September 6, 2000
RWM

Droplet enhancement of surface heat transfer

The idea is to inject droplets at a small angle to the free surface as shown in the figure. If the droplets are small enough and low enough speed perpendicular to the surface no splash is predicted to occur. The surface tension energy and the kinetic energy go into a convective vortex, which propagates into the interior and dissipates. In the process, cool interior liquid is brought to and near the surface and hot surface liquid is convected into the interior. While this process is completely laminar and predictable rather than turbulent, it nevertheless has the effect of enhancing surface heat transfer.

We want to make an estimate of the required spacing of droplets and the effective heat transfer enhancement, which is the objective of this note.

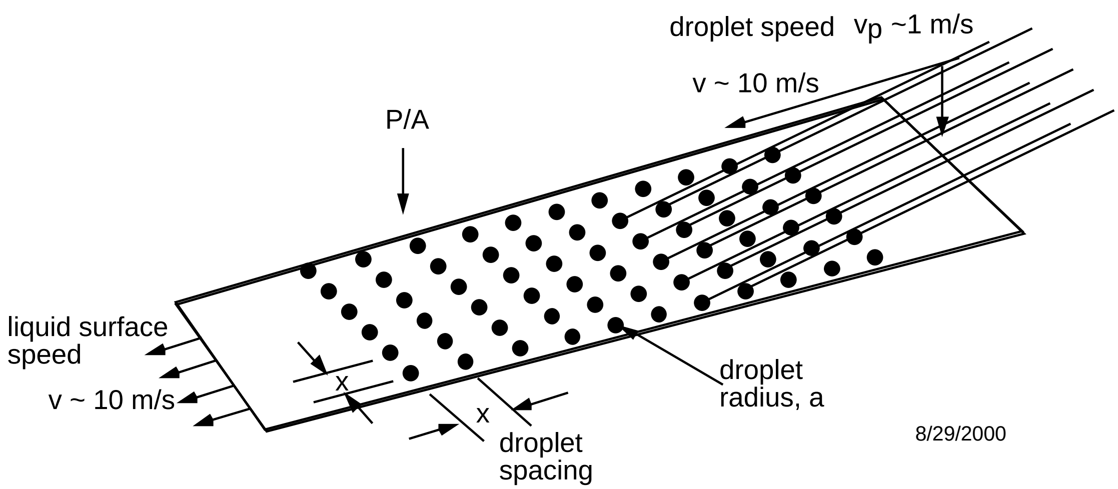

Fig. 1. Droplets are sprayed onto the free liquid surface at an equal spacing, x.

Once the droplet touches the surface, it merges with the aid of surface tension and forms a vortex which dissipates in a time τ

$$\tau = 0.36\frac{a^2}{\nu} = 0.18\frac{a}{v_\perp}R_e \qquad (1)$$

Where this decay time is based on a cylindrical vortex[1].

The Reynolds number is

$$R_e = \frac{2\ v_\perp a}{\nu} = \frac{2\rho v_\perp a}{\eta} \tag{2}$$

Where η, the viscosity, is 0.012 Pa•s at 550 °C, ν is the kinematic viscosity, 6.0 $\times 10^{-6}$ m^2/s, and ρ is 2000 kg/m^3 for flibe.

The heat transfer at the surface due to convection can be written as

$$\frac{P}{A} = nk_B D \frac{\partial T}{\partial x} \tag{3}$$

D is the diffusion coefficient. The usual conduction heat transfer equation is

$$\frac{P}{A} = k \frac{\partial T}{\partial x} \tag{4}$$

This equation can be modified to include convection.

$$\frac{P}{A} = k_{eff} \frac{\partial T}{\partial x} \tag{5}$$

Where k_B is Boltzman's constant, k is the thermal conductivity, 1.06 J/mK, and k_{eff} is the effective thermal conductivity including convective heat transfer near the surface.

$$k_{eff} = k + nk_B D = k(1+F) \tag{6}$$

$$nk_B = \frac{A}{M}\rho k_B = \frac{6.025 \cdot 10^{26}}{98.4} 2000 \cdot 1.3804 \cdot 10^{-23} = 1.01 \cdot 10^6 \frac{J}{m^3 K}$$

The radius of the vortex embedded in the liquid with significant convective motion is larger than a, the radius of the droplet before impact. The diffusion coefficient is estimated as a characteristic distance Na squared divided by the characteristic time from Eq. 1. To accounts for the space between droplet impacts, where little convection or diffusion is happening we multiply by $(Na/x)^2$.

$$D = \frac{(Na)^2}{\tau} \left(\frac{Na}{x}\right)^2 \tag{7}$$

We will use N=2 for our examples. Based on Damir Juric's work (Fig. 5 of draft paper) we stretch by hoping a Re=40 and radius of 0.12 mm will give no splash. The lifetime of the vortex is then 0.86 ms and the spacing is x=vt=10 m/s × 0.4 ms = 8.6 mm.

$$D = \frac{(2 \cdot 1.2 \cdot 10^{-4})^2}{0.86 \cdot 10^{-3}} (\frac{2 \cdot 1.2 \cdot 10^{-4}}{8.6 \cdot 10^{-3}})^2 = 5.2 \cdot 10^{-8}\, m^2 / s$$

$$n k_B D = 1.0 \cdot 10^6 \cdot 4 \cdot 10^{-6} = 0.053\, W / mK$$

$$k_{eff} = k + nDk_B = k(1 + F) = 1.06 + 0.053 = 1.06(1 + F)$$

$$F = 0.05$$

If the characteristic distance rather than being Na were the decay distance, Δx

$$\Delta x = v_\perp \tau = 1\, m / s \cdot 0.86\, ms = 0.86\, mm$$

$$D = \frac{(0.86 \cdot 10^{-3})^2}{0.86 \cdot 10^{-3}} (\frac{0.86 \cdot 10^{-3}}{8.6 \cdot 10^{-3}})^2 = 0.86 \cdot 10^{-5}\, m^2 / s$$

$$n k_B D = 1.0 \cdot 10^6 \cdot 0.86 \cdot 10^{-5} = 8.7\ W / mK$$

The vortex propagation speed used here was approximated by the droplet speed. It should be corrected. Sergey, would you improve this along the lines you told me?

$$k_{eff} = k + nk_B D = k(1 + F) = 1.06 + 8.6 = 1.06(1 + F)$$

$$F = 8.2$$

In the first example the thermal conduction is enhanced by only 5% whereas in the second example the enhancement factor of 1+F is 9 times. This example gives either no encouragement in the first case or lots of encouragement in the second case for enhanced heat transfer by droplet injection and sets the challenge forth of arranging for 0.12 mm radius droplets to be injected (rained down) at a spacing of 9 mm. Numerical simulation studies could be carried out to determine the "no splash" parameters range and then with heat flux on the droplets and free surface, the average surface temperature with convection could be calculated all from first principles. Such simulations should be virtually definitive because there are no unknowns, contrary to turbulent processes.

1. Theoretical hydromechanics, by N.E. Kochin, I.A. Kibel' and N.V. Roze. Translated from the 5th Russian edition by D. Boyanovitch. Edited by J.R.M. Radok. 1964.