Dynamic Recovery of Power Networks using Dynamic Mode Decomposition

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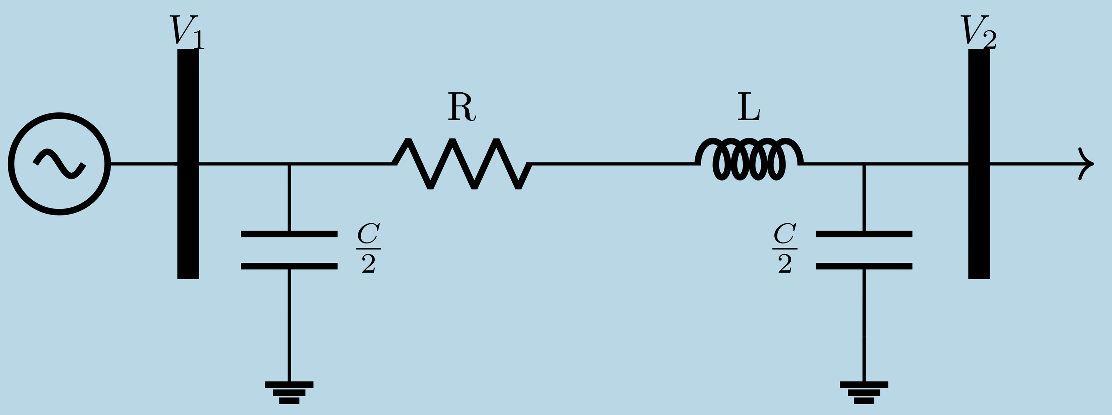
Problem

We want to recover a linear model from data, ideally PMU measurements, that describes the rotor dynamics in a power network.



Power Network as a Graph

We follow the approach provided in [1, 2]. They consider the power network as a connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with buses as nodes $\mathcal{V} = \{1, 2, \ldots, n\}$ and transmission lines as edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Generally, a bus can host different combinations of generators and loads, or it may even be a simple junction node. We assume that each bus hosts a generator, otherwise we may use Kron reduction as in [3], so that the power network is modeled as a completely connected graph.

Swing Equation for Networks

Recall that the linearized Swing Equations for networks (SEN) are of the form

$$\mathbf{M}\boldsymbol{\delta}'' + \mathbf{D}\boldsymbol{\delta}' + \mathbf{L}\boldsymbol{\delta} = \mathbf{b},$$

where $\mathbf{M}$ and $\mathbf{D}$ are the diagonal matrices of inertia and damping coefficients, and $\mathbf{L} \in \mathbb{R}^{n\times n}$ is the susceptance Laplacian matrix $\mathbf{L} = \mathbf{L}^T \geqslant 0$ whose (i,j)-th entry is given by

$$(\mathbf{L})_{i,j} = \begin{cases} -b_{i,j}, & \text{if } (i,j) \in \mathcal{E}, \\ \sum_{(i,j)\in\mathcal{E}} b_{i,j}, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

This model can be written as a first order system of differential equations:

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}' \end{bmatrix}' = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{L} & -\mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}' \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix},$$

equivalently,

$$\begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}' \end{bmatrix}' = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{L} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}' \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{b} \end{bmatrix}}_{\mathbf{B}}$$

Considering

$$\mathbf{x} = \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\delta}' \end{bmatrix},$$

the SEN take the form

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad \text{with} \quad \mathbf{u} = 1.$$

References

[1] B. Safaee and S. Gugercin, *Structure-preserving Model Reduction of Parametric Power Networks*, 2021 American Control Conference (ACC), 2021, pp. 1824-1829, doi: 10.23919/ACC50511.2021.9483168.

[2] M. K. Singh and V. Kekatos, *Optimal Power Flow Schedules with Reduced Low-Frequency Oscillations*, Power Systems Computation Conference (PSCC), June, 2022, Porto, Portugal, (to appear also in the Elsevier Electric Power Systems Research)

[3] T. Ishizaki, A. Chakrabortty and J. Imura: *Graph-Theoretic Analysis of Power Systems*, in Proceedings of the IEEE, vol. 106, no. 5, pp. 931-952, May 2018, doi: 10.1109/JPROC.2018.2812298.

Dynamic Mode Decomposition (DMD)

DMD is a data-driven modeling approach where the data is usually sampled with homogeneous time step h, i.e., $t_{k+1} = t_k + h$. The data, m snapshots each containing n state-variables, is arranged into matrices $\mathbf{X}_1, \mathbf{X}_2 \in \mathbb{R}^{n\times(m-1)}$ that are used to recover dynamical modes. For details see [4].

