STABILITY ANALYSIS FOR THE HARTMANN FLOW WITH INTERFACIAL SLIP

VETCHA Naveen , SMOLENTSEV Sergey, ABDOU Mohamed
Fusion Science and Technology Center, UCLA, USA
e-mail: vnaveen@ fusion.ucla.edu (Naveen Vetcha)

Abstract: In the present research we study the effect of the slip length on the stability of the Hartmann flow by modifying the Orr-Sommerfeld problem to account for the slip effect. A MATLAB code based on Chebyshev collocation method is modified to solve the eigenvalue problem for the reference slip flow. The analysis is carried out for both symmetric and asymmetric slip conditions. It has been found that the interfacial slip significantly increases the critical Reynolds number even for very small slip lengths compared to the thickness of the Hartmann layer.

1. Introduction

Interfacial slip has recently been identified as an important phenomenon in the so called Dual Coolant Lead Lithium (DCLL) blanket, where the eutectic alloy lead-lithium flows in contact with an insulating ceramic insert made of silicon carbide (SiC) in presence of a strong plasma-confining magnetic field [1]. Recent experimental studies [2-4] also suggest that the slip between the flowing liquid metal and SiC insert will likely occur. An analytical solution has been derived for the Hartmann flow with the interfacial slip in [1] by introducing a slip length. In the same study it is also observed that slip does not affect the Hartmann layer thickness. The classic "SM82" model [5] for quasi-two-dimensional flow is also modified in [1] for the flow with interfacial slip and it is observed that the flow demonstrates more irregular behavior as the slip length increases.

In the present study we are interested in the influence of slip on the stability of the Hartmann flow. For the case of non MHD flows this type of analysis is carried out for plane Poiseuille flow [6-7] for both symmetric and asymmetric slip conditions. The study by Lauga *et. al.* [6] concluded that the slip strongly stabilizes the flow, while an extended study by Ling *et. al.* [7] showed that slip plays a dual role by either stabilizing or destabilizing the flow depending on the slip length. For the case of the Hartmann flow with no-slip Lock [8] carried out stability analysis and found that the critical Reynolds number Re_c becomes larger than 10^6 when the Hartmann number Ha is larger than 20. Takashima [9] reexamined the problem in greater detail and found that the critical values vary as $Re_c = 48311.016\ Ha$ and, critical wave number $\alpha_c = 0.161531\ Ha$.

In the present research we study the effect of the slip length on the stability of the Hartmann flow by modifying the Orr-Sommerfeld problem to account for the slip effect. A MATLAB code based on Chebyshev collocation method [10] is modified and validated first against available results for both hydrodynamic [11] and MHD flows [8, 9] without the slip and then applied to solve the eigenvalue problem for the reference slip flow. The analysis is carried out for both symmetric and asymmetric slip conditions. The results presented in the form of neutral stability curves are compared with analogous results for hydrodynamic flows with and without the slip and also with the classic Hartmann problem.

2. Mathematical formulation

We consider fully developed pressure-driven (pressure gradient *dP/dx*) steady flow of an incompressible, viscous, electrically conducting liquid (ρ, ν, σ are the fluid density, kinematic viscosity and the electrical conductivity) between two infinite parallel plates in the presence of a uniform external magnetic field B_0 perpendicular to the plates. The origin is taken midway between the plates with *x*-axis in the direction of flow and the *y*-axis perpendicular to the flow. The plates at $y^* = \pm b$ (superscript '*' is used for dimensional quantities) are assumed to be electrically non- conducting. For this kind of two-dimensional flow only the stream wise component of the velocity $U^*(y^*)$ and induced magnetic fields $b_x^*(y^*)$ are non zero governed by two equations

$$0 = 1 + \frac{d^2U}{dy^2} + Ha\frac{db_x}{dy}, \tag{1}$$

$$0 = \frac{d^2b_x}{dy^2} + Ha\frac{dU}{dy}. \tag{2}$$

Here, the equations are written in dimensionless form. The velocity is scaled by $[U] = b^2\rho^{-1}\nu^{-1}(-dP/dx)$; the induced magnetic field is scaled by $[b_x] = [U]\mu_0\sqrt{\sigma\rho\nu}$ (μ_0 is the magnetic permeability); and half of the distance between the plates *b* is used as length scale.

