THEORETICAL STUDY OF MIXED CONVECTION IN POLOIDAL FLOWS OF DCLL BLANKET

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An approach is developed to study the stability of mixed convection in the poloidal flows of the DCLL blanket. Modified Orr-Sommerfeld equations are derived and then solved using a numerical code based on a pseudo-spectral method. The stability analysis has been performed for the flows in the front blanket ducts, where the forced flow is upwards; *showing that in the DCLL blanket conditions, all disturbances associated with the buoyant flows in the front ducts will likely be damped by a strong toroidal magnetic field.*

I. INTRODUCTION

In the dual-cooled lead-lithium (DCLL) blanket, which is considered in the US for testing in ITER and for using in DEMO, the eutectic alloy lead-lithium (PbLi) circulates slowly (at ~ 10 cm/s) for power conversion and tritium production (Fig. 1). In this blanket, the most of heat deposition and tritium breeding occur in the poloidal ducts facing the plasma (front ducts) where the forced flow is superimposed with buoyant flows, resulting in the mixed convection flow regime. The preliminary analysis of mixed convection under the ITER and DEMO blanket conditions[1] has shown that buoyant flows in the long poloidal ducts are comparable with or can even dominate over forced flows, thus affecting heat transfer and tritium transport. The goal of the present study is to identify conditions when the mixed flows in the poloidal ducts become unstable, as significant changes in heat and tritium transport are likely to occur if the flow in the blanket transients to being unstable.

The stability problem for poloidal flows is attacked here analytically using a linear stability theory based on the modified Orr-Sommerfeld equations that take into account specific DCLL blanket conditions. In these conditions, the flow is known to become quasi-two-dimensional (Q2D) (*e.g.*, Ref. 2) due to a kind of magnetic diffusion associated with a strong toroidal magnetic field. This striking feature allows using Q2D flow equations, known as SM82 equations.[2] First, the basic flow solution of the SM82 equations is derived analytically. Second, the modified Orr-Sommerfeld equations are derived by imposing small perturbations on the basic solution. Then, the associated eigenvalue problem is solved numerically using a pseudo-spectral method.

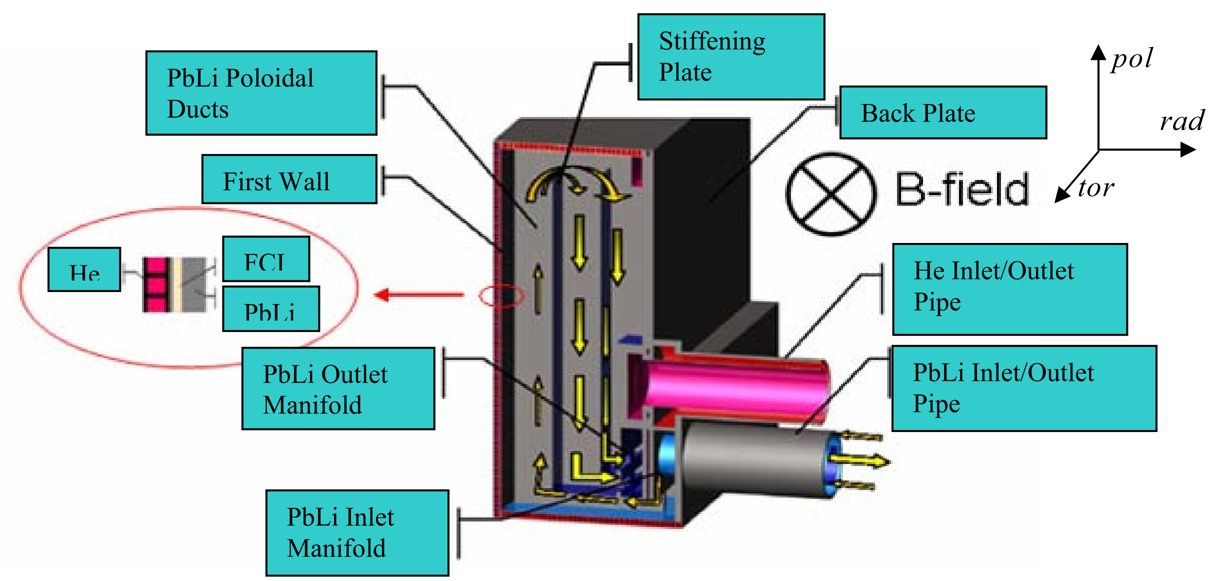

Fig. 1. Sketch of the US DCLL DEMO blanket module.

