Subordinated Processes for Solar Irradiance Simulation

Cait Berry and Will Kleiber

Department of Applied Mathematics, University of Colorado at Boulder
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Applied Mathematics
UNIVERSITY OF COLORADO **BOULDER**

Problem Setup

Overall Goal: Capture variability of one-second solar irradiance data using subordinated Gaussian processes for use in understanding how distributed energy resources like rooftop solar affect grid operations.

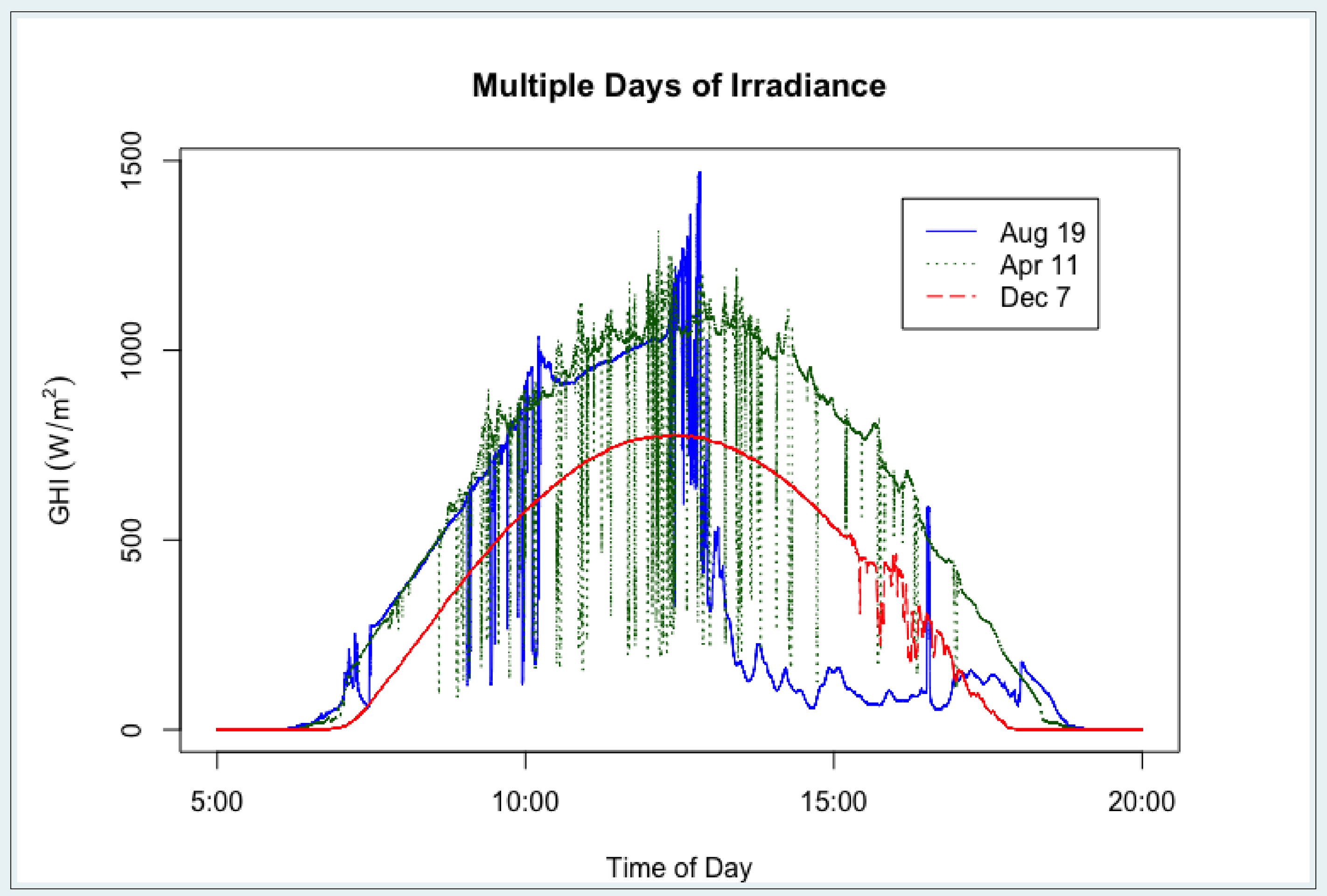

Figure: Observed global horizontal irradiance (GHI) every second for three select days of the year.