Considering that the flow satisfies the slip conditions on both surfaces, with slip lengths $\square_1$ and $\square_2$ at $y^* = +b$ and $y^* = -b$, respectively. If we define the dimensionless slip lengths as $\lambda_1 = \square_1/b$ and $\lambda_2 = \square_2/b$, then the boundary conditions in the presence of slip are

$$y = +1:\; U + \lambda_1\frac{dU}{dy} = 0, \qquad b_x = 0 \tag{3a}$$

$$y = -1:\; U - \lambda_2\frac{dU}{dy} = 0, \qquad b_x = 0 \tag{3b}$$

The solution for the velocity is derived in the following form

$$U(y) = (K_1\,K_2\sinh(Ha\,y) + K_2\cosh(Ha\,y) + 1)\,C \tag{4}$$

where

$$K_1 = \frac{-(\lambda_1 - \lambda_2)Ha\,\sinh(Ha)}{2\sinh(Ha) + (\lambda_1 + \lambda_2)Ha\,\cos(Ha)},$$

$$K_2 = \frac{-2}{2\cosh(Ha) + (\lambda_1 + \lambda_2)Ha\sinh(Ha) + (\lambda_1 - \lambda_2)K_1\,Ha\,\cosh(Ha)},$$

$$C = -\frac{1}{K_2\,Ha\sinh(Ha)}$$

To study the stability of the flow with the basic velocity (4), we write the total velocity as the sum of the basic flow and small perturbations. As the total and basic flow both satisfy the slip conditions the boundary conditions for the perturbed flow are also of the form of eq. (3). To derive the modified Orr-Sommerfeld equation we follow the procedure in [9] and get the differential equation:

$$\left(\overline{U} - {}^{\beta}\!/_{\alpha}\right)\left(D^2 - \alpha^2\right)\phi - D^2\overline{U}\phi = \frac{1}{i\alpha \operatorname{Re}}\left[\left(D^2 - \alpha^2\right)^2 \phi - Ha^2 D^2\phi\right] \tag{5}$$

with the boundary conditions

$$y = +1: \ \phi = D\phi + \beta_1 D^2\phi = 0 \tag{6a}$$

$$y = -1: \ \phi = D\phi - \beta_2 D^2\phi = 0 \tag{6b}$$

where $\overline{U} = U(y)/U(y=0)$, $\beta=\beta_r+i\beta_i$ is a complex number with β_i being the wave amplification factor, α is the wave number, D is the differentiation matrix, ϕ is the stream function and $Re(=U^*(y^*=0)b/\nu)$ is the Reynolds number. Equation (5) and (6) govern an eigenvalue problem which has to be solved fosr the eigenvalue β.

A MATLAB code based on Chebyshev collocation method [10] is used to calculate the eigenvalues. The code is modified to solve the equation (5) and also the no-slip boundary conditions are changed into more general slip boundary conditions (Eq. (6)). The code is then validated against available non MHD [11] and MHD cases [9].

3. Results and Discussions

We restrict the analysis to symmetric slip ($\lambda_1 = \lambda_2 = const.$) and asymmetric slip ($\lambda_1 = const.$, $\lambda_2 = 0$) cases. The flow is said to be linearly unstable if there exists at least one eigenvalue with a positive imaginary part, $\beta_i > 0$. The neutral curves $\beta_i(\alpha, Re) = 0$ for different values of *Ha* and for both symmetric and asymmetric slip are shown in fig 1. The area inside the curves corresponds to linearly unstable flow regime, whereas outside the curves the flow is considered linearly stable. For the case of Ha = 0 the symmetric boundary slip (fig 1a) appears to significantly shift the neutral curves towards larger values of *Re*, indicating a strong stabilizing effect. The same behavior can be seen for the case of asymmetric slip but it is less pronounced fig 1b. In both symmetric and asymmetric cases when *Ha* is increased to 10, the value of Re_c increases enormously (about two orders of magnitude greater that that of *Ha* = 0) due to the stabilizing effect of the magnetic field. As shown in fig 1c for the case of symmetric slip even very small slip length (about one order of magnitude less than that for *Ha* = 0 case) has tremendous influence on stabilizing the flow. Interestingly, the stabilizing effect does not appear to be that strong for the case of asymmetric slip as observed in fig 1d. The slip effect becomes stronger at *Ha* = 20, as can be seen in fig 1e, for the case of symmetric slip just by increasing the slip length from 0.001 to 0.002 increases Re_c by one order of magnitude. For the case of asymmetric slip (fig 1f) there is no or very small change in Re_c as the slip length is increased from no-slip. Same behavior as that of *Ha* = 20 is observed for higher values of *Ha* (not shown here) indicating that only symmetric slip has strong stabilizing effect on Hartmann flow.

