# Computation of Film Flow in Toroidal Field Gradient

Donghong Gao, Neil Morley

# 2D model considering the field gradient effect

## Governing Equations:

$$\frac{\partial B_i}{\partial t} + u\frac{\partial B_i}{\partial x} + v\frac{\partial B_i}{\partial y} = \frac{1}{\mu_m \sigma_e}\nabla^2 B_i - \left(u\frac{\partial B_a}{\partial x} + v\frac{\partial B_a}{\partial y}\right) - \frac{\partial B_a}{\partial t} \quad \longrightarrow \textbf{induction}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\nabla^2 u + g_x - \frac{1}{\rho \mu_m}(B_a + B_i)\frac{\partial B_i}{\partial x} \quad \longrightarrow \textbf{x-mom}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{1}{\rho}\nabla^2 u + g_y - \frac{1}{\rho \mu_m}(B_a + B_i)\frac{\partial B_i}{\partial y} \quad \longrightarrow \textbf{y-mom}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad \longrightarrow \textbf{continuity}$$

## Features of the Model:

2D model: 2D geometry, x-y plane velocity, spanwise (z-direction) B field.
Induced equation reveals: source terms come from field spatial and temporal variation.
Hartmann layers (removed to infinity) effect is ignored.
Important role of Bi: represents currents. Bi can be considered as stream-function of currents

# Numerical Schemes

## Projection method for Navier-Stokes

First Step:

$$\frac{\vec{V}^* - \vec{V}^n}{\Delta t} = -\vec{V}^n.\nabla\vec{V}^n + \frac{1}{\rho^n}\nabla\cdot(\mu\tau^n) + \vec{g} + \frac{1}{\rho^n}\vec{F}^n_{sv} - \frac{1}{\rho^n\mu_m}(B_a^{n+1} + B_i^{n+1})\nabla B_i^n$$

Second Step:

$$\left.\begin{array}{l} \dfrac{\vec{V}^{n+1} - \vec{V}^*}{\Delta t} = -\dfrac{1}{\rho^n}\nabla p^{n+1} \\ \nabla\cdot\vec{V}^{n+1} = 0 \end{array}\right\} \quad \nabla\cdot\left[\frac{1}{\rho^n}\nabla p^{n+1}\right] = \frac{\nabla\cdot\vec{V}^*}{\Delta t}$$

## Free surface tracking — VOF method

$$f(x,y,t) = \begin{cases} 1, & \text{in the fluid} \\ 0, & \text{in the gas side (void)} \\ >0 \text{ and } <1 & \text{at the free surface} \end{cases}$$

Advection equation:

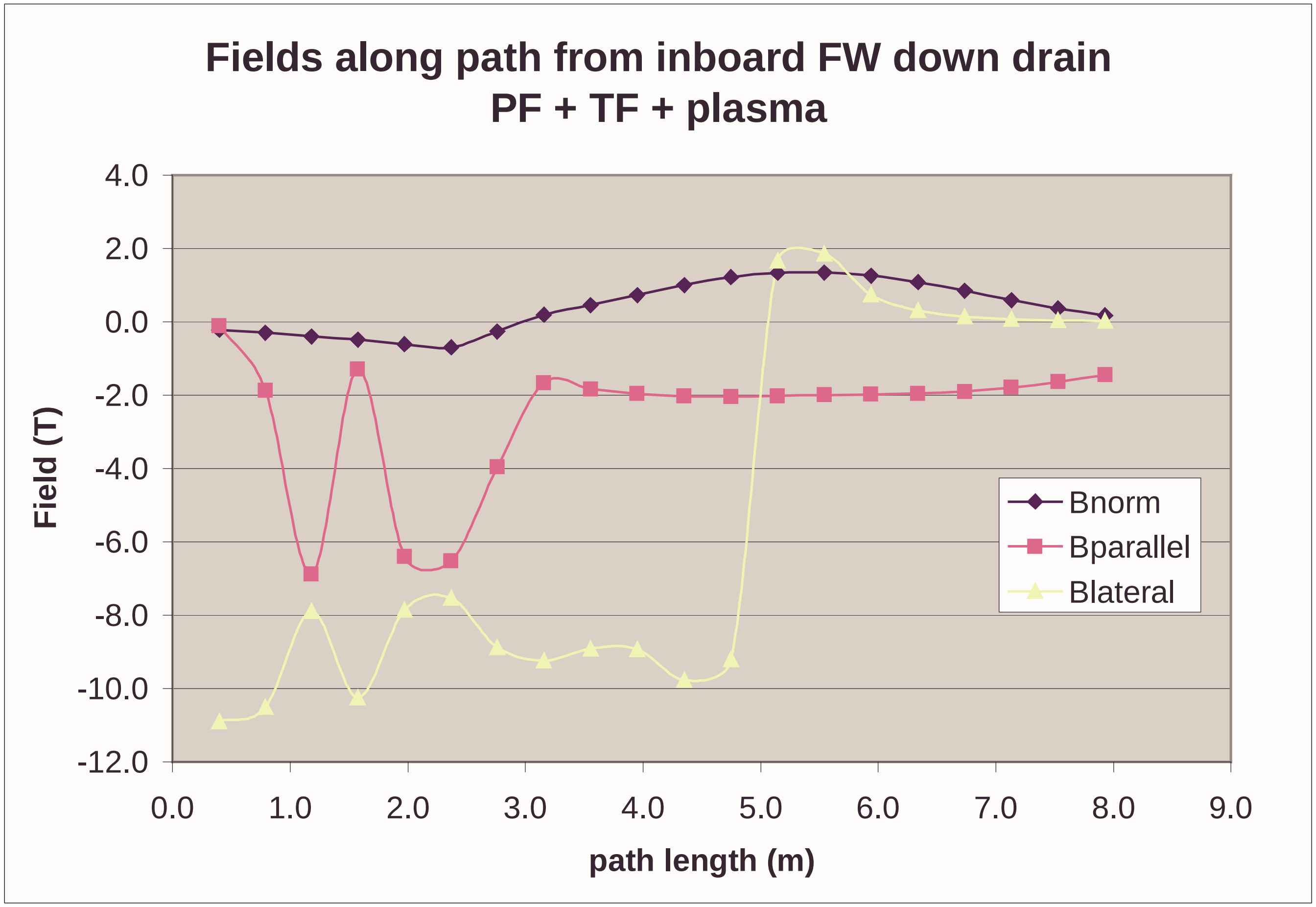
$$\frac{\partial f}{\partial t} + (\vec{V}\cdot\nabla)f = 0$$

