

# MAT329–Project\*

## Convolving Again and Again†

This project gives an example of the so-called *smoothing properties* of the convolution operation.

**Project.** Let  $f$  be a periodic function of period 1 that is defined by

$$f(x) = \begin{cases} 1 & \text{if } 0 < x \leq 1/2 \\ 0 & \text{if } 1/2 < x < 1 \end{cases}$$

over  $(0, 1)$ .

- (a) Compute  $f * f$  and  $f * f * f$  by hand.
- (b) Use your favorite computer software (such as Maple, Mathematica, MATLAB, or Sage) to compute and plot the functions  $f$ ,  $f * f$ , and  $f * f * f$  over  $(0, 1)$ . Notice how the smoothness improves as we take more convolutions.

**Caution:** You might find a “convolution” command in the library of your software. However, before using any built-in command you should make sure that its definition agrees with what you have in mind. For the purposes of this project you may need to code an appropriate convolution command using the definition

$$(f * g)(x) = \int_0^1 f(t)g(x - t) dt.$$

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\*Due on Friday, March 2

†Adapted from *Harmonic analysis: from Fourier to wavelets*, Pereyra, Ward.