded and other systems were experimented with such as Diffusionless Lorenz, Rucklidge, and Double Scroll. However, Moore-Spiegel was the only system out of these that allowed for a similar oscillation and bandwidth as the other systems.

After finding appropriate compression factors for each system, frequency modulation and normalization were used to simulate transmitted signals. The following frequency modulation formula was applied to each of the x variables to generate the transmitted chaos based frequency modulated signal s(t).

$$s(t) = Asin(2\pi K \int_{-\infty}^{\infty} x(t)dt)$$

Where A = 1, K= $\frac{1}{2}$, and x(t) was the normalized waveform. The following figure displays the simulated transmitted signals for all of the systems over 40 μs.

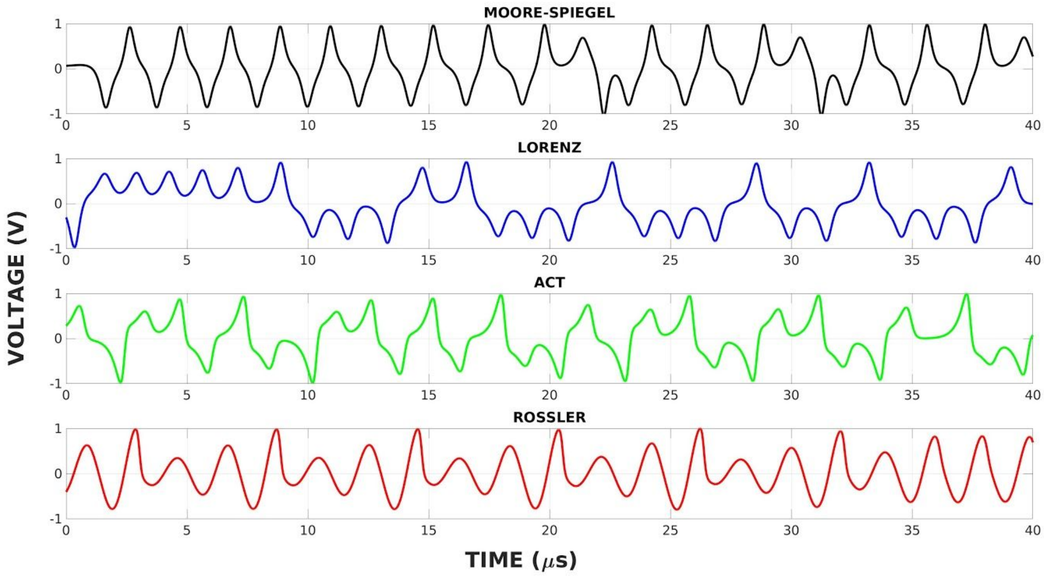


Figure 1: Voltage of x-state variables over time for each chaotic system

As evident from the figure, all of the transmitted are oscillating at similar rates. With these signals generated, correlations were taken of each signal.

Correlations

In a radar communication system, range resolution is dependent on the bandwidth of the transmitted signal. In order to find the bandwidth of a signal, we consider the width of the mainlobe of the signal's correlation at -3dB. The correlation is generated by:

$$s(t) = \int_{-\infty}^{\infty} s(t)s(t-\tau)dt$$

With the correlation found, the inverse of the length is taken as the bandwidth of the signal. With the bandwidth, range resolution is calculated by:

$$\Delta r = \frac{c}{2\beta}$$

Where c is the speed of light in meters per second and β is the bandwidth of the signal in Hz. In order to have a higher resolution, it is necessary to decrease Δr, and this is possibly by increasing the bandwidth. As mentioned in the previous section, in order to have a fair comparison among all of the systems in terms of range and cross range, a fixed bandwidth must be considered. Additionally, the autocorrelation provides another piece of information. The autocorrelation of signals indicates the differences between the mainlobe and the sidelobes of each system. If the sidelobes are much lower than the mainlobe, by at least -13 dB, there is a clear difference between the actual target and how it would stand out from external noise. While -13 dB is a necessary threshold for the sidelobes to be under in order to initially be able to distinguish between the target and the noise, the further below -13 dB the sidelobes are indicates higher resolution. Thus, while all systems were modified to have the same bandwidth at -3 dB, the rest of the correlations provide an initial observation to be made about the capability for higher range resolution. The auto-correlation results for the systems is shown below.