## Surface Tension —CSF model

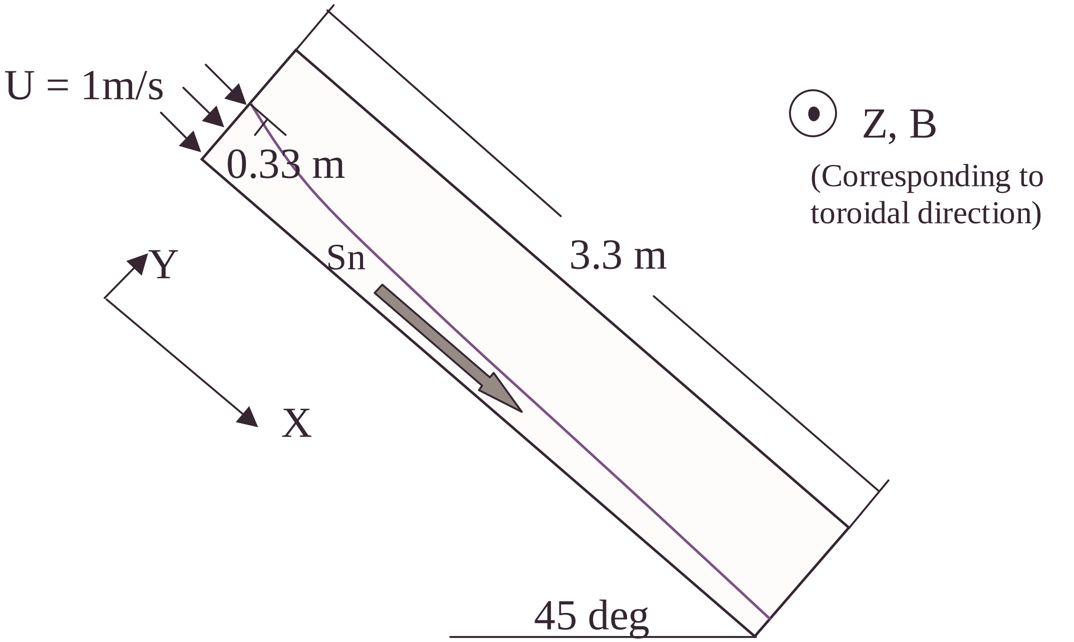
$$\vec{F}_{sv}(x,y) = \sigma\,\kappa(x,y)\,\nabla f(x,y), \qquad \vec{n} = -\nabla f, \qquad \hat{n} = \vec{n}/|\vec{n}|$$

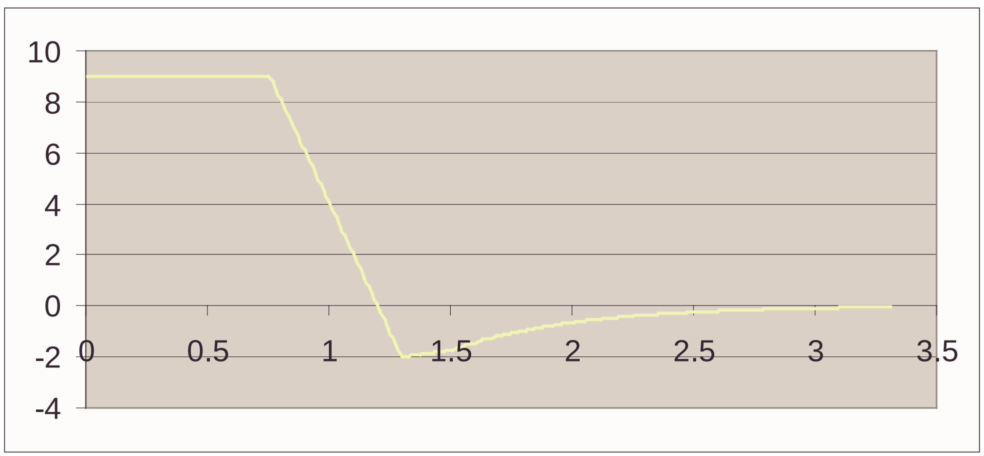
$$\begin{aligned} \kappa &= (\nabla\cdot\hat{n}) = -\frac{1}{|\vec{n}|}\left[(\frac{\vec{n}}{|\vec{n}|}\cdot\nabla)|\vec{n}| - (\nabla\cdot\vec{n})\right] \\ &= -\frac{1}{|\vec{n}|}\left[\frac{n_x^2}{|\vec{n}|^2}\frac{\partial n_x}{\partial x} + \frac{n_x n_y}{|\vec{n}|^2}\left(\frac{\partial n_x}{\partial y} + \frac{\partial n_y}{\partial x}\right) + \frac{n_y^2}{|\vec{n}|^2}\frac{\partial n_y}{\partial y}\right] \end{aligned}$$