$\mathbf{x}' = \mathbf{A}\mathbf{x}$ — Continuous time model ⟷ $\mathbf{x}_{k+1} = \mathcal{A}\mathbf{x}_k$ — Discrete time model

DMD framework

$$\mathbf{X}_1 = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix}$$

$$\mathbf{X}_2 = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_{m} \\ | & | & & | \end{bmatrix}$$

$$\mathbf{X}_1 \approx \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^*$$

$$\mathcal{A} \approx \mathbf{X}_2\mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^*.$$

$$\mathbf{A} = \frac{1}{h}\ln(\mathcal{A})$$

We recover an estimate of $\mathbf{A}$ from data which is optimal in the least square sense, since

$$\mathcal{A} = \arg\min_{\mathbf{M}\in\mathbb{R}^{n\times n}} \|\mathbf{X}_2 - \mathbf{M}\mathbf{X}_1\|_F$$

[4] J. Nathan Kutz, Steven L. Brunton, Bingni W. Brunton, and Joshua L. Proctor. 2016. *Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems.* SIAM-Society for Industrial and Applied Mathematics, Philadelphia, PA, USA.

Least Square Structure Imposed (LS-ioDMD)

We try to recover the dynamics for the SEN, but the DMD matrices may not preserve the structure of the matrices $\mathbf{M}^{-1}\mathbf{L}$ and $\mathbf{M}^{-1}\mathbf{D}$, thus we use the input-output DMD approach (ioDMD). For details see [5, 6], and then we apply a regression to enforce the structure that we want.

$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ — Continuous time model ⟷ $\mathbf{x}_{k+1} = \mathcal{A}\mathbf{x}_k + \mathcal{B}\mathbf{u}_k$, $\mathbf{y}_k = \mathcal{C}\mathbf{x}_k$ — Discrete time model

ioDMD framework

$$\mathbf{X}_1 = [\,\mathbf{x}_1 \;\; \mathbf{x}_2 \;\; \cdots \;\; \mathbf{x}_{m-1}\,]$$

$$\mathbf{X}_2 = [\,\mathbf{x}_2 \;\; \mathbf{x}_3 \;\; \cdots \;\; \mathbf{x}_{m}\,]$$

$$\mathbf{U}_1 = [\,\mathbf{u}_1 \;\; \mathbf{u}_2 \;\; \cdots \;\; \mathbf{u}_{m-1}\,]$$

$$\mathbf{Y}_1 = [\,\mathbf{y}_1 \;\; \mathbf{y}_2 \;\; \cdots \;\; \mathbf{y}_{m-1}\,]$$

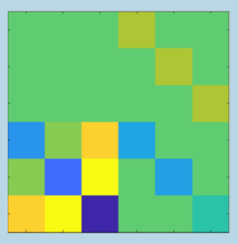
$$\mathbf{X}_1 \approx \mathbf{U}_r\boldsymbol{\Sigma}_r\mathbf{V}_r^*$$

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} \approx \begin{bmatrix} \mathbf{U}_r^*\mathbf{X}_2 \\ \mathbf{Y}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_r\mathbf{V}_r \\ \mathbf{U}_1 \end{bmatrix}^\dagger$$

