

Comparison of the core flow approximation and full solution approach for MHD flow in non-symmetric and multiple adjacent ducts

K.A. McCarthy, A.Y. Ying, N.B. Morley and M.A. Abdou

Mechanical, Aerospace and Nuclear Engineering Department, University of California, Los Angeles, CA 90024-1597, USA

The core flow approximation method has been used extensively in the past several years to model MHD flow at high Hartmann number and interaction parameter. This method assumes that inertial and viscous forces and the induced magnetic field are negligible, thus transforming the governing equations to a set of linear equations. The resulting equations are much easier to solve than the original set of equations, and as a result this method has been applied to several geometries. Benchmarking to experimental results has been done with excellent results for a limited number of geometries, but more investigation is necessary to determine the validity and applicability of the core flow approach. In this paper, results of the core flow analysis are compared with results from a two-dimensional full solution analysis (which included inertial and viscous terms) to further evaluate the core flow method. In particular, comparison is made for a rectangular duct geometry with an oblique magnetic field, a rectangular duct with a magnetic field parallel to one set of walls but without symmetry about the plane between the two side walls, and multiple adjacent rectangular ducts where the magnetic field is parallel to one set of walls. Special attention is paid to assumptions used in the core flow approximation dealing with the electric current flowing in the side layers. These comparisons are done for fully developed flow, and the results show that the core flow approximation can predict flow in these geometries with accuracy that is within design limits.

1. Introduction

While liquid metal blankets have many attractive features, magnetohydrodynamic (MHD) effects complicate their use. It is necessary to be able to model MHD flow in order to evaluate liquid metal blanket designs. The equations which describe MHD flow include Maxwell's equations and the Navier–Stokes equation, and solving these equations without any simplifying assumptions is a very difficult task, and has been done to a very limited extent for relatively high magnetic fields (see, e.g. refs. [1] and [2]). An alternative method which has been developed is the core flow method which is based on research first done by Kulikovskii [3]. This method assumes that inertial and viscous effects are negligible. The Navier–Stokes equation is then linear, and no longer has terms which may be many orders of magnitude larger than other terms. Additionally, certain characteristics of the equations can be taken advantage of which allow one to integrate out one dimension of the equations without losing the three-dimensional characteristics. The equations are then two-dimensional, and considerably easier to solve.

The core flow method has been applied to several geometries, including circular ducts, rectangular ducts,

and multiple rectangular ducts (see e.g. refs. [5–7]). The equations and details of the solution method can be found in the aforementioned papers. Benchmarking to experimental results has shown the core flow approximation to be highly accurate for the geometries tested (see e.g. refs. [8] and [9]).

There are many geometries, however, for which there are no experimental results with which to compare. Among these are rectangular ducts with an oblique magnetic field, rectangular ducts lacking certain planes of symmetry, and multiple duct geometries. These are geometries which may be commonly encountered in liquid metal blanket designs, and are therefore important to understand. The applicability of the core flow approximation to these geometries is examined by comparing results of 2-D analyses with results from a 2-D full solution code developed by Hashizume and co-workers [10]. This code has been shown to be highly accurate by comparing to analytic results [11].

2. Rectangular duct with an oblique magnetic field

In an effort to compare the core flow approximation to a full solution method for the case of a square duct