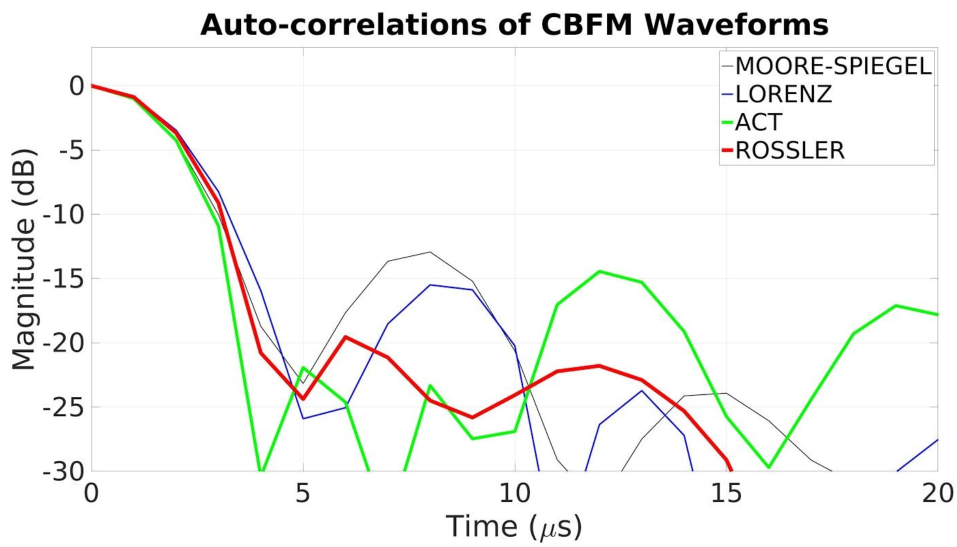


Figure 2: Autocorrelations of waveforms for each system

As the figure shows, all of the systems have sidelobes below -13 dB. Thus, they would most likely all be efficient to implement in a radar system. Out of all the systems, the Rossler system has the lowest sidelobes overall. However, Lorenz and ACT are both on a similar level. Moore-Spiegel appears to have the highest as it is the only system whose sidelobes reach above -15 dB.

Frequency Spectrums

An additional way generate the bandwidth of the systems and ensure they are consistent among the systems is by converting the transmitted signals from the time domain to the frequency domain using the Fourier Transform. The frequency spectrums for each of the systems are shown in the following figure.

