Stabilization of kink modes by lithium streams

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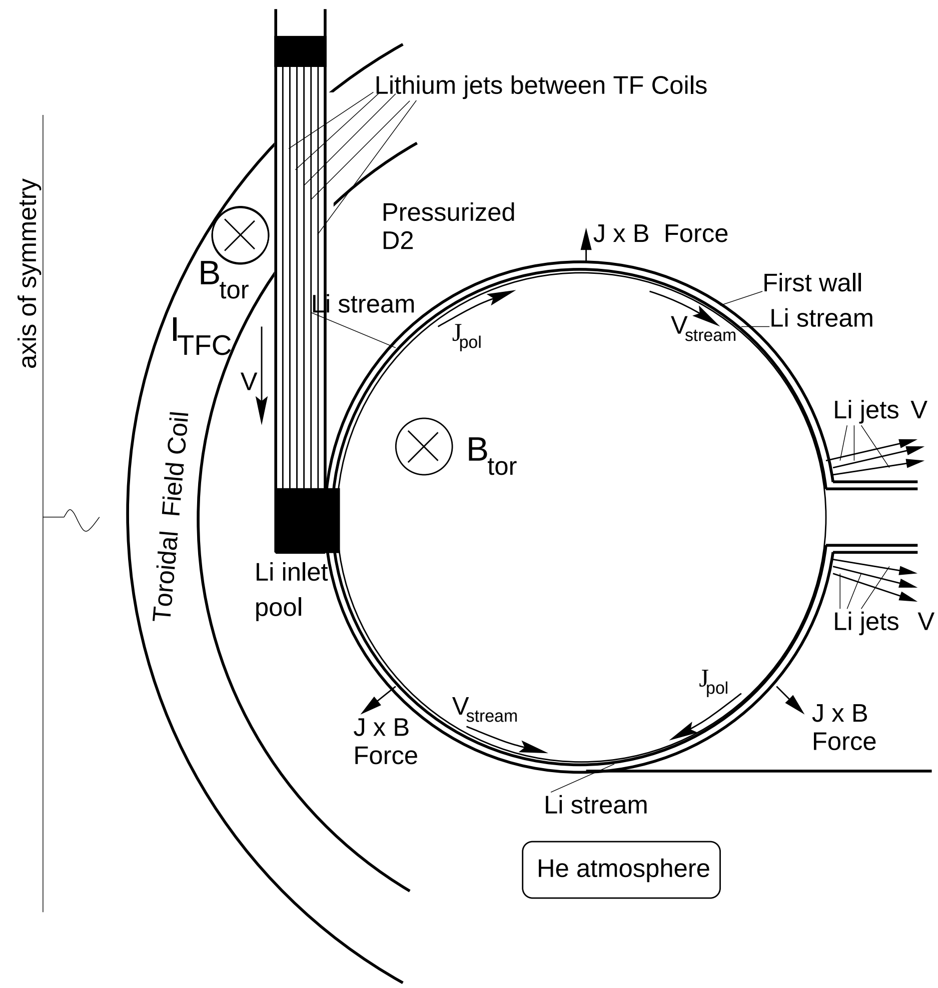
1. Two effects of lithium streams on stability

1. situating an effective conducting shell right at the plasma boundary
2. affecting resistive wall modes by an axisymmetric $m = 1$ pattern of a fast flow of conducting media



- The first effect, the most essential, has its straightforward analog in the theory of resistive (solid) wall modes (Zakharov, Putvinskii, 1987) and has been extensively analyzed in the literature.
- The second still effect has no direct analogs in existing theory.

Here I present the basic theory of interaction of lithium streams with free-boundary kink modes.