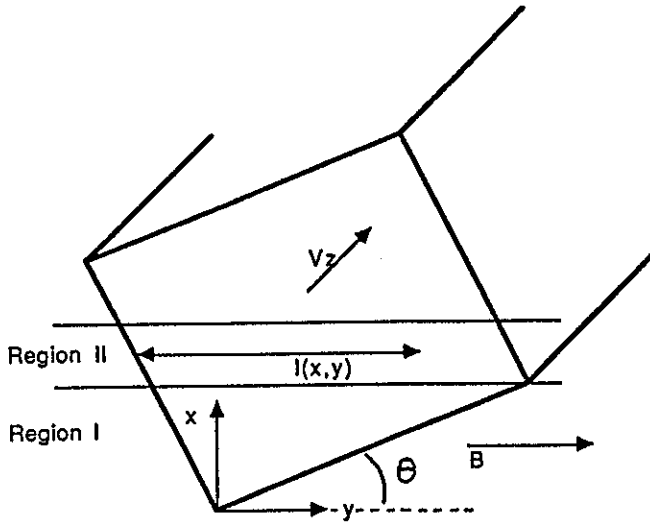


Fig. 1. Rectangular duct with oblique magnetic field geometry.

with an obliquely incident magnetic field, an analytic solution for fully developed flow yielding the current and potential was constructed from the basic core flow MHD equations and the thin wall boundary condition for the geometry shown in fig. 1. The velocity profile is then obtained via the component of Ohm's law perpendicular to the magnetic field. The governing equations are:

$$\phi(x, y) = \phi_L(x) - J_y(x)l(x, y), \tag{1}$$

$$v(x, y) = -\frac{\partial \phi}{\partial x} - J_{x0}. \tag{2}$$

Region I:

$$J_y(x) = \frac{A_1 I_1(2\sqrt{C_2 x})}{\sqrt{x}} - \frac{C_3}{C_2}, \tag{3}$$

$$\phi_L(x) = \frac{A_1 \sqrt{x} I_1(2\sqrt{C_2 x})}{\Phi C_2 \cos \theta} + \frac{C_4 x^2}{2} + A_2 x. \tag{4}$$

Region II:

$$J_y(x) = A_3 \cosh[C_1(x_m - x)] - J_{x0} \tan \theta, \tag{5}$$

$$\phi_L(x) = \frac{1}{2 \cos \theta} \left(A_3 \cosh[C_1(x_m - x)] + \frac{J_{x0} x}{\Phi \cos \theta} \right) + A_4, \tag{6}$$

where ϕ_L is the potential along the left wall, Φ is the wall conductance ratio, $l(x, y)$ is the distance parallel

to the field from the left wall to a point (x, y) , and J_{x0} is the constant current perpendicular to the field that is chosen to normalize the average velocity to 1. The constants C_{1-4} and A_{1-4} are functions of θ, Φ , and j_{x0} where the C 's are groupings that appear in the core flow differential equations, and the A 's are integration constants chosen to satisfy boundary conditions at the region boundary [12].

At zero angle of incidence of the magnetic field, the full solution of the MHD equations predicts the formation and shape of a side layer on the walls parallel to the field. As the angle is increased, the side layer turns into a shear layer than peaks along the region boundary. This shear phenomena is also predicted by the core flow approximation and at large enough angles the core flow solution provides reasonably accurate flow information. This angle above which the core solution is qualitatively very close to the full solution is a function of the Hartmann number (Ha) and the wall conductance ratio.

Tests were run in an effort to determine this angle at two different wall conductance ratios and a variety of fusion relevant Hartmann numbers. Table 1 gives the results where the approximate values of the angles were determined through visual comparison of the

Table 1
Approximate angles for different Φ and Ha

Φ	Ha = 100	Ha = 1000	Ha > 5000
0.05	angles very close to 45° only	at angles of ~10° the peak velocity predicted by the core solution is within 25% of the peak velocity predicted by the full solution; greater than 10° even better	same as Ha = 1000 but angle $\geq 8^\circ$
0.01	same as 0.05	angles of ~20° or greater give core solution peak velocities within 25% of full solution	angles > 10° give core solution peak velocity within 25% of full solution peak velocity