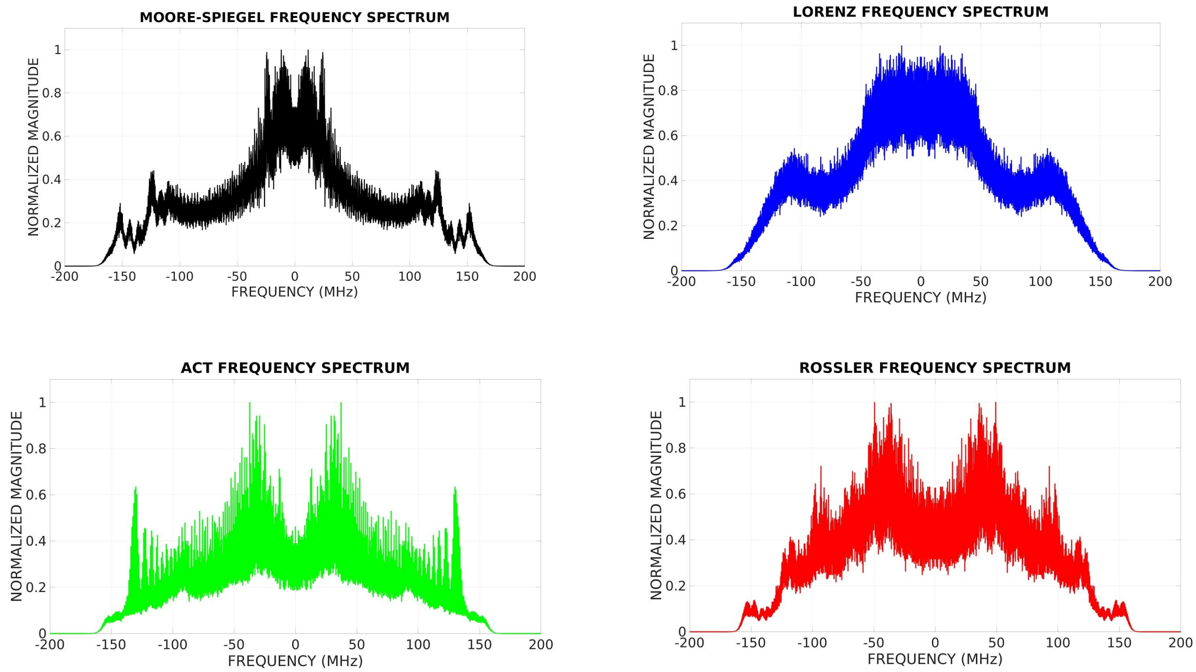


Figure 3: Transmitted waveforms in the frequency domain

All of the systems in the figure appear to have large frequencies from 0 to roughly 150 MHz, making the bandwidth 150 MHz.

Thus, four transmitted signals with similar oscillations and bandwidths were generated. With these signals, the following step was to incorporate how these signals act in reaching a target and are processed by a receiver.

Ambiguity Functions

When the signals are transmitted, they take time to propagate through the air, reach a target, and be received by the receiver. Thus, there will be some time delay present in the received signal. Additionally, because the target the signals reach is moving at some velocity, the received signal will be at a different frequency than the transmitted signal. The difference between the transmitted signal and the received signal is referred to as a Doppler frequency. With this in mind, the received signal will look as such:

$$r(t) = s(t - \tau)e^{j2\pi f_d}$$

Where τ is the delay and f_d is the Doppler shift. When transmitting a signal that detects a signal point, to find the delay and Doppler shift of the received signal, the ambiguity function is considered. The ambiguity function mixes the transmitted signal with the received signal and returns a peak on a 2-dimensional plane where the delay and Doppler shift occur. The ambiguity function is calculated as such:

$$Amb(\tau, f_d) = \left| \int_{-\infty}^{\infty} s(t)s(t - \tau)e^{j2\pi f_d t} dt \right|^2$$

This provides information about the time delay and Doppler shift of a single point. This operation is useful when considering a more complex target, which is discussed in the following section.

Signature Analysis

After setting up the ambiguity function to reveal information about a single point, a more complex target was considered to generate images of a complete target. A full scale 73x65 meter Boeing 737 airplane was used to demonstrate imaging for each system. Twenty five points were taken that would portray the contour of the target. As each point consists of a time delay and a shift in Doppler frequency, the received signal is the summation of the received signal for each point with a different time delay and Doppler shift. The received signal is given as shown:

$$r(t) = \sum_{k=1}^{N} s(t - \tau_k) e^{j(2\pi f_c t + 2\pi f_{d_k})t} dt$$

To then generate an image of the target with the received signal, the range and cross range of the target are found. The range indicates the distance along the x-axis of the target from the radar's antenna. The cross-range provides the y-axis information of the target. Upon reception of the signal, the range and cross-range can be evaluated through their mathematical relationships with the delay, τ, and Doppler frequency f_d. The range is evaluated as a function of the time delay τ by:

$$r = \frac{2\tau}{c}$$

The cross-range can be expressed as a function of f_d by:

$$r_c = \frac{f_d \lambda}{2\Omega}$$

Where λ is the wavelength of the signal, and Ω is the total angular speed in rad/s given by:

$$\Omega = \frac{\Delta\theta}{T_{int}}$$

Where T_{int} is the integration time of the transmitted signal, and $\Delta\theta$ is the total angular rotation given by:

$$\Delta\theta = \frac{\beta}{f_c}$$

Where β is the bandwidth, and f_c is the carrier frequency. Thus, by processing the received signal to obtain the delay and Doppler shift of each point on the target, the range and cross-range of the target can be found and an image is generated. In order to find the delay and Doppler shift of each point on the target, the signature analysis is implemented. In signature analysis, a bank of matched filters is designed. In each of the matched filters, a reference signal is created by mixing the transmitted signal with an estimated Doppler frequency. Based on the target being an airplane, the bank of matched filters was designed to iterate through each potential Doppler frequency in the range of -100f_d to 100f_d where $f_d = \frac{1}{Tint}$. When each reference signal is created, the reference signal replaces the transmitted signal in the ambiguity function. When the reference signal's Doppler frequency is equivalent to one of the received signal's Doppler frequencies, the signature analysis will show a peak at the delay and the Doppler frequency of a point on the received signal. After iterating through each point of the received signal and finding the time delay and Doppler shift of each point, the image of the target is generated.