The obtained numerical data are plotted in the form of neutral curves, which can be used to predict transitions from a steady mixed flow to the unsteady one, where heat and mass transport are strongly controlled by large coherent vortical structures.

II. PROBLEM FORMULATION

The flow investigated in this paper is mixed convection flow of liquid metal PbLi in the presence of a transverse (toroidal) magnetic field B_0, which is driven by an external pressure gradient and also by a buoyancy force associated with the volumetric heating $\dot{q}$ due to neutrons. The volumetric heating changes rapidly in the radial direction (y) and is approximated here with the following formula:

$$\dot{q} = q_0 e^{-(y+a)/l},$$

where the ratio $a/l \equiv m$ is hereafter referred to as the "shape parameter".

The present analysis is limited to the front ducts of the DCLL blanket, where the forced flow is upwards. A schematic of this system is shown in Fig. 2 and typical dimensionless parameters are summarized in Table I for both the DEMO and ITER blanket.

Table I. Basic Dimensionless Flow Parameters in the Poloidal Flow in the Front Duct for ITER and DEMO (for definition of *Ha*, *Re* and *Gr*, see Section III)

Parameter	ITER	DEMO
Ha	6500	12,000
Re	30,000	60,000
Gr	7.0×10^{9}	2.0×10^{12}
m	0.3	1.0

Such flows, generally unsteady, are governed by a system of Q2D equations, written in the Boussinesq approximation, in terms of the velocity components $U(t,x,y)$ and $V(t,x,y)$, the pressure $P(t,x,y)$, and the temperature $T(t,x,y)$:[1]

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) - \frac{U}{\tau} - g - g\beta(T_0 - T), \quad (1)$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) - \frac{V}{\tau}, \quad (2)$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}\right) = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \dot{q}, \quad (3)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \quad (4)$$

Here, $\rho, \nu, \sigma, \kappa, C_p, \beta$ are the fluid density, kinematic viscosity, electrical conductivity, thermal conductivity, specific heat and volumetric thermal expansion coefficient respectively. g is the acceleration due to gravity, L is the duct length, $2a \times 2b$ are the duct cross-sectional dimensions, τ is the Hartmann braking time, and T_0 is the mean bulk temperature at the flow inlet. In the present analysis, we assume ideal electrical and thermal insulation. In the real blanket conditions, near-perfect insulation can be achieved using the so-called flow channel insert.[3] In the conditions of near-perfect electrical insulation, $\tau = b(\rho/\sigma\nu)^{1/2}$ (Ref. 2). In what follows, a modified form of the energy equation is used by introducing a new function $\theta(t,x,y)$, such that:

$$T(t,x,y) = T_0 + \frac{\overline{q}}{\rho C_p U_0}x + \theta(t,x,y), \quad (5)$$

where U_0 stands for the mean bulk velocity, and $\overline{q} = (2a)^{-1}\int_{-a}^{a}\dot{q}(y)dy$ is the mean volumetric heating.