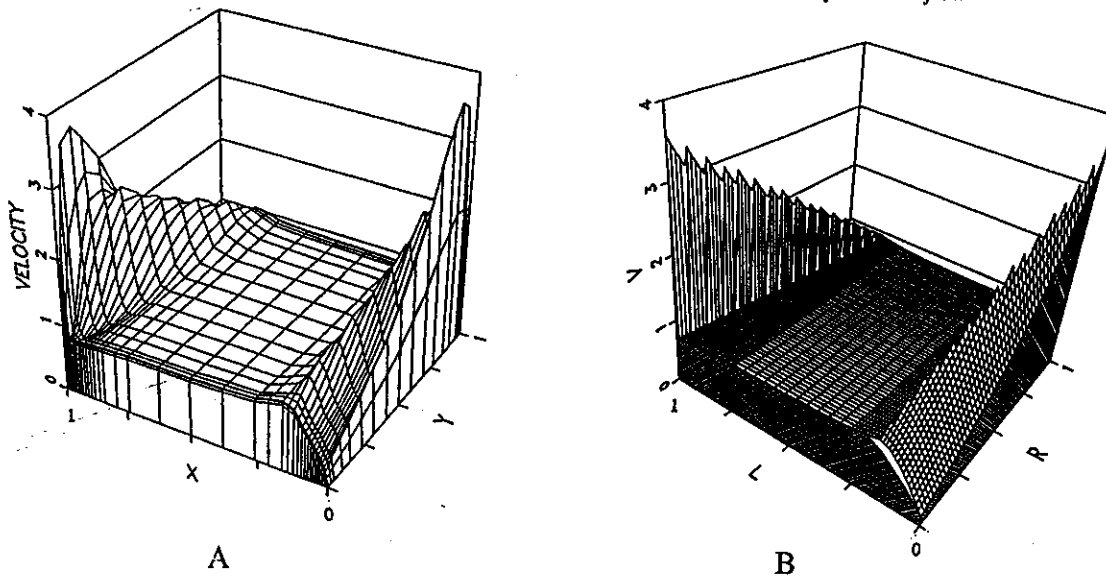


Fig. 2. (A) Full and (B) core solution velocity profiles: good agreement at $Ha = 5000$, $\Phi = 0.05$ and $\theta = 10^\circ$.

velocity profiles. Figure 2 shows a typical set of velocity plots used in the comparison.

3. Rectangular duct with field parallel to one set of walls

When the magnetic field is parallel to a wall at more than a point, side layers form. These side layers can carry a significant percentage of the core flow and returning core currents. If the resistance of the side walls is of the same order of magnitude as the resistance of the side layers, a significant fraction of the core current will return through the side layers. In this case it is necessary to incorporate this effect in the core flow solution. This can be done by assuming that a certain fraction of the core current enters the side wall (and the remainder enters the side layer), where this fraction depends on Ha and Φ . An expression for this fraction was calculated by Walker [13] and also by Tillack [14]. However, in both these analyses, symmetry about the $y = 0$ plane shown in fig. 3 is assumed. When this symmetry is present, half of the current entering the side layer will go towards corner a , and the other half will go towards corner b . However, if this symmetry is not present, this assumption is not strictly valid.

In order to determine what effect the current direction has on the core flow variables, the core solution with various assumptions of current directions is compared with the full solution. There are three different assumptions: (1) all current enters the side wall (this corresponds to $Ha = \infty$), (2) half the current enters the

duct wall from the side layer through corner a and half through corner b , and (3) the fraction of current from the side layer which enters corners a and b depends on their relative wall conductance ratios. In assumption 3, the fraction of current entering corner a will be $\Phi_2/(\Phi_4 + \Phi_2)$ and the fraction of current entering corner b will be $\Phi_4/(\Phi_2 + \Phi_4)$. This expression is based on the assumption that more current will tend to go toward the wall with the lower resistance. The wall with the lower resistance will be the wall with the larger wall conductance ratio. Table 2 compares the pressure gradients resulting from the different current direction assumptions for the case $\Phi_1 = \Phi_2 = \Phi_3 = 0.01$ and $\Phi_4 = 0.001$.