With the increase in magnetic field strength the critical wave number α_c increases. However, α_c is observed to decrease with the increase of slip length indicating that as the slip length increases perturbations with smaller wavelengths are suppressed.

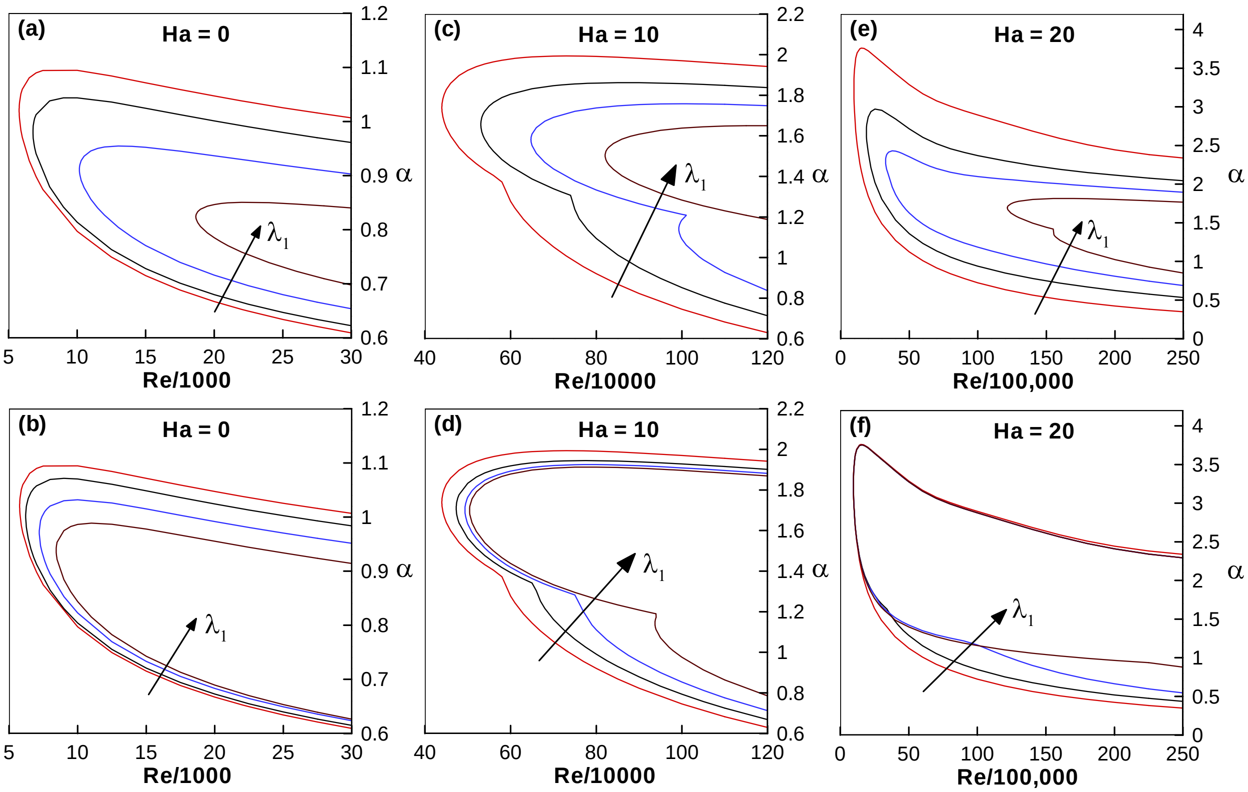

Figure1: Neutral stability curves for the symmetric (top) and asymmetric (bottom) slip cases. (a, b) values of λ_1= 0, 0.01, 0.02 and 0.03, (c, d, e, f) values of λ_1= 0, 0.001, 0.0015 and 0.002.