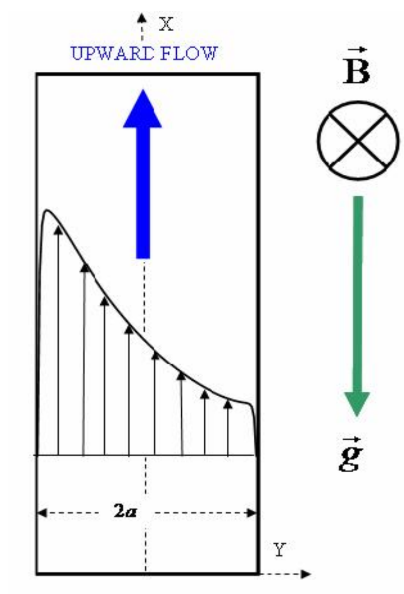

Fig. 2. Sketch illustrating the forced flow direction with respect to the gravity vector and magnetic field.

III. LINEAR STABILITY ANALYSIS

In the linear stability analysis, infinitesimal disturbances are imposed on steady laminar basic flow. The basic flow has only one velocity component: $\overline{U}(y)$. By introducing the above said disturbances, the velocity, pressure and temperature fields can be written as

$$U=\overline{U}+u', V=v', P=\overline{P}+p', \theta=\overline{\theta}+\theta', \tag{6}$$

where the prime denotes an infinitesimal disturbance which is a function of x, y and t. The basic flow is a solution of Eqs. (1-4). However, the resulting motion in Eq. (6) also has to satisfy these equations. Inserting Eq. (6) in the governing equations and ignoring all small terms quadratic in perturbations, we obtain

$$\frac{\partial u'}{\partial t}+\overline{U}\frac{\partial u'}{\partial x}+\frac{\partial \overline{U}}{\partial y}v'=-\frac{\partial p'}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u'}{\partial x^2}+\frac{\partial^2 u'}{\partial y^2}\right) +\frac{Gr}{Re^2}\theta'-\frac{Ha}{Re}\left(\frac{a}{b}\right)^2 u', \tag{7}$$

$$\frac{\partial v'}{\partial t}+\overline{U}\frac{\partial v'}{\partial x}=-\frac{\partial p'}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v'}{\partial x^2}+\frac{\partial^2 v'}{\partial y^2}\right) -\frac{Ha}{Re}\left(\frac{a}{b}\right)^2 v', \tag{8}$$

$$\frac{\partial \theta'}{\partial t}+\overline{U}\frac{\partial \theta'}{\partial x}+\frac{\partial \overline{\theta}}{\partial y}v'=\frac{1}{RePr}\left(\frac{\partial^2 \theta'}{\partial x^2}+\frac{\partial^2 \theta'}{\partial y^2}-u'\right), \tag{9}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}=0. \tag{10}$$

Here, $Re=U_0 a/\nu$, $Gr=g\beta\overline{q}a^5/\kappa\nu^2$, $Ha=B_0 b\sqrt{\sigma/\rho\nu}$ and $Pr=\rho C_p\nu/\kappa$ are the Reynolds, Grashof , Hartmann and the Prandtl number respectively.

By expanding the solution in normal modes, the disturbances can be represented by

$$\psi(x,y,t)=\varphi(y)e^{i(\alpha x-\beta t)}, \tag{11}$$

$$\theta'(x,y,t)=h(y)e^{i(\alpha x-\beta t)}, \tag{12}$$

where $\psi(x,y,t)$ is the stream function for the two-dimensional perturbation. Here α is real, so that $\lambda=2\pi/\alpha$ is the wavelength of the perturbation. The quantity β is complex: $\beta=\beta_r+i\beta_i$. The ratio β_r/α is the propagation velocity of the flow perturbation, also known as the "phase velocity". The parameter β_i (amplification factor) determines whether a perturbation is amplified or damped with time: the flow is stable, neutrally stable or unstable for $\beta_i<0$, $\beta_i=0$ or $\beta_i>0$, respectively. The amplitude functions $\varphi(y)$ and $h(y)$ are set to be only dependent on y, since the basic flow is only dependent on y. Equation (11) yields the components of the perturbation velocity as

$$u'=\frac{\partial\psi}{\partial y}=\frac{\partial\varphi}{\partial y}e^{i(\alpha x-\beta t)}, \tag{13}$$

$$v'=-\frac{\partial\psi}{\partial x}=-i\varphi(y)e^{i(\alpha x-\beta t)}. \tag{14}$$