# Case Study: Divertor in toroidal magnetic field gradient



# Geometry of the computation cases

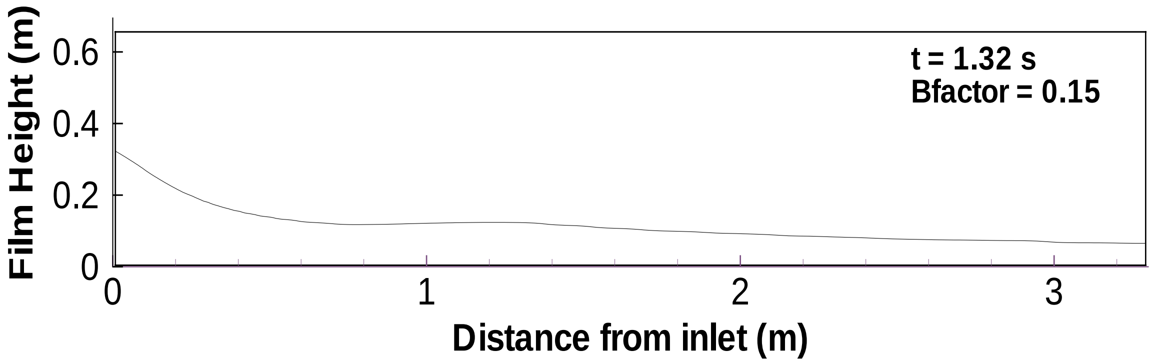


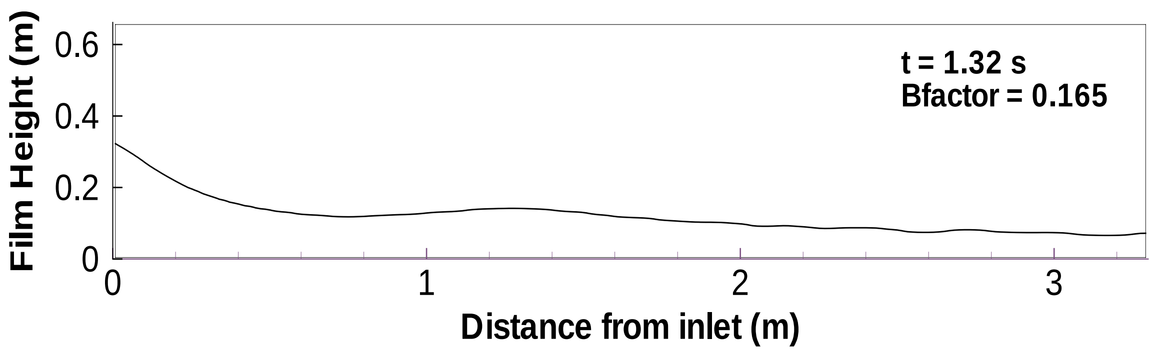



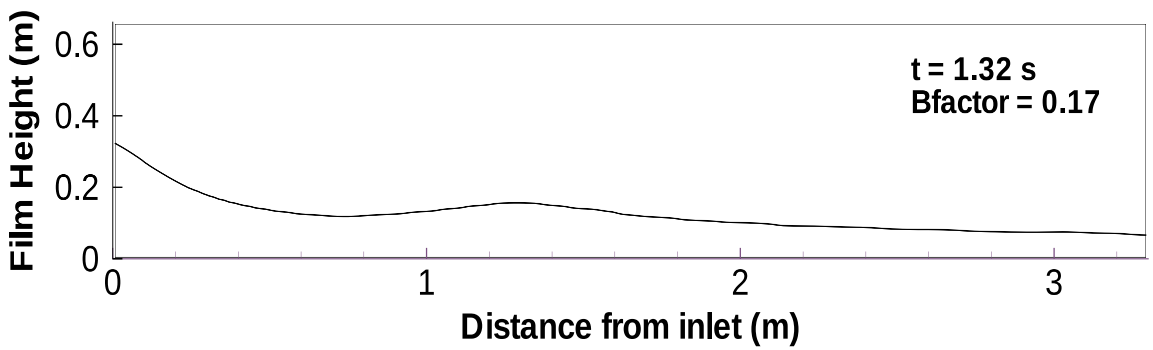

In computation: The toroidal magnetic field profile is shown on the left. The film flow is unstable, to study how the flow behavior varies with the field gradient, in each case it multiply a factor.

magnetic field = the-profile * Bfactor

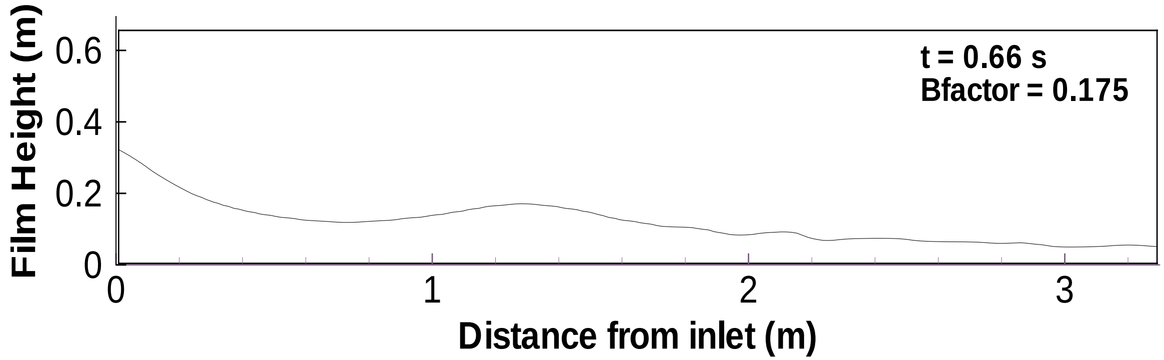
# Increase of Magnetic field gradient causes unstable flow (Non-conducting-wall, no applied currents)

