

# STUDY OF MHD MIXED CONVECTION IN THE DCLL BLANKET CONDITIONS

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**Abstract:** Magnetohydrodynamic mixed convection associated with a non-uniform volumetric heating in conditions relevant to a Dual-Coolant Lead-Lithium blanket of a fusion reactor has been studied analytically and numerically in the quasi-two-dimensional approximation for buoyancy-assisted and buoyancy-opposed flows.

## 1. Introduction

In the Dual-Coolant Lead-Lithium (DCLL) blanket, the eutectic alloy lead-lithium (PbLi) circulates slowly ( $\sim 10$  cm/s) for power conversion and tritium breeding. The poloidal flows in the blanket are strongly affected by a plasma-confining magnetic field and by a volumetric heating caused by the interaction of neutrons with the liquid metal (LM) [1]. The resultant flow is a superposition of the buoyant and forced flows known as mixed convection. The goal of the present study is to address magnetohydrodynamic (MHD) mixed convection in the blanket-relevant conditions using a quasi-two-dimensional (Q2D) approximation for buoyancy-assisted and buoyancy-opposed flows (Fig. 1).

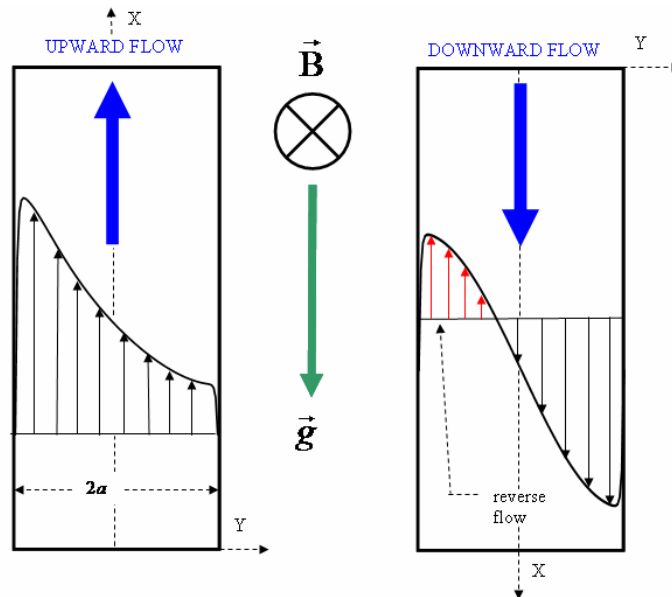


Figure 1. Sketch illustrating the forced flow direction with respect to the gravity vector and the magnetic field.

## 2. Problem formulation

We consider LM flows ( $\rho$ ,  $\nu$ ,  $\sigma$ ,  $k$ ,  $C_p$ ,  $\beta$  are the fluid density, kinematic viscosity, electrical conductivity, thermal conductivity, specific heat, and the volumetric thermal expansion coefficient) in a vertical rectangular duct ( $L$  is the duct length,  $2a \times 2b$  are the cross-sectional dimensions) in a strong transverse uniform magnetic field ( $B_0$ ) under gravity conditions ( $g$  is the acceleration due to gravity). A volumetric heating, which drives the buoyancy flows, can be approximated as  $\dot{q}(y) = q_0 \text{Exp}\{-\frac{y+a}{l}\}$ . Such flows, generally unsteady, are governed by a system of Q2D equations [2], written in the Boussinesq approximation, in terms of the velocity components  $U(t, x, y)$  and  $V(t, x, y)$ , the pressure  $P(t, x, y)$ , and the temperature  $T(t, x, y)$ :

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \frac{U}{\tau} + g(-1)^n + g(-1)^n \beta(T_0 - T), \quad (1)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) - \frac{V}{\tau}, \quad (2)$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \dot{q}, \quad (3)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \quad (4)$$