Inserting Eqs. (12), (13) and (14) in Eqs. (7), (8) and (9), and eliminating the pressure, the following coupled ordinary differential equations are found for the amplitude functions $\varphi(y)$ and $h(y)$:

$$i\alpha \mathrm{Re}((\overline{U}-\beta/\alpha)(\varphi''-\alpha^2\varphi)-\overline{U}''\varphi)=\varphi^{iv} -2\alpha^2\varphi''+\alpha^4\varphi+\frac{Gr}{\mathrm{Re}}h'-Ha\left(\frac{a}{b}\right)^2(\varphi''-\alpha^2\varphi), \tag{15}$$

$$i\alpha \mathrm{Re}\,\mathrm{Pr}((\overline{U}-\beta/\alpha)h-\overline{\theta}'\varphi)=h''-\alpha^2 h-\varphi', \tag{16}$$

where prime denotes differentiation with y. Equations (15) and (16) show that instability can appear either due to inflection points in the basic velocity profile (Eq. 15) or due to temperature variations (Eq. 16). The latter, however, introduces weaker instability mechanism compared to the Kelvin-Helmholtz (inflectional) instability associated with the inflection points. This simplifies the problem by eliminating the need for solving Eq. (16).

IV. NUMERICAL METHOD

The stability analysis is now reduced to an eigenvalue problem for the perturbation differential equation, Eq. (15) without the term with h', with the following boundary conditions:

$$y=-1: u'=v'=0: \varphi=0, \varphi'=0, \tag{17a}$$

$$y=+1: u'=v'=0: \varphi=0, \varphi'=0. \tag{17b}$$

For a given basic flow $\overline{U}(y)$, Eq. (15) contains six parameters, namely $Re, Gr, Ha, \alpha, \beta_i$ and β_r. Of these parameters, Reynolds, Grashof and Hartmann numbers of the basic flow can be considered as given, and in addition the wavelength $\lambda = 2\pi/\alpha$ of the perturbation can also be taken as given. Therefore, for every set of these parameters, the differential equation with the specified boundary conditions yields an eigenfunction $\varphi(y)$ and a complex eigenvalue β. The special case with $\beta_i = 0$ gives neutral disturbances.

A MATLAB code[5] developed on the basis of a pseudo-spectral method, which eliminates spurious eigen values,[6] is used to solve the present eigenvalue problem. The code has been validated against the available literature results for the stability of a plane Poiseuille[7] and Hartmann[8] flows.

V. RESULTS AND DISCUSSIONS

The basic velocity profiles are obtained for different values of parameters Re, Gr and Ha using the analytical solution derived in Ref. 4 (Fig. 3). In all computations, the shape parameter *m* was set at *m*=1.0 that corresponds to the DEMO blanket conditions (see Table I). The velocity profiles show clearly two inflection points near the walls $y = \pm a$. The results of stability calculations using the numerical method described in Section IV are plotted in the form of a "neutral stability" curve obtained from the condition $\beta_i = 0$, which separates the stable and unstable solutions. The point on this curve where the Hartmann number is largest gives the critical Hartmann number. Above the critical Hartmann number all possible perturbation modes are damped, while below some modes are amplified.

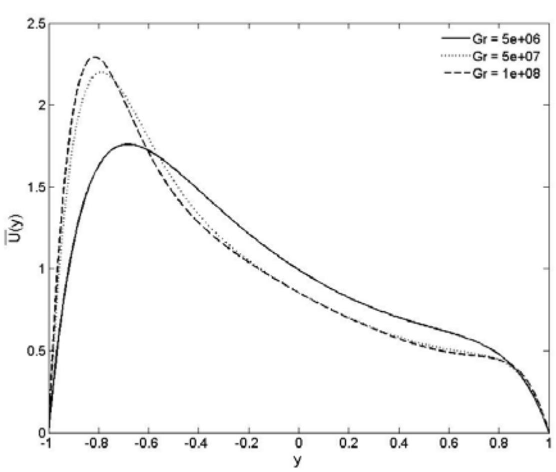

Fig. 3. Variation of basic velocity profile for different *Gr* at $Ha = 50$, $Re = 10{,}000$.