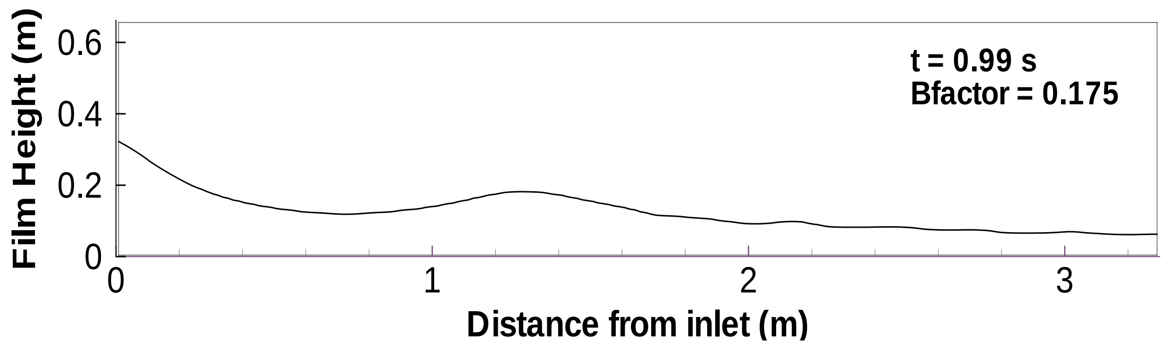


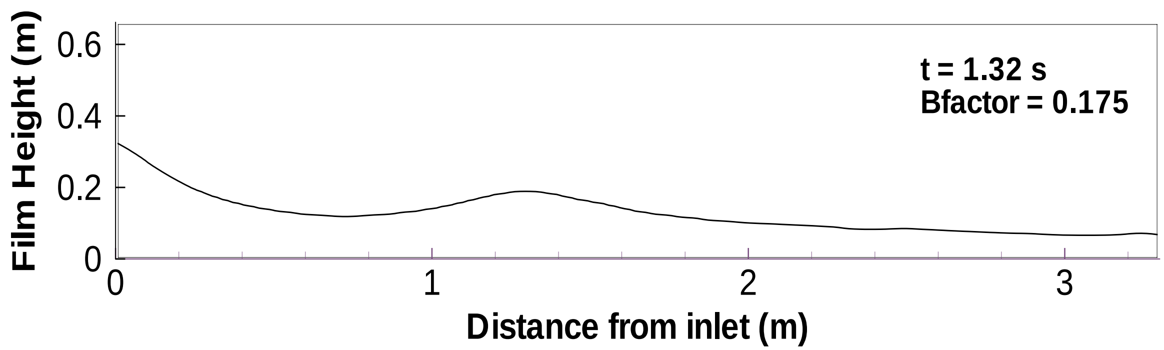



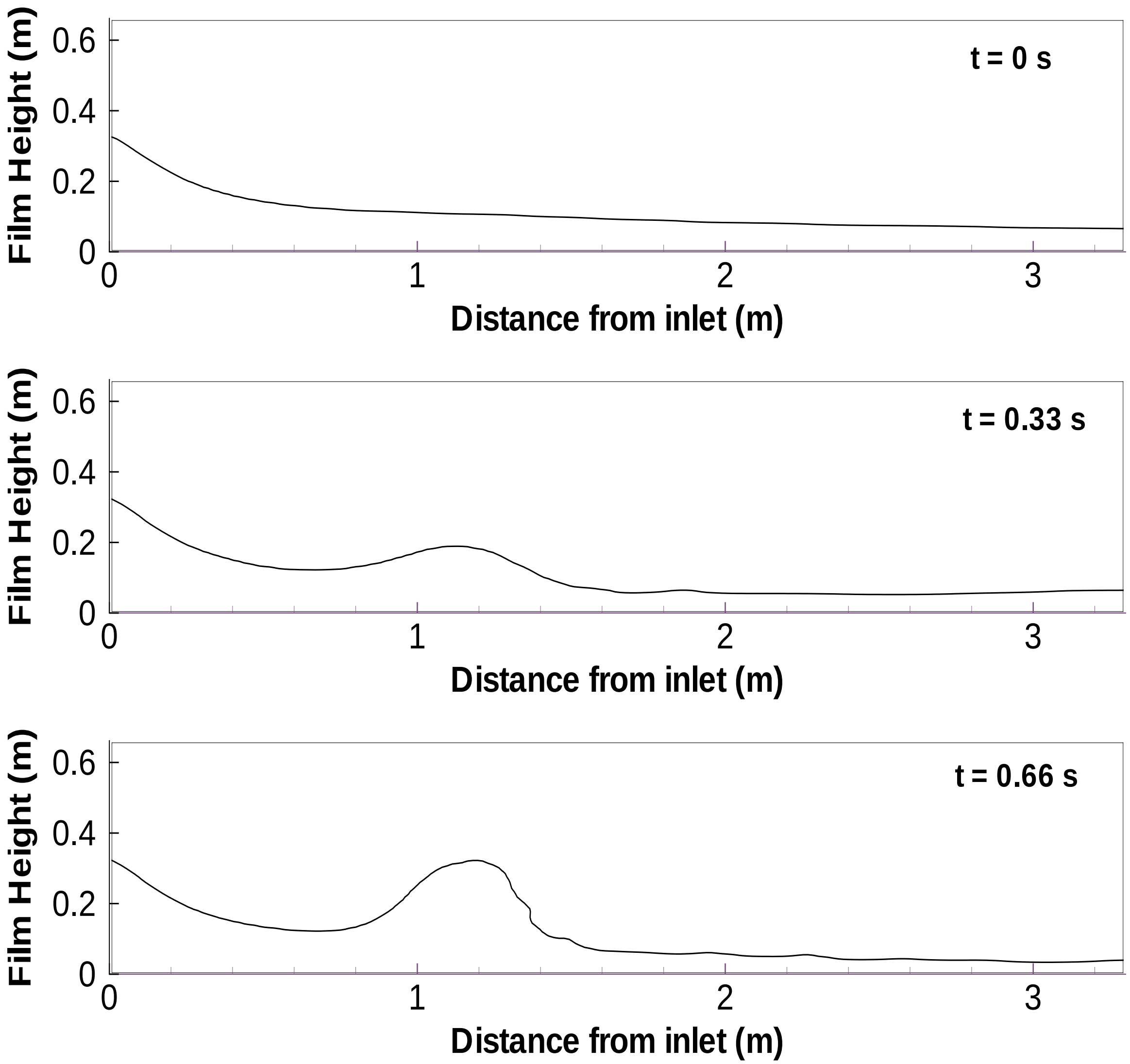
# Critical point of Non-conducting-wall, no applied currents
# Bfactor ≅ 0.175