Data

- One-second resolution GHI (W/m^2) measurements from a set of pyranometers in Hawaii
- Clear Sky GHI (CSGHI) is calculated from NSRDB data using spline interpolation for each day of the season (June, July, August).
- The Clear sky index (CSI) be defined as:

$$\text{CSI}(t) = \frac{\text{GHI(t)}}{\text{CSGHI(t)}} \tag{1}$$

  at time point t.
- Modeling is done with transformed data:

$$Z(t) = \log(\text{CSI(t)}) \tag{2}$$

Model

Let $f_Z(\cdot;\Theta)$ and $F_Z(\cdot;\Theta)$ be the pdf and cdf, respectively, of $Z(\cdot)$. Define $W(\cdot)$ as

$$W(t) = \Phi^{-1}(F_Z(Z(t);\Theta)) \tag{3}$$

where Φ is the cdf of a Gaussian random variable.
Then our model is $X(t)$:

$$X(t) = W(\beta S(t)) \tag{4}$$

where $X(t)$ is a subordinated Gaussian process (SGP) with:

- $W(\cdot)$ is a mean zero Gaussian process with Matérn covariance function with parameters (ν, ρ)
- $S(\cdot)$ is a subordinator, a non-negative and non-decreasing process, defined below
- $\beta > 0$ is a scaling parameter

The subordinator: $S(t) = \sum_{\ell=1}^{N} \alpha_\ell \phi_\ell(t)$
where:

- $\phi_\ell(\cdot)$ are I-spline basis functions
- α_ℓ are random variables drawn from estimated distributions
- $N = \#$ of knots $+$ degree $+ 1$

Estimation Approach

1. Aggregate $Z(\cdot)$ for June/July/August data, assume each observation z is an independent sample from pdf $f_Z(\cdot;\Theta)$ to be given by:

$$f_Z(z;\Theta) = \begin{cases} (0.995)\sum_{k=1}^{3} \lambda_k f_k(z;\theta_k) & z \leq .995 \text{ percentile} \\ (0.005) f_4(z;\theta_4) & z > .995 \text{ percentile} \end{cases} \tag{5}$$

   Using EM, estimate the MLE for $\Theta = (\lambda_1, \lambda_2, \lambda_3, \theta_1, \theta_2, \theta_3, \theta_4)$ where θ_k is a vector of parameters for the distribution functions f_k, which are Gauss, Gauss, Gamma, and Pareto respectively.
2. Estimate $\{\alpha_1, ..., \alpha_N\}$ for the I-spline using a local variance model
3. For $X(t)$ a SGP subordinated by $S(t)$, estimate parameters $\boldsymbol{\theta} = \{\beta, \nu, \rho\}$ using approximate Bayesian computation (ABC) via:

$$\underset{\theta}{\text{argmin}} \int \left(\widehat{P_X}(\omega) - \widehat{P_Z}(\omega)\right)^2 \mathrm{d}\omega$$

   where $\widehat{P_X}$ and $\widehat{P_Z}$ are empirical spectral densities of a simulated and observed irradiance time series, respectively.