Here  $\tau$  is the Hartmann braking time, and  $T_0$  is the mean bulk temperature at the flow inlet. Parameter  $n=1$  on the RHS of Eq. (1) corresponds to buoyancy-assisted flows (forced flow is upwards), while  $n=2$  to buoyancy-opposed flows (forced flow is downwards). In what follows we assume that the duct walls are perfectly insulating (both electrically and thermally). In the conditions of near-perfect electrical insulation,  $\tau = b(\rho/\sigma\nu)^{1/2}$  [2]. The associated boundary conditions include no-slip velocity and zero heat flux at the walls  $y = \pm a$ , along with periodic conditions at  $x = 0, L$ . In what follows a modified form of Eqs. (1-4) is also used by introducing:

$$T(t, x, y) = T_0 + \frac{\bar{q}}{\rho C_p U_0} x + \theta(t, x, y), \quad (5)$$

where  $U_0$  stands for the mean bulk velocity, and  $\bar{q} = (2a)^{-1} \int_{-a}^a \dot{q}(y) dy$  is the average volumetric heating.

### 3. Fully-established steady flow

In the fully-established steady flow conditions, by applying (5) and writing the pressure as  $P(x) = P_0 - \rho[G - g(-1)^n]x - g(-1)^n \rho \beta \frac{\bar{q}}{\rho C_p U_0} \frac{x^2}{2}$  ( $G$  is a new constant to be determined), Eqs. (1-4) are reduced to a set of two ODEs for  $U(y)$  and  $\theta(y)$ . After non-dimensionalization by introducing characteristic scales  $[U] = U_0$ ,  $[\theta] = \Delta T = \bar{q} a^2 / k$ ,  $[y] = a$ , and  $[G] = \nu U_0 / a^2$  these equations read:

$$\tilde{U}'' - Ha \left( \frac{a}{b} \right)^2 \tilde{U} - (-1)^n \frac{Gr}{Re} \tilde{\theta} = -\tilde{G}, \quad \text{and} \quad \tilde{U} = \tilde{\theta}'' + \frac{2m}{1 - e^{-2m}} e^{-m(\tilde{y}+1)}. \quad (6, 7)$$

Here,  $Ha = B_0 b \sqrt{\frac{\sigma}{\nu \rho}}$ ,  $Gr = \frac{g \beta \Delta T a^3}{\nu^2}$  and  $Re = \frac{U_0 a}{\nu}$  are the Hartmann, Grashof and the

Reynolds number correspondingly, and  $m = \frac{a}{l}$  is the ‘‘shape’’ parameter. By eliminating

$\tilde{U}(y)$ , Eqs. (6) and (7) can be combined into one of the fourth order for  $\tilde{\theta}(y)$ :

$$\tilde{\theta}^{IV} - Ha \left( \frac{a}{b} \right)^2 \tilde{\theta}'' - (-1)^n \frac{Gr}{Re} \tilde{\theta} = -\tilde{G} + \frac{2m}{1 - e^{-2m}} e^{-m(\tilde{y}+1)} \left[ Ha \left( \frac{a}{b} \right)^2 - m^2 \right], \quad (8)$$

with the following four boundary conditions

$$\tilde{\theta}' \Big|_{\tilde{y}=\pm 1} = 0, \quad \tilde{\theta}'' \Big|_{\tilde{y}=-1} = -\frac{2m}{1 - e^{-2m}}, \quad \tilde{\theta}'' \Big|_{\tilde{y}=+1} = -\frac{2m}{1 - e^{-2m}} e^{-2m}. \quad (9)$$

Further analysis for the fully established mixed convection depends on the solution to the characteristic equation:

$$\lambda^4 - Ha \left( \frac{a}{b} \right)^2 \lambda^2 - (-1)^n \frac{Gr}{Re} = 0, \quad (10)$$

in particular on the sign of the discriminant  $D = Ha^2 \left( \frac{a}{b} \right)^4 + (-1)^n 4 \frac{Gr}{Re}$ , which depends

on the ratio of the two characteristic length scales:  $\delta_1 \sim Ha^{-1/2}$  associated with the thickness of the MHD side layer, and  $\delta_2 \sim Gr^{-1/4}$  relevant to the hydrodynamic boundary layer in purely buoyant flows. In the case of the buoyancy-assisted flow, the discriminant can be either positive or negative, while it is always positive in the case of the buoyancy-opposed flows. The full analytical solution includes exponents  $e^{\pm \lambda(\tilde{y}+1)}$ ,  $e^{-m(\tilde{y}+1)}$  and four integration constants, which are found from Eqs. (9). The solution is lengthy and not presented here. Simplified form of the full solution is given in the next section.