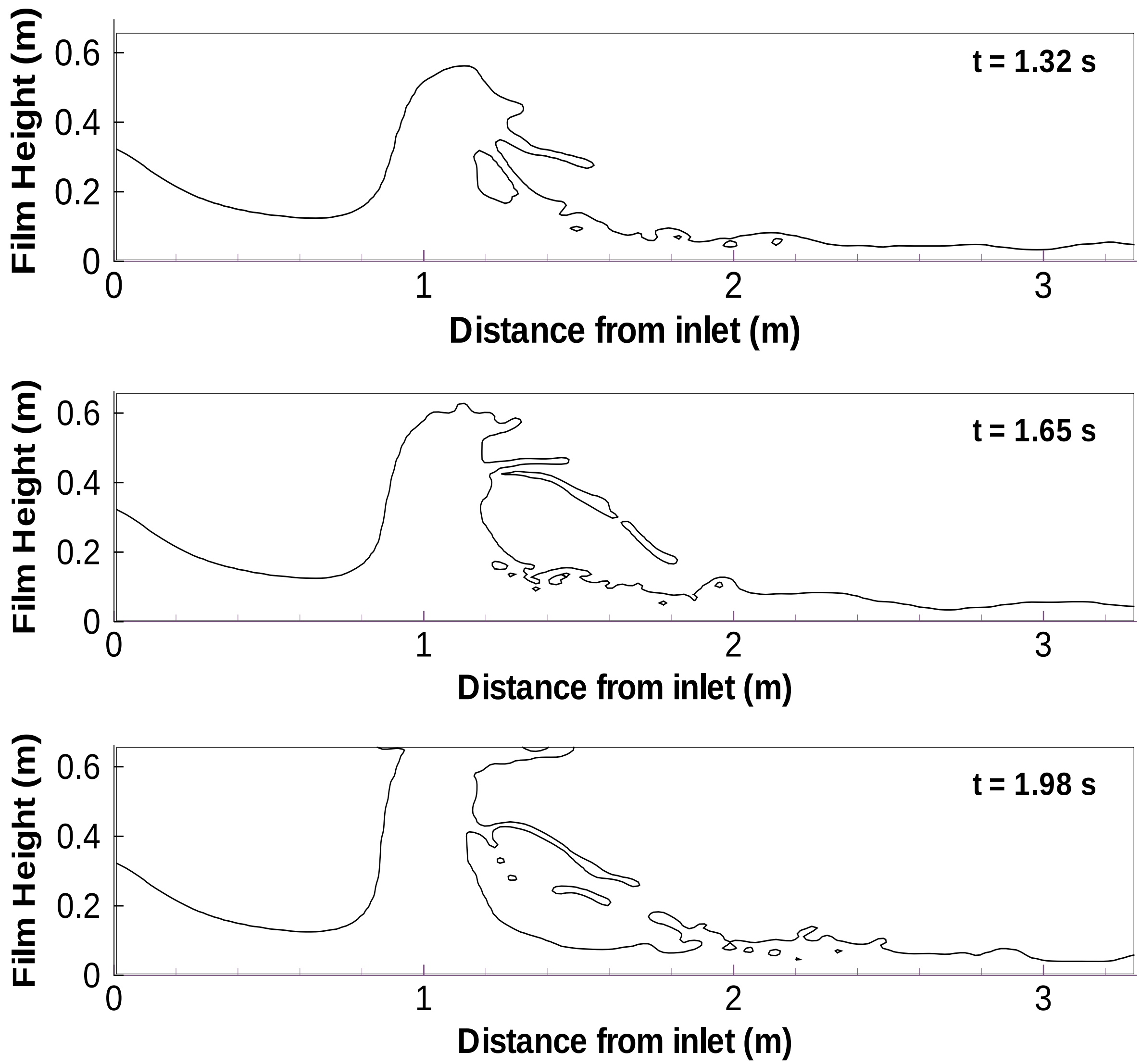



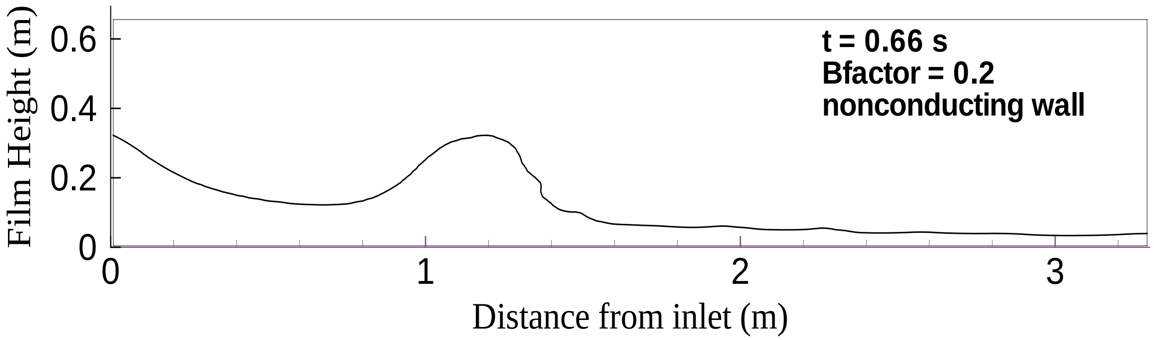
# How the unstable flow develops (Bfactor=0.2)
# (non-conducting wall, no applied current)