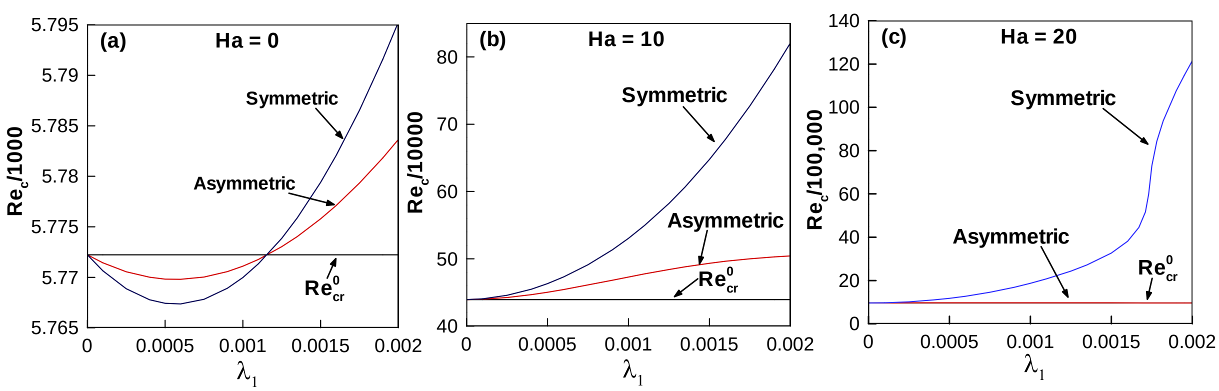

Figure2: Variation of critical Reynolds number with non dimensional slip length for the symmetric and asymmetric slip cases.

Figure 2 shows the variation of Re_c with slip length for the cases with and without magnetic field. In the absence of magnetic field (fig 2a), it appears that slip length below certain value destabilizes the flow (Re^0_{cr} is the no-slip value). Once the magnetic field is applied this behaviour is not observed anymore. Slip always causes flow stabilization. For the case of asymmetric slip with the increase of Ha, the variation of Re_c from that of no slip value decreases and for sufficiently strong magnetic field, Re_c remains equal to that of no slip value (fig 2c).

4. Conclusions

The effect of slip on the stability of Hartmann flow is analyzed by performing a linear stability analysis. It is observed that slip strongly stabilizes the flow even for very small values of slip length; this behaviour is different from that of non MHD case where slip plays a dual role of either stabilizing or destabilizing the flow depending on slip length. For strong enough magnetic field only symmetric slip is causing the flow stabilization. The stabilizing effect of the slip on the MHD flow is strong even though the slip length is very small compared to the thickness of the Hartmann layer.

5. References

[1] Smolentsev S.: MHD duct flows under hydrodynamic "slip" condition. Theor. Comput. Fluid Dyn. 23 (2009) 557-570.

[2] Pint B.A.; More K.L.; Meyer H.M.; Distefano J.R.: Recent progress addressing compatability issues relevant to fusion environments. Fusion Sci. Technol. 47 (2005) 851-855.

[3] Morley N.B.; Medina A.; Abdou M.A.: Measurements of specific electrical contact resistance between SiC and lead-lithium eutectic alloy, TOFE 18, San Francisco, September 28-October 2 (2008) Book of Abstracts, P2.66.

[4] Changho P.; Kazuyuki N.; Yamamoto Y.; Konishi S.: Compatability of materials for advanced blanket with liquid LiPb, TOFE 18, San Francisco, September 28-October 2 (2008) Book of Abstracts, P1.67.

[5] Sommeria J.; Moreau R.: Why, how, and when, MHD turbulence becomes two-dimensional. J. Fluid Mech. 118 (1982) 507-518.

[6] Lauga E.; Cossu C.: A note on the stability of slip channel flows. Phy. of fluids 17 (2005) 088106,1-4.

[7] Ling R.; Jian-Guo C.; Ke-Qin Z.: Dual role of wall slip on linear stability of plane poiseuille flow. Chin. Phys. Lett. 25-2 (2008) 601-603.

[8] Lock R.C.: The stability of the flow of an electrically conducting fluid between parallel planes under transverse magnetic field. Proc. Roy. Soc. Lond. A 233 (1955) 105-125.

[9] Takashima M.: The stability of the modified plane Poiseuille flow in the presence of a transverse magnetic field. Flu. Dyn. Research. 17 (1996) 293-310.

[10] Schmid P. J.; Henningson D.D.: Stability and Transition in shear flows, Springer, New York, 2001.

[11] Orszag S.: Accurate solution of the Orr-Sommerfeld stability equation. J. Fluid Mech. 50 (1971) 689-703.