Signature Analysis with Noise

In the previous sections, the received signal assumed to be exactly the same as the transmitted signal with some time delay and Doppler shift. However, in a realistic setting, information will be lost in the received signal due to noise. The actual received waveform should then look like:

$$r(t) = s(t) + \eta(0, \sigma^2)$$

Where $\eta(0, \sigma^2)$ is a Gaussian noise signal with mean = 0 and variance σ^2. To simulate the influence of noise on the received signal, the power of both the received signal and the noise are considered.The power of the signal is calculated by:

$$P_{avg} = \frac{A^2}{2}$$

With constant A = 1V. The power of the noise can be written as:

$$P_{noise} = \sigma^2$$

Thus, the Signal-to-Noise ratio on the dB scale can be written as:

$$SNR_{dB} = 10\log_{10}(\frac{A^2}{2\sigma^2})$$

With this equation, different SNR can be created to determine how much noise is required to make the received signal incomprehensible. This is possible by using the above equation to solve for the standard deviation of the noise, where standard deviation = σ. The standard deviation is calculated by:

$$\sigma = \sqrt{\frac{P_{avg}}{10^{\frac{SNR_{dB}}{10}}}}$$

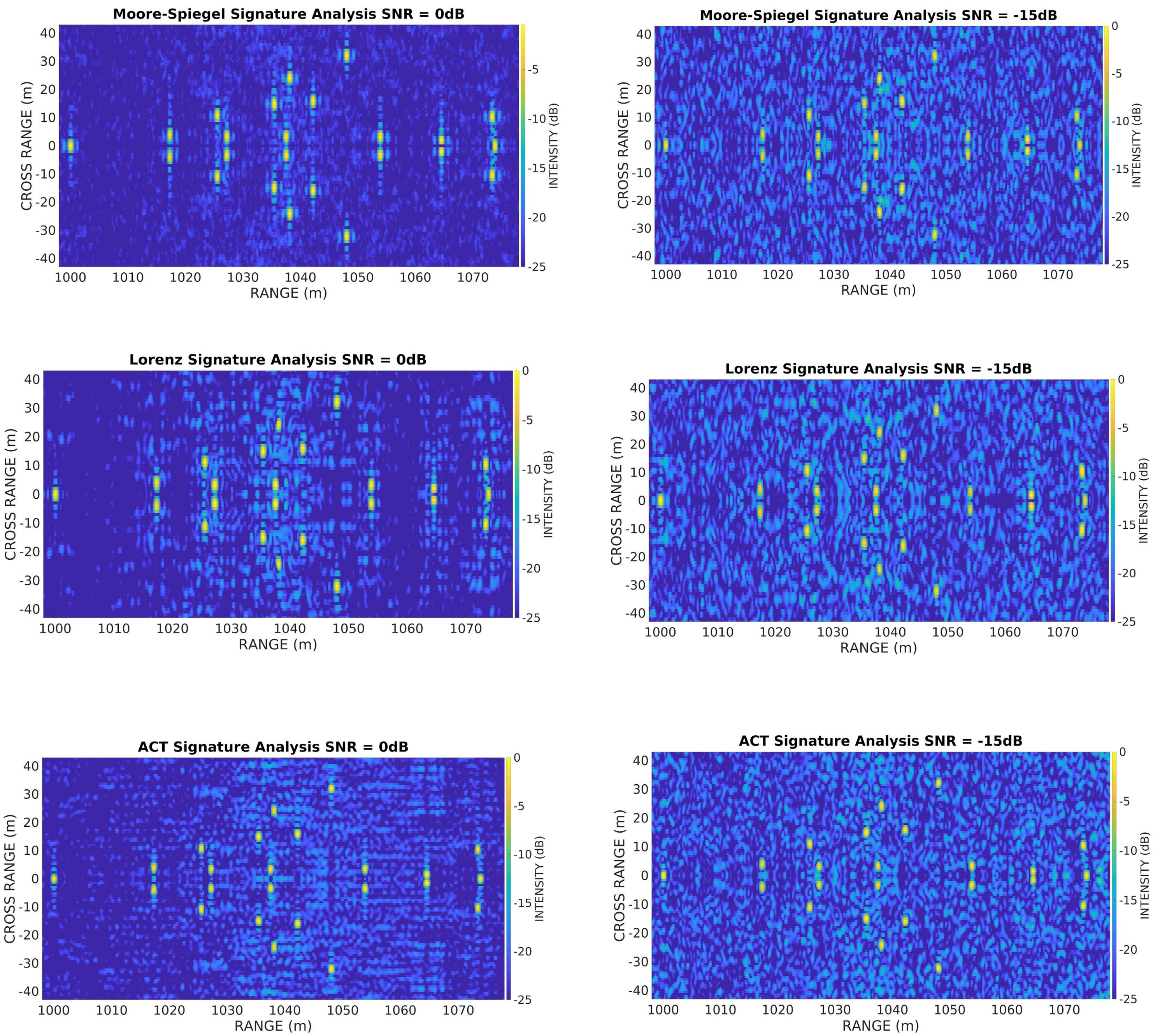
Then, by varying different SNR levels, a new standard deviation will occur, and the new received signal will be as so:

$$r(t) = s(t) + \sigma\eta(0,1)$$

By adding different levels of noise to the received signal, we would be able to see how the quality of each of the hotspots degrades with the inclusion of external interference. After implementing signature analysis on the received signal with different levels of noise, the level at which the target was still distinguishable from the noise was found to be at -15 dB.

Results of Signature Analysis without and with Noise

The following figure shows the results of the signature analysis for each chaotic system. The images were generated with an SNR of 0 dB and -15 dB.



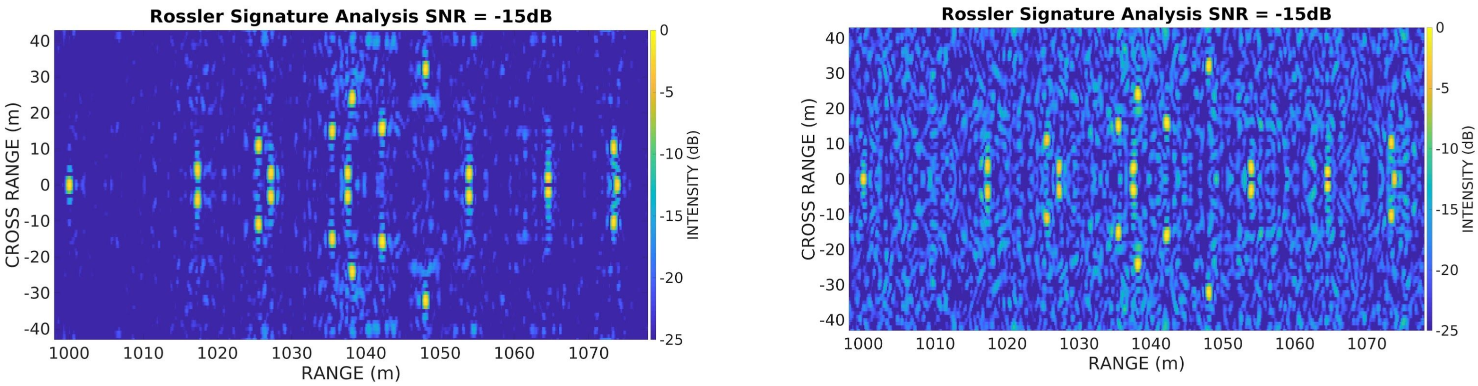


Figure 4: Signature Analysis of all systems

From this figure, all of the systems are capable of producing high range resolution images. Each point is distinguishable from each of the others, and no two points overlap each other. Without the presence of noise, the Rossler system appears to have the highest resolution. It displays the thinnest peaks with the least amount of spillover from the sidelobes. While all of the systems appear to be efficient if implemented in a radar system, the Rossler system would most likely be the best from a qualitative analysis of the images.