Results

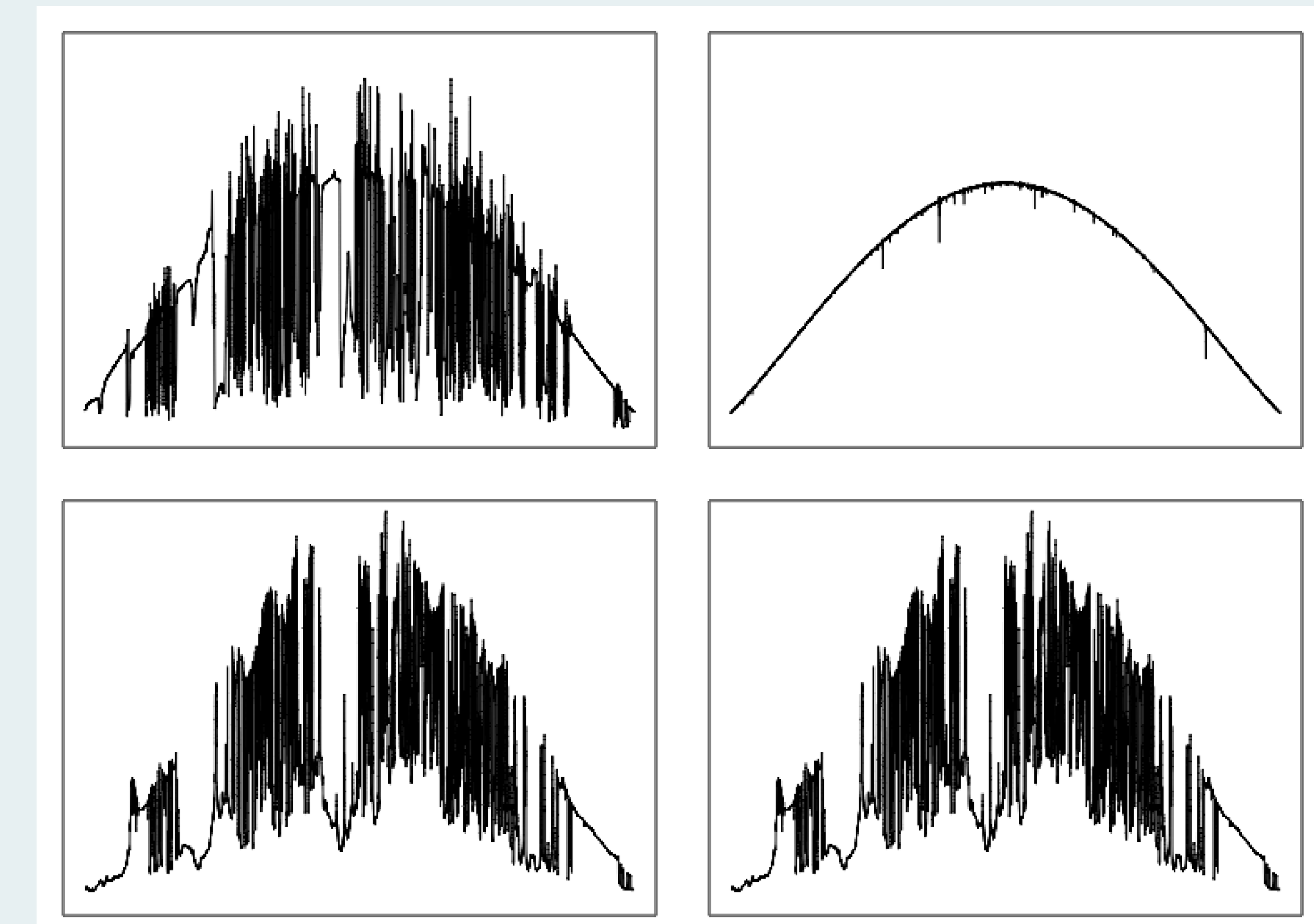

Figure: Simulations from proposed SGP (top left) and a GARCH model (top right); real data (bottom row)