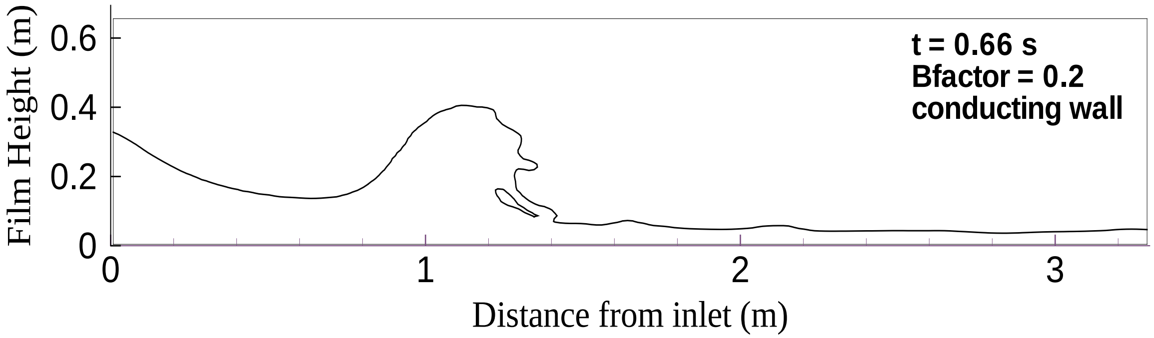


## How the unstable flow develops (Bfactor=0.2)—continued

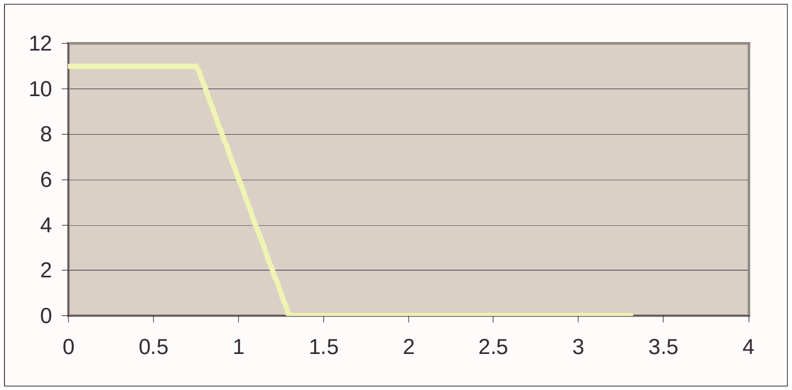


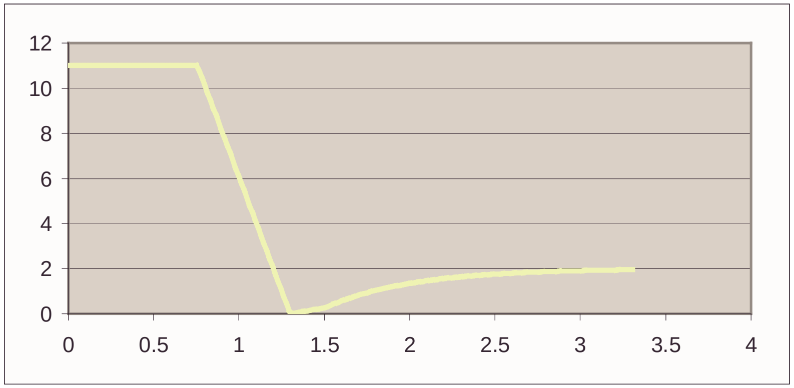
# Conducting wall effect under no-applied currents
# (make the flow more unstable)

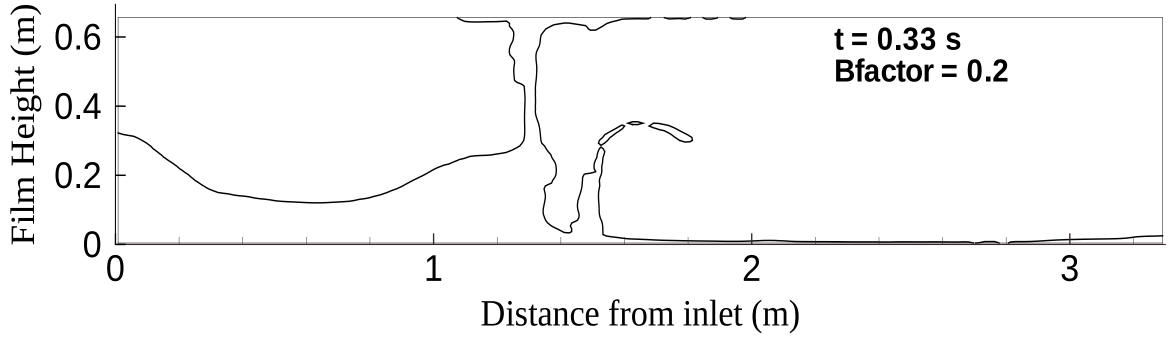


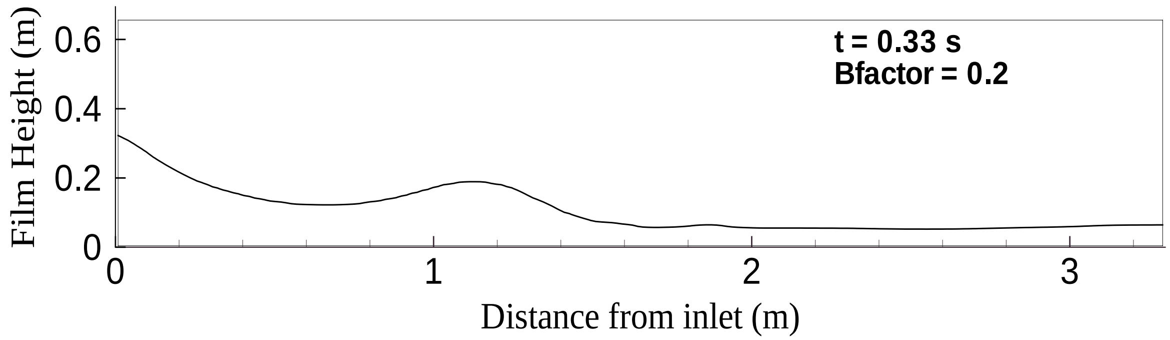
# A Little different of B-profile results in highly different flows (non-conducting wall and no-applied currents)