Figure 4 shows neutral stability curves for $Re = 10{,}000$ at three different Grashof numbers. When the Grashof number is increased from $5e+06$ to $5e+07$ the critical Hartmann number increases, showing that the flow becomes more unstable. Further increase from $Gr = 5e+07$ to $1e+08$ shows a decrease in the critical Ha. This is due to the change of the velocity profile at high Gr. Namely for this set of Gr and *Re*, the inflection points move very close to the wall (see Fig. 3) so that the flow in the vicinity of the inflection points experiences more viscous damping; a lesser magnetic field strength is enough for stabilizing the flow.

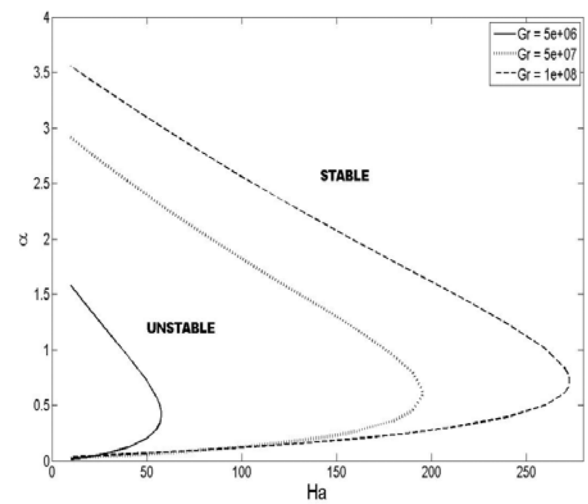

Fig.4. Neutral stability curves for different *Gr* at *Re*= 10,000

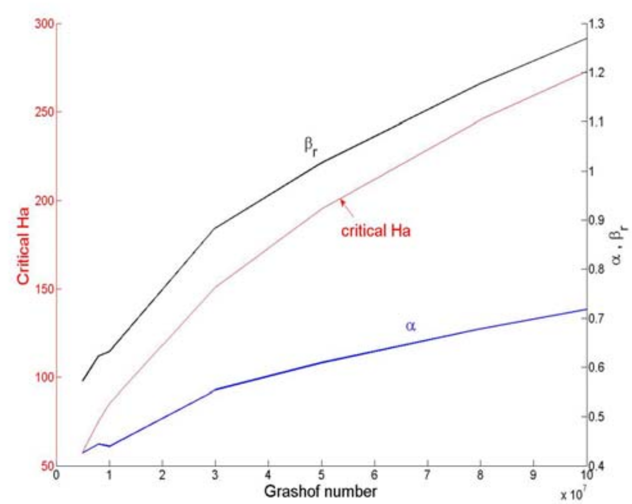

Fig.5. Variation of critical *Ha*, *α*, β_r with *Gr* at *Re* = 10,000

Figure 5 shows variation of critical Ha, α and β_r as a function of Gr. As explained above the critical Ha first increases with Gr and then starts decreasing after a particular value of Gr, while both α and β_r increase monotonously.

IV. CONCLUSIONS

An approach for accessing conditions when the MHD mixed convection in long vertical ducts becomes unstable has been developed for flows in a strong transverse magnetic field. Using this approach, a linear stability analysis has been performed under conditions relevant to the US DCLL blanket for the front duct, where the liquid metal flows upwards. The computations performed at Re=10,000 and Gr from 5×10^6 to 10^8 have demonstrated that all instabilities in such a flow are damped at relatively low Hartmann numbers: critical $Ha \sim 10^2$. Extrapolating these results to the blanket conditions, where the Hartmann number is sufficiently higher, suggests that the mixed flows in the front ducts of the DCLL blanket are stable.

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