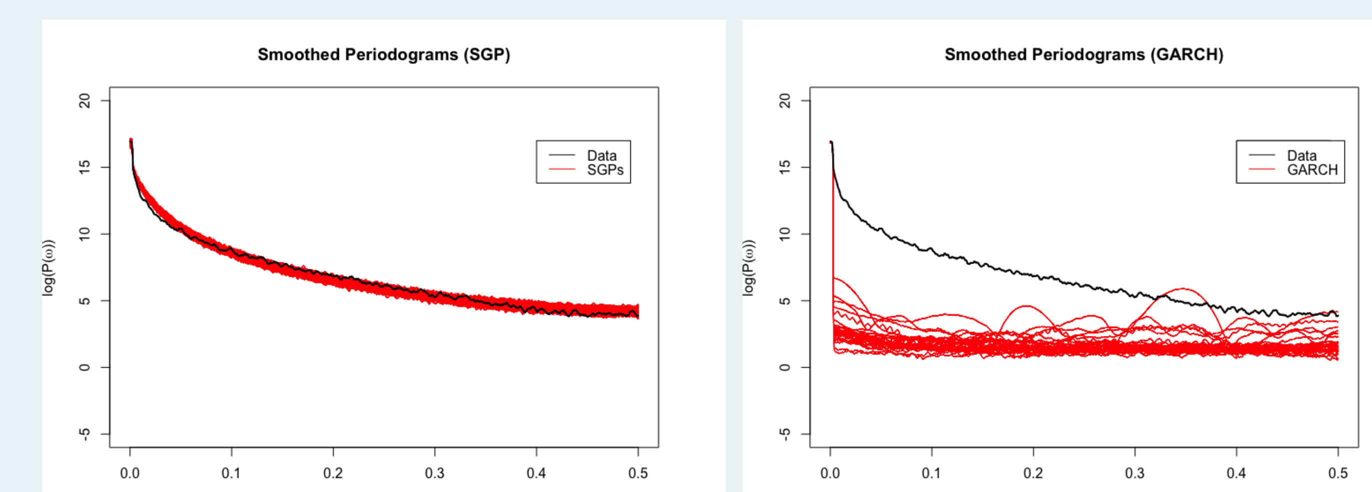

Figure: Smoothed periodograms of simulations (red) compared with that of a day of data (black) for the SGPs (left) and a GARCH model (right)

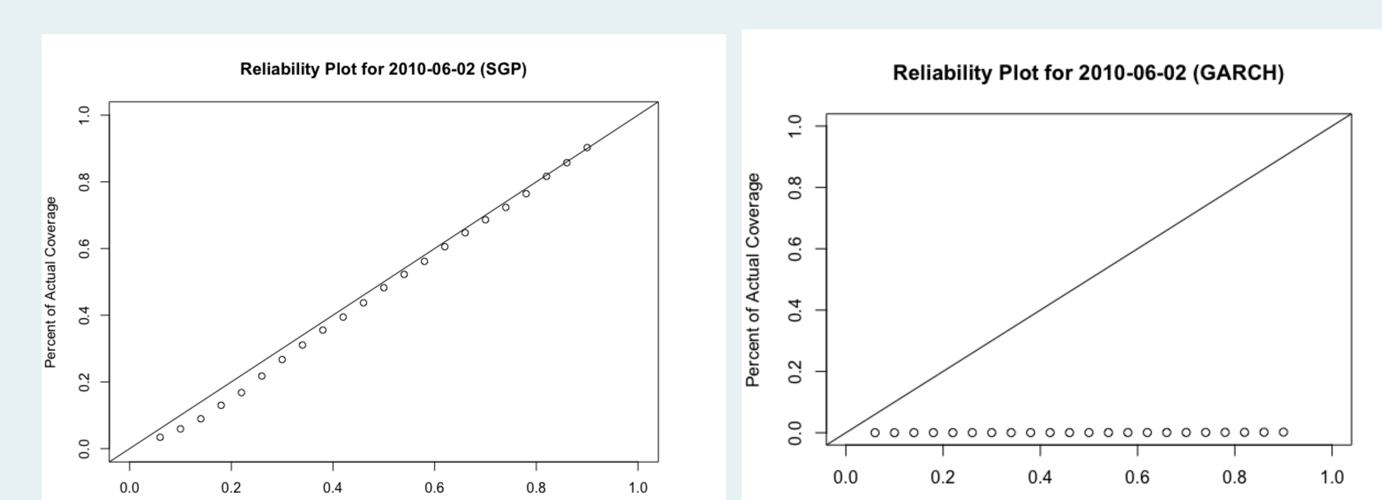

Figure: Reliability plots based on 300 simulations of the SGPs (left) and a GARCH model (right)