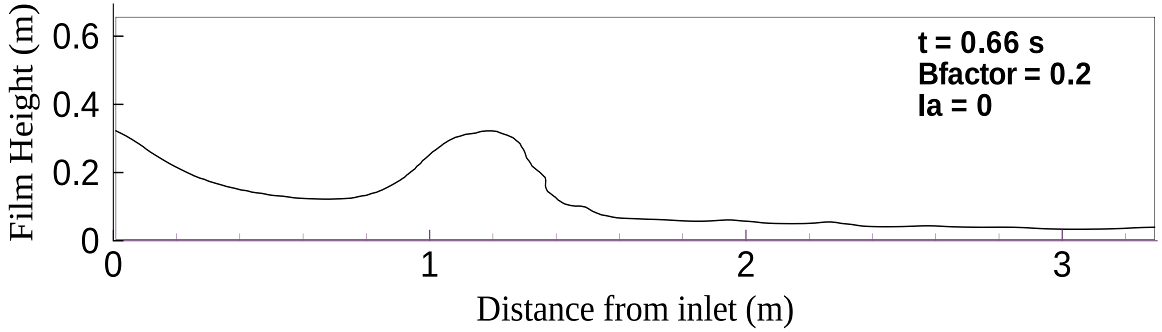



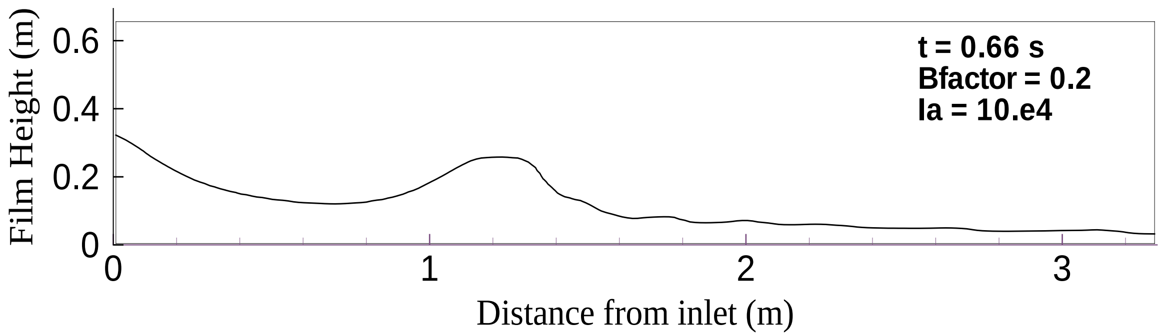

First B-profile

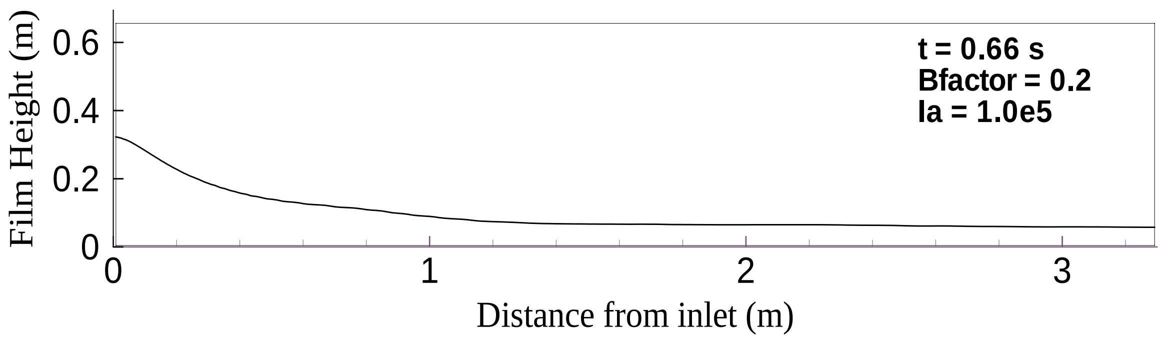



Second B-profile

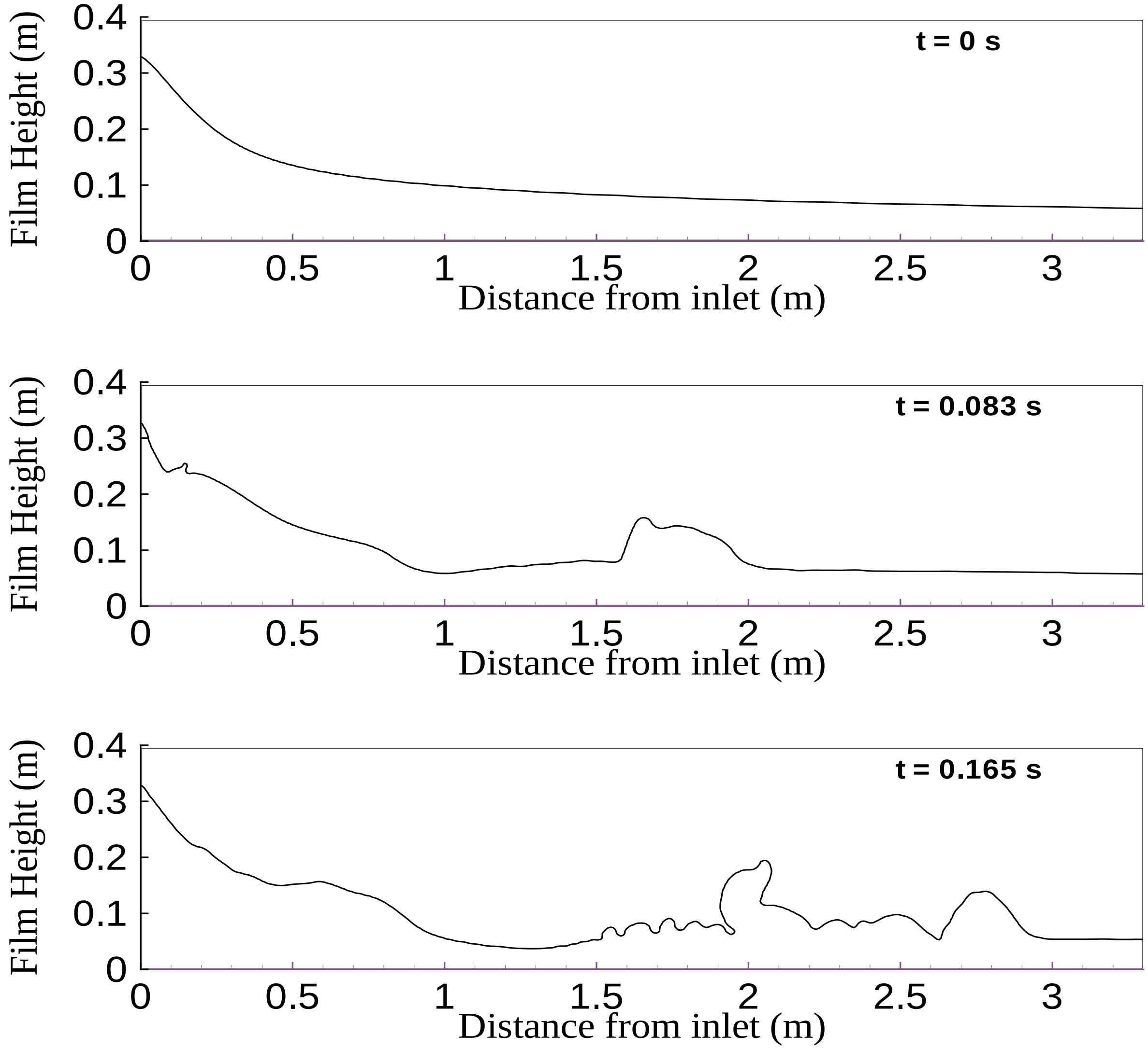
# Applied currents effect — propulsion (non-conducting wall)