Entropy

While it is possible to visibly observe the differences in quality between the different images, to quantify the results the entropy is calculated on the normalized signature analysis. As it is normalized, all of the values in the image should either be a 0 if a peak is not present at a delay and frequency, or a 1 if a peak is present. By taking the log with a base of 2 of each value in the image, then every peak with a value of 1 will be a 0, and all non-peaks in an image can be set to 0 as a log will return an infinite number. This operation is referred to as entropy and is calculated by:

$$H = \frac{-1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} I(i,j) * log_2(I(i,j))$$

Where I is the image and i and j represent each row and column of the image. Under the scenario discussed above where a peak indicates a 1, and a non-peak indicates a 0, the total entropy should result in 0. However, due to spillover in the sidelobes along with the inclusion of noise, there will be some non-zero value. Thus, by finding the entropy for each system, we are able to determine which system has the least spillover of sidelobes in having the least entropy. For each system, entropy was calculated over various SNR values.

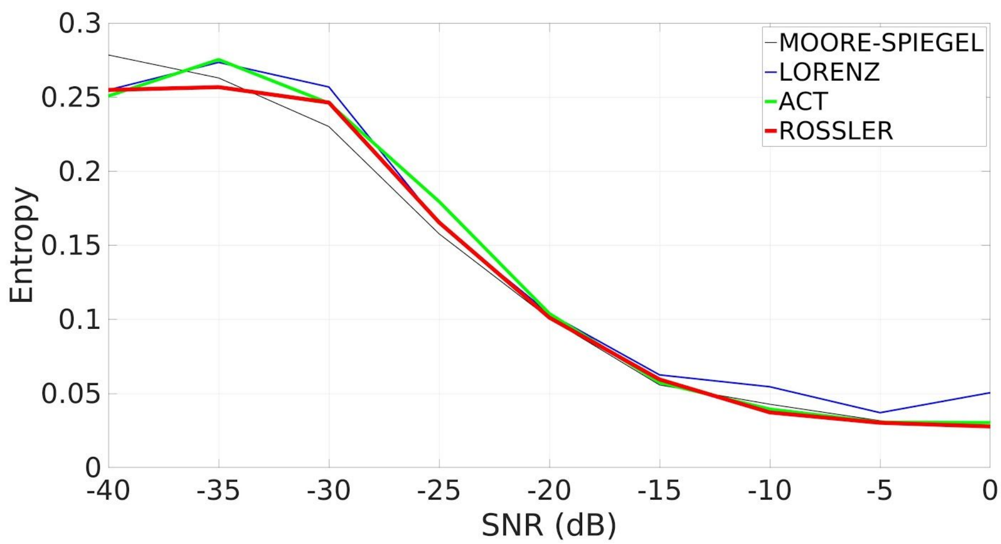


Figure 4: Entropy of all systems

As shown in the image, all of the systems demonstrate similar entropy values over various SNR levels. However, Moore-Spiegel and Lorenz demonstrate higher entropy values as Moore-Spiegel starts with a higher entropy, and Lorenz ends with a lower entropy value. The Rossler system appears to have the most consistently lowest entropy.

Conclusion

Overall, these simulations allowed us to determine that all of these systems would generate high range resolution images in a radar system. Out of all of the systems, the simulations have shown that the Rossler system would most likely perform the best out of the rest of the systems. In the autocorrelations, the Rossler system demonstrated the lowest sidelobes overall. Additionally, the Rossler system in signature analysis had the thinnest peaks indicating higher range resolution. When the SNR was set to 0 dB, the image was surrounded by the least amount of noise. Lastly, the Rossler system had a consistently low entropy, indicating less spillover from the sidelobes.

Future Work

These simulations allowed us to determine which signals had good metrics when transmitted and received without noise. Future work will consist of implementing a "smart jammer" [1]. This will be useful in determining whether chaotic signals, like ultrawideband bandlimited random noise signals, are difficult for a jammer to predict the transmitted signal.

Works Cited

[1] Garmatyuk, Dmitriy S., and Ram M. Narayanan. "ECCM capabilities of an ultrawideband bandlimited random noise imaging radar." *IEEE Transactions on Aerospace and Electronic Systems* 38.4 (2002): 1243-1255.

[2] Pappu, Chandra S., et al. "Analysis of chaotic FM system synchronization for bistatic radar." *Radar Sensor Technology XIX; and Active and Passive Signatures VI*. Vol. 9461. International Society for Optics and Photonics, 2015.