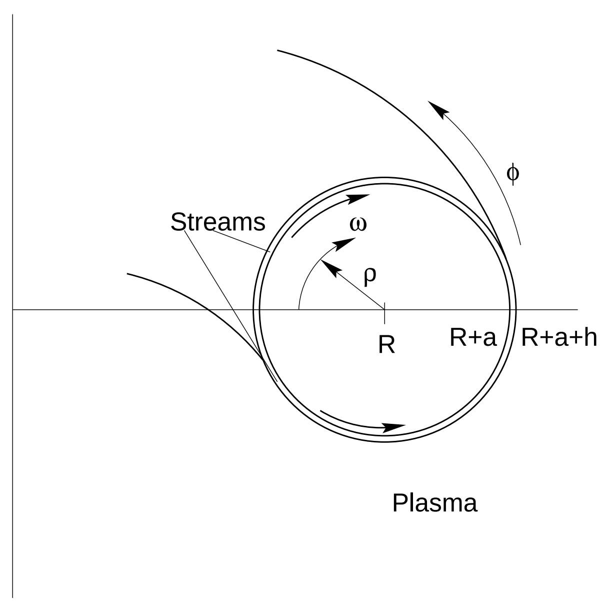
All new features can be explained based on large aspect ratio approximation for the circular cross-section plasma.

2. Streams and the kink modes for $\beta = 0$

Perturbation of the magnetic field $\tilde{\mathbf{B}}$ can be represented as

$$\begin{aligned} &\tilde{\mathbf{B}} \simeq \nabla\psi \times \mathbf{e}_\varphi, \quad \tilde{B}_\rho = \frac{1}{\rho}\psi'_\omega, \quad \tilde{B}_\omega = -\psi'_\rho, \\ &\psi(\rho,\omega,\varphi) = \sum_m \psi^p_m(\rho) e^{im\omega - in\varphi + \gamma t}, \quad \rho < a \\ &\psi(\rho,\omega,\varphi) = \sum_m \psi^e_m(\rho) e^{im\omega - in\varphi + \gamma t}, \quad \rho > a \end{aligned} \tag{2.1}$$

(ρ, ω, φ is a local coordinate system).



Stream function i of electric current in the stream

$$\tilde{\mathbf{j}} \equiv \nabla i \times \mathbf{e}_\rho = \frac{i'_\varphi}{R}\mathbf{e}_\omega - \frac{i'_\omega}{a}\mathbf{e}_\varphi \qquad (2.2)$$

is determined by the Ohm's law

$$\gamma\psi\mathbf{e}_\varphi + \nabla\phi^E + (V\nabla\psi)\mathbf{e}_\varphi = -\frac{\nabla i \times \mathbf{e}_\rho}{\sigma}. \qquad (2.3)$$

Neglecting plasma pressure, $p_{pl} = 0$, resistive wall modes are described by dispersion relation

$$\sum_m a\Delta'_m\psi_m e^{im\omega - in\varphi + \gamma t} = \mu_0 h i'_\omega = \tau_{res}\gamma\psi + Rev(\omega)\psi'_\omega, \qquad (2.4)$$

where

$$\begin{aligned} &\tau_{res} \equiv \mu_0\sigma a h, \quad Re \equiv \mu_0\sigma h V_{max}, \quad v(\omega) \equiv \frac{V(\omega)}{V_{max}}, \\ &\Delta'_m \equiv \frac{\psi'^e_m(a+h) - \psi'^p_m}{\psi^p_m(a)}. \end{aligned} \qquad (2.5)$$

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In Fourier space the eigenvalue problem is

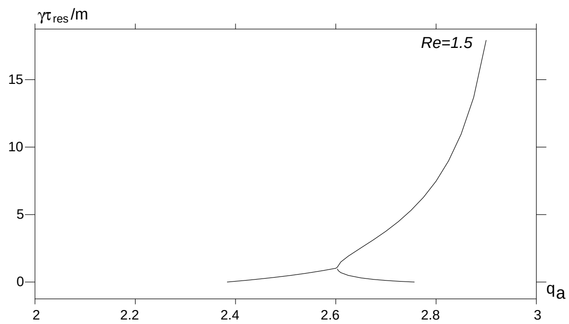
$$(a\Delta'_m - \tau_{res}\gamma)\psi_m = Re \sum_k i(m - 2k - 1)v_{2k+1}\psi_{m-2k-1}. \qquad (2.6)$$

Lithium streams add a

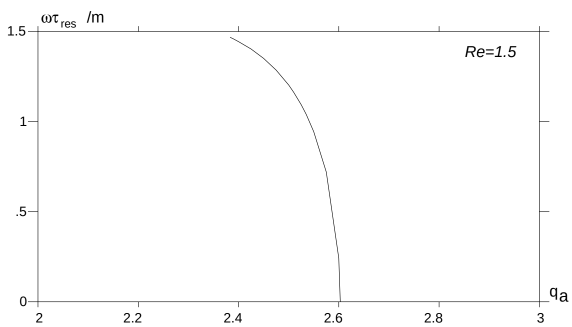
stabilizing ANTI-HERMITIAN coupling

into dispersion relation.