#### 4. Approximate solution for the core region

Equation (6) can further be simplified by neglecting the second derivative  $\tilde{U}''$ , because, at high  $Ha$ , friction in the boundary layers is negligible in comparison with Hartmann damping. Besides, the boundary layers, which are very thin, do not carry any significant flow rate. The corresponding approximate solution is valid in the core (the region between the two boundary layers at the walls  $y = \pm a$ ). For buoyancy-assisted flows,

$$\tilde{\theta}(\tilde{y}) = \frac{2m^2}{r(r^2 - m^2)(1 - e^{-2m})} \left[ \frac{e^{-2m} \cosh[r(\tilde{y} + 1)] - \cosh[r(\tilde{y} - 1)]}{\sinh(2r)} \right] + \frac{2me^{-m(\tilde{y}+1)}}{(r^2 - m^2)(1 - e^{-2m})} - \frac{1}{r^2}, \quad (11)$$

$$\tilde{U}(\tilde{y}) = 1 + r^2 \tilde{\theta}(\tilde{y}). \quad (12)$$

For buoyancy-opposed flows,

$$\tilde{\theta}(\tilde{y}) = -\frac{m^2}{r(r^2 + m^2)(1 - e^{-2m})} \left[ \frac{\sin(r\tilde{y})}{\cos(r)} (1 + e^{-2m}) + \frac{\cos(r\tilde{y})}{\sin(r)} (1 - e^{-2m}) \right] - \frac{2me^{-m(\tilde{y}+1)}}{(r^2 + m^2)(1 - e^{-2m})} + \frac{1}{r^2}, \quad (13)$$

$$\tilde{U}(\tilde{y}) = 1 - r^2 \tilde{\theta}(\tilde{y}). \quad (14)$$

It is noticeable that there are only two dimensionless parameters:  $r = \sqrt{\frac{Gr}{HaRe(a/b)^2}}$ , and

$m$ . The shape parameter  $m$  affects the steepness of the heating profile. It is fully determined by the interaction of neutrons with the LM. Parameter  $r$  is more related to the liquid metal flow itself as it carries information on the contribution of various forces acting on the flow. In typical DCLL blanket conditions,  $r \sim 40$ ,  $m \sim 1$  for the front, and  $r \sim 5$ ,  $m \sim 0.2$  for the return ducts. Figure 2 shows velocity profiles computed with the full and simplified solutions, demonstrating a good match within the core.

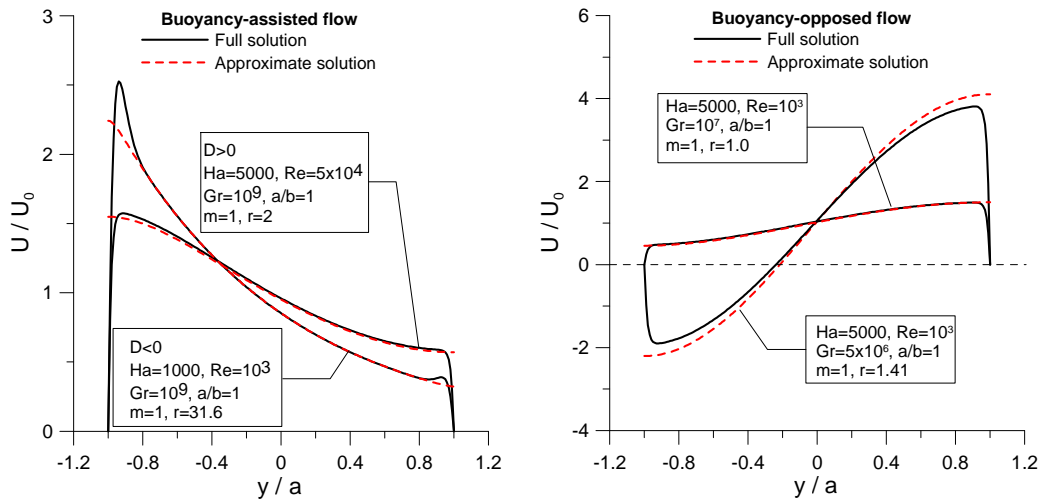


Figure 2. Typical velocity profiles in buoyancy-assisted and buoyancy-opposed flows. Comparison between full and approximate solutions.