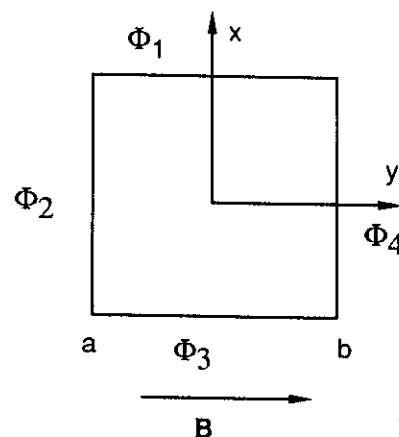


Fig. 3. Rectangular duct with magnetic field parallel to one set of walls.

Table 2
Comparison of pressure drops calculated using the full solution and the core solution

Ha	Δp for core solution assumption			full solution Δp
	1	2	3	
1000	-4.414×10^{-3}	-4.958×10^{-3}	-5.157×10^{-3}	-6.234×10^{-3}
5000	-4.414×10^{-3}	-4.822×10^{-3}	-4.966×10^{-3}	-5.452×10^{-3}
10000	-4.414×10^{-3}	-4.758×10^{-3}	-4.879×10^{-3}	-5.328×10^{-3}

It is evident from table 2 that assumption 1 is the least accurate as far as the pressure drop is concerned (the percent error in the pressure drop ranges from 17 to 29% with this assumption, depending on Ha). This is because assuming that all current returns through the side walls means that more fluid will be in the side layers, thus decreasing the predicted pressure drop. This is evident from fig. 4 which compares the velocity profiles predicted from MH2D with the core flow predictions based on the three assumptions. Assumption 3 is the most accurate, the pressure drop ranging from 8 to 17% depending on the Hartmann number for the wall conductions considered. The core flow approximation is expected to be more accurate at higher Ha because as Ha increases, viscous effects, which are neglected in the core flow solution, become less significant.

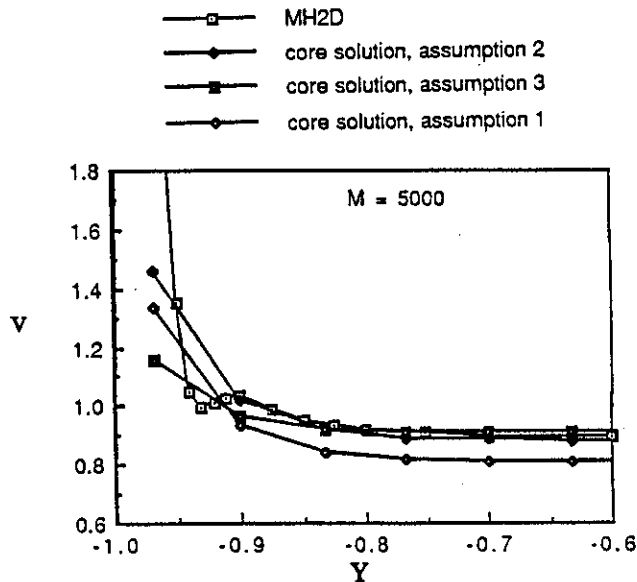


Fig. 4. Comparison of velocity profiles from MH2D and core flow approximation.

4. Multiple rectangular ducts

Another geometry in which the nonsymmetry described in the preceding section can exist is the multiple duct geometry shown in fig. 5. An analysis of a multiple duct geometry was done by McCarthy [15] in which it is assumed that all core current returns in the side walls. This is valid if the resistance of the side layers is much larger than the resistance of the side wall, however if this is not true, the current which returns in the side layer must be accounted for.