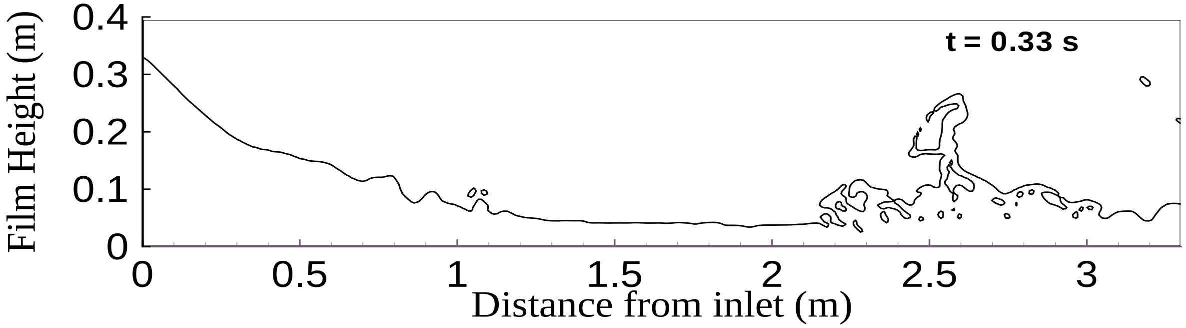



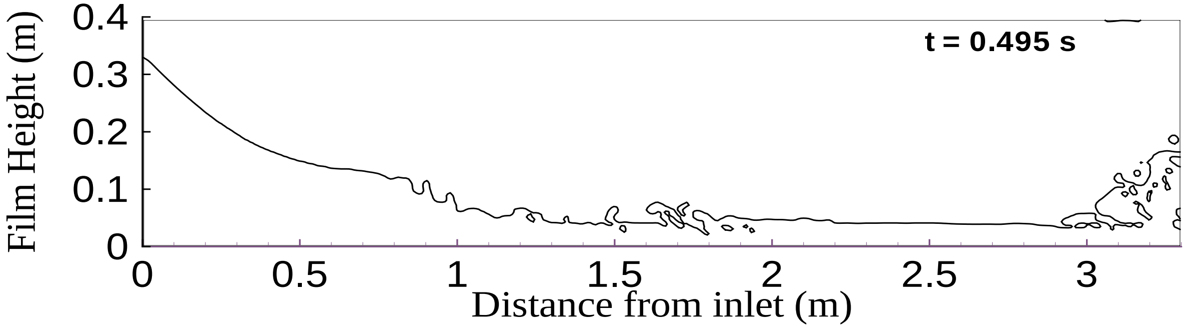
# Increase of field gradient: Bfactor=0.5
# Applied current Ia=2.5e5, non-conducting wall

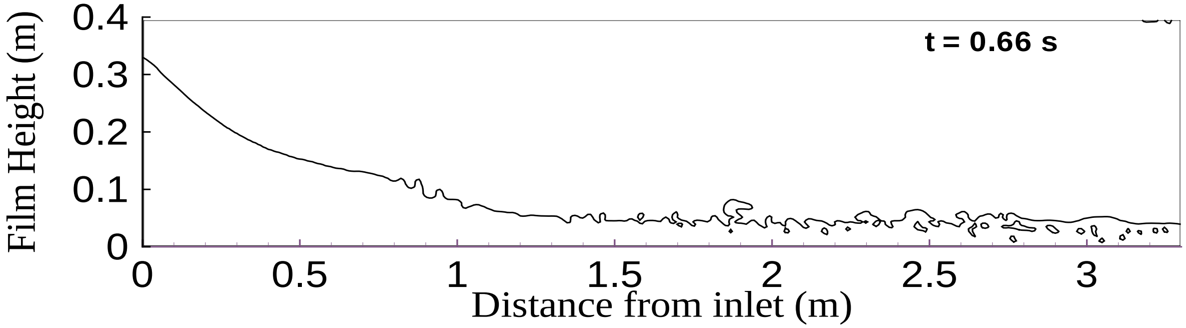


# Bfactor=0.5 (continued)

## Applied current Ia=2.5e5, non-conducting wall



# Bi contours at Bfactor=0.5 (continued)
# Applied current Ia=2.5e5, non-conducting wall