## 5. Computations of unsteady flows

The velocity profiles in Fig. 2 have inflection points and thus are subject to Kelvin-Helmholtz (inflectional) instability. In numerical computations of unsteady flows governed by Eqs. (1-4) an equivalent formulation based on the vorticity and the streamfunction is used. The full fully-established steady solution is used as the initial condition. The computations are advanced in time until the non-linear saturation is achieved. The code is based on the Fast Fourier Transform (FFT) algorithm, allowing for implementation of periodic boundary conditions in  $x$ . Some results computed with the code are shown in Fig. 3. The flow exhibits a procession of isolated vortices at the locations associated with the inflection points in the basic velocity profile. In the buoyancy-assisted flow, there are two rows of vortices, while there is only one in the buoyancy-opposed flow.

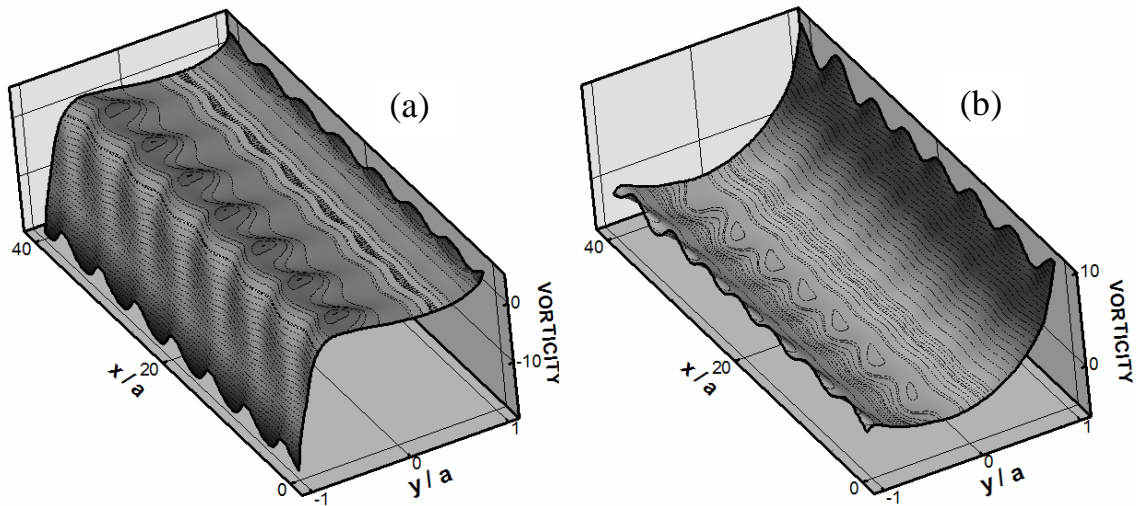


Figure 3. Vorticity distribution in unsteady MHD mixed convection. (a) Buoyancy-assisted flow:  $Ha=10$ ,  $Re=10^3$ ,  $Gr=10^6$ ,  $Pr=0.01$ ,  $a/b=1$ ,  $m=1$ , and  $r=10$ . (b) Buoyancy-opposed flow:  $Ha=10$ ,  $Re=10^3$ ,  $Gr=3 \times 10^4$ ,  $Pr=0.01$ ,  $a/b=1$ ,  $m=1$ , and  $r=1.73$ .

## 6. Conclusions

Analytical solution for fully-established steady mixed convection in the DCLL blanket conditions have been obtained in the Q2D approximation using the full and simplified equations. Both solutions agree well within the core region for buoyancy-assisted and buoyancy-opposed flows. Unsteady flows have been computed using a FFT-based code, showing inflectional instability.

## 7. References

- [1] Smolentsev, S.; Moreau, R., Abdou, M.: Characterization of Key Magnetohydrodynamic Phenomena in PbLi Flows for the US DCLL Blanket. Submitted to Fusion Engineering and Design, 2008.
- [2] Sommeria, J.; Moreau, R.: Why, How and When MHD Turbulence Becomes Two-Dimensional?, J. Fluid Mech. 118 (1982) 507-518.