Comparisons of the full solution code and the core flow solution code with assumptions 1–3 were made to determine the accuracy of the assumptions. Each duct was square, $Ha = 5000$, and $\Phi_1 = 0.001$, $\Phi_2 = 0.01$, $\Phi_3 = 0.1$, $\Phi_4 = 1.0$, and $\Phi_{sw} = 0.01$. Table 3 shows the pressure gradient for the full and core flow solution codes. Again, assumption 3 is the most accurate with a percent error of 12%, while assumption 1 resulted in a

Table 3

Comparison of the pressure drop from the full solution and the core solution for a multiple duct geometry

Δp for core solution assumption			full solution Δp
1	2	3	
-0.01395	-0.01903	-0.02121	-0.02398

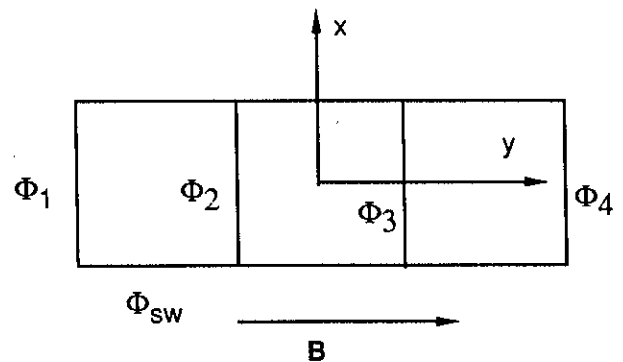


Fig. 5. Multiple duct geometry.

percent error of 12%, while assumption 1 resulted in a percent error of 42%. As in the preceding example, assumption 1 is inaccurate because it underpredicts the core velocity, thus underpredicting the pressure drop.

5. Conclusion

The core flow solution method is a powerful tool for analyzing MHD flow because of the small amount of CPU time needed as compared to the full solution method. While this method has been shown to be highly accurate in certain geometries and parameter ranges, the accuracy of the method in many situations still needs to be investigated. The results of this paper show that the accuracy of the core solution method in rectangular ducts with an obliquely incident magnetic field depends on Ha , Φ , and the angle of incidence. For example, the core solution method is most accurate in rectangular ducts with obliquely incident magnetic fields when the angle of incidence is greater than or equal to 10° when $\Phi = 0.05$ and $Ha = 1000$. This angle is larger for smaller Ha or smaller Φ .

For rectangular duct geometries with a magnetic field parallel to one set of walls, it was shown to be particularly important in many cases to include the effect of the core current which returns through the side layers. This has an effect on the velocity profile and pressure drop. The most accurate assumption on the direction of current return in the side layer was shown to depend on the ratio of the wall conductance ratios of the walls perpendicular to the magnetic field.

The comparison presented here is for fully developed flow. It is also necessary to compare developing flows. Additionally, in order to get a more accurate assessment of the assumptions presented here, it is necessary to see what kind of effect the somewhat different velocity profiles have on heat transfer. This is an area of application of the core flow solution method which still needs much investigation.

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ANALYSIS OF THIN FILM LIQUID METAL PROTECTION OF FUSION REACTOR LIMITER/DIVERTOR SURFACES

Neil B. Morley, Mark S. Tillack, and Mohamed A. Abdou
 University of California
 Mechanical, Aerospace, and Nuclear Engineering Department
 Los Angeles, CA 90024
 (213) 206-1230

ABSTRACT

In an effort to prolong the lifetime of impurity control components, the idea of protecting the contact surface from erosion and radiation damage with a thin film of liquid metal has been advanced. This flowing, liquid metal film could also be used to remove the high heat fluxes incident on limiter or divertor surfaces, thus eliminating problems with thermal stresses in the components as well. In order to determine the attractiveness and feasibility of such a concept, the heat transfer characteristics of a thin film of liquid metal are examined when the film is exposed to a large, one-sided heat flux incident on the free surface. The method developed yields the temperature at any location in the film and is used to determine, for a given design and space-dependant heat flux, the film velocity required to keep the maximum film temperature below whatever T_{max} limit is imposed.