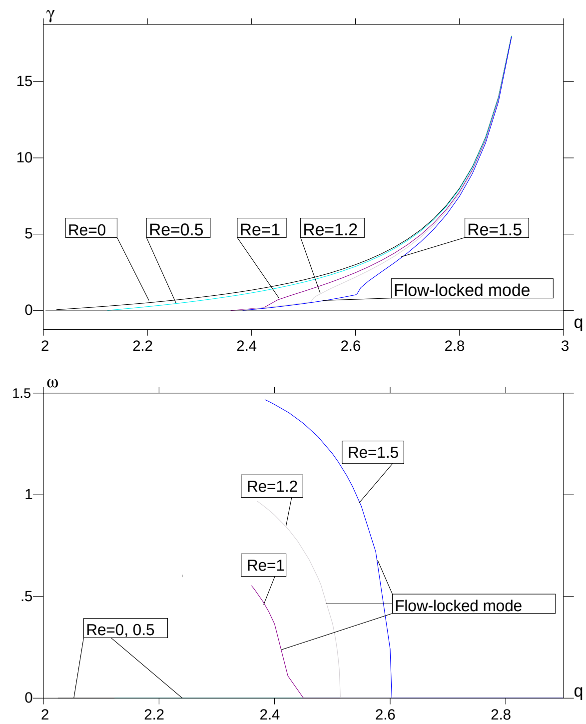
Stability of a uniform current distribution



Real part of the eigenvalues



Imaginary part of eigenvalues



Uniform current distribution

For 3 harmonic model stabilizing contribution of flow is evident

$$a\Delta'_m - \tau_{res}\gamma + \frac{m(m+1)(Re)^2|v_1|^2}{a\Delta'_{m+1} - \tau_{res}\gamma} + \frac{m(m-1)(Re)^2|v_1|^2}{a\Delta'_{m-1} - \tau_{res}\gamma} = 0. \quad (2.7)$$

For finite pressure plasmas

$$\begin{aligned}(a\Delta'_m - \tau_{res}\gamma)\psi_m =& Re\sum_k i(m-2k-1)v_{2k+1}\psi_{m-2k-1} \\ &+ \frac{a}{R}\beta_p \frac{m}{m-nq}(\psi_{m-1}+\psi_{m+1}).\end{aligned} \tag{2.8}$$

Ballooning effect adds a destabilizing HERMITIAN contribution.

For 3 harmonic model stabilizing contribution of flow is evident

$$\begin{aligned}&a\Delta'_m - \tau_{res}\gamma \\ &+ \frac{m(m+1)(Re)^2|v_1|^2 - \frac{a^2}{R^2}\frac{m^2}{(m-nq)^2}\beta_p^2}{a\Delta'_{m+1} - \tau_{res}\gamma} \\ &+ \frac{m(m-1)(Re)^2|v_1|^2 - \frac{a^2}{R^2}\frac{m^2}{(m-nq)^2}\beta_p^2}{a\Delta'_{m-1} - \tau_{res}\gamma} = 0\end{aligned} \tag{2.9}$$

the effect is reduced to replacement

$$\begin{aligned}m(m+1)(Re)^2|v_1|^2 &\rightarrow m(m+1)(Re)^2|v_1|^2 - \frac{a^2}{R^2}\frac{m^2}{(m-nq)^2}\beta_p^2, \\ m(m-1)(Re)^2|v_1|^2 &\rightarrow m(m-1)(Re)^2|v_1|^2 - \frac{a^2}{R^2}\frac{m^2}{(m-nq)^2}\beta_p^2.\end{aligned} \tag{2.10}$$

Again the effect of lithium streams is stabilizing.