$$\mathbf{A} = \frac{1}{h}\ln(\mathcal{A})$$

$$\mathbf{B} = (\mathcal{A} - \mathbf{I})^{-1}\mathbf{A}\mathcal{B}$$

We recover an estimate of $\mathbf{A}$ and $\mathbf{B}$ from data and impose the dynamic structure using least squares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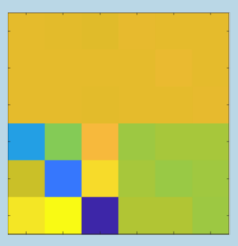
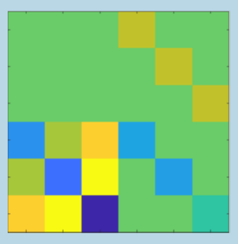
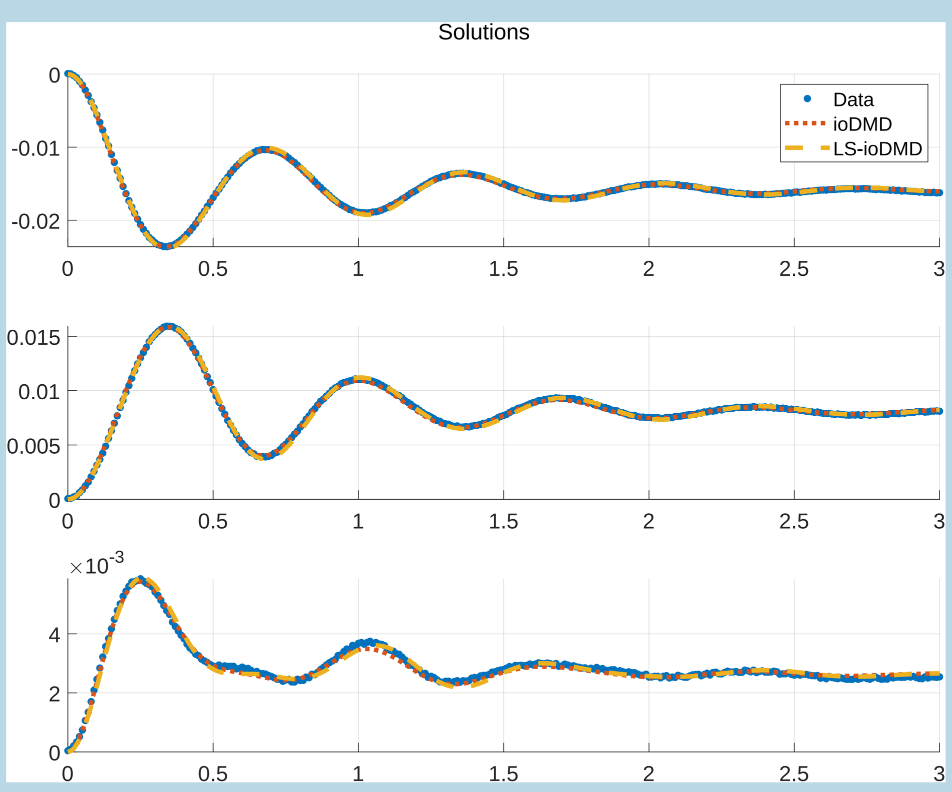
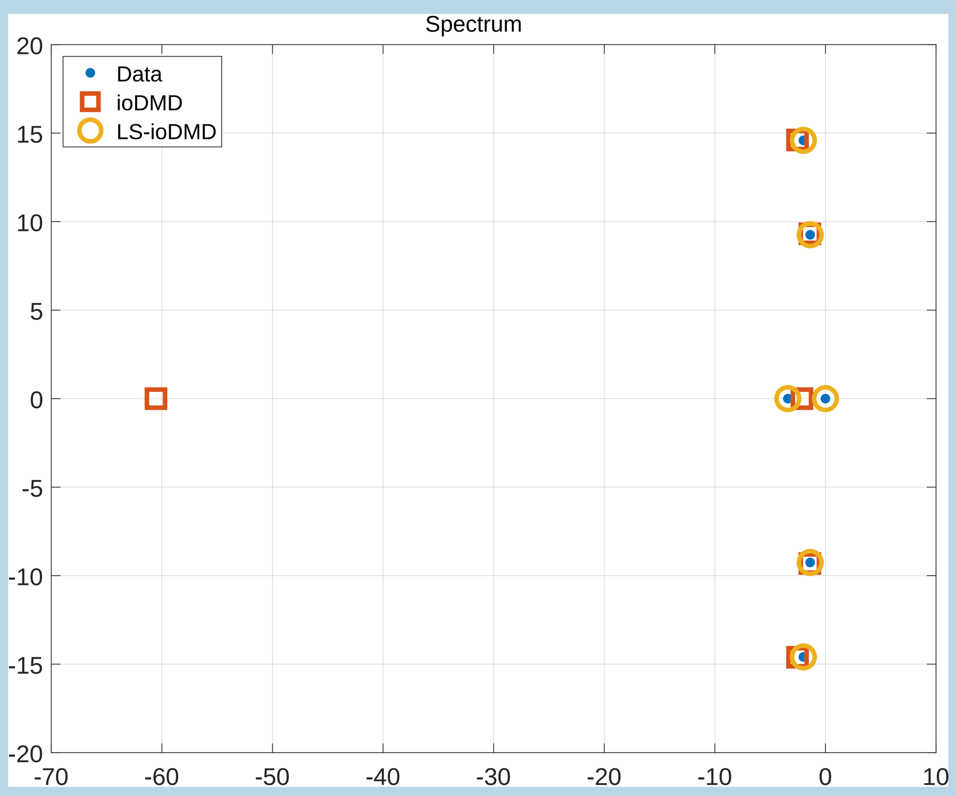

ioDMD


LS-ioDMD

The figures below show the quality of the recovered dynamic response for a synthetic 3 generator network, and where the eigenvalues are located.





[5] J. Annoni, P. Gebraad and P. Seiler, *Wind farm flow modeling using an input-output reduced-order model*, 2016 American Control Conference (ACC), 2016, pp. 506-512, doi: 10.1109/ACC.2016.7524964.

[6] Benner, P., Himpe, C. & Mitchell, T. *On reduced input-output dynamic mode decomposition.* Adv Comput Math 44, 1751–1768 (2018). https://doi.org/10.1007/s10444-018-9592-x