In addition, the behavior of the film flow at the required velocity is examined in order to determine if such a flow is possible. This analysis is accomplished by using a one-dimensional model of the film height, developed from the basic set of MHD equations, to show the design conditions that allow for a stable film. The analytical method is applied to ITER-type limiter and divertor configurations, resulting in required film velocities ($v < 5$ m/s for the cases examined) and allowable values of the design parameters (channel size, wall conductivity, and substrate angle) that yield a stable film, capable of removing all incident heat.

I. INTRODUCTION

The concept of using a thin film of liquid metal (LM) to protect limiters and divertors was originally advanced in an effort to eliminate the erosion and radiation damage problems associated with the use of solid materials as plasma contact surfaces. In these devices, flowing liquid metal films form a continually renewed surface that suffers no ill effects due to sputtering erosion caused by incident energetic particles. Preliminary designs of such devices were presented in the INTOR study¹, and since then several variations have been proposed, some of which are tailored for use in ITER.^{2,3}

Thin film LM protection schemes are generally divided into the two main categories of slow and fast films; where fast films have a flow rate sufficient to remove all incident energy, while slow films serve only to protect the contact surface from the energetic particle flux exiting the plasma. Fig. 1 shows examples of a fast film limiter and divertor.

In the context of these classifications, fast films then have the additional advantage that the need for a separate coolant is eliminated and thus thermal stresses in the limiter/divertor

structure are significantly reduced. Due to the higher velocities required, however, MHD effects and substrate corrosion by the LM may limit their effectiveness in a fusion reactor situation.

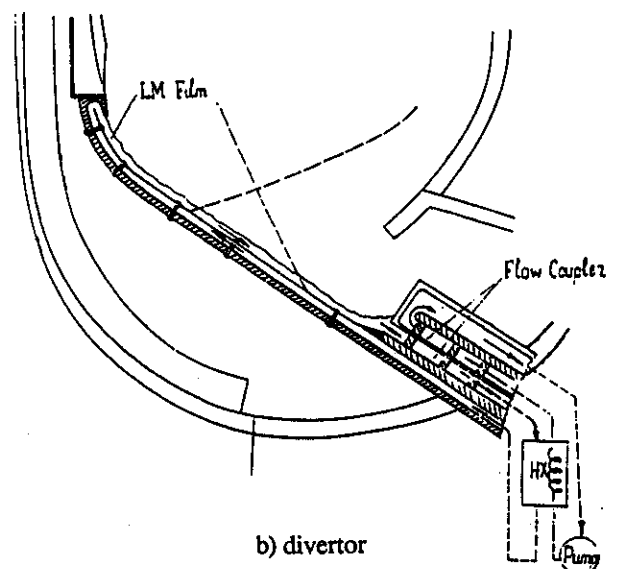
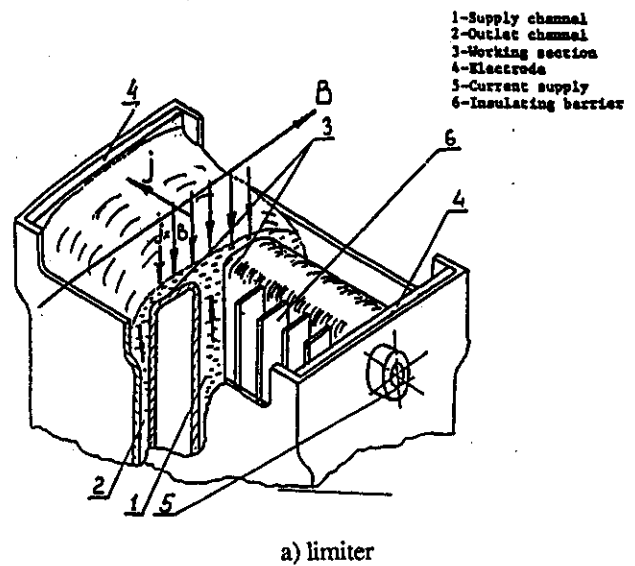


Fig. 1. Examples of a Fast Film Limiter and Divertor²