DEVELOPMENT AND ADJUSTMENT OF "K-ε" TURBULENCE MODEL FOR MHD CHANNEL FLOWS WITH LARGE ASPECT RATIO IN A TRANSVERCE MAGNETIC FIELD

S.Smolentsev and M.Abdou
UCLA, MAE Dept., 44-114 Engineering IV, Los Angeles, CA 90095-1597, USA

T.Kunugi
Kyoto University Dept. of Nuclear Engineering, Yoshida, Sakyo, Kyoto, 606-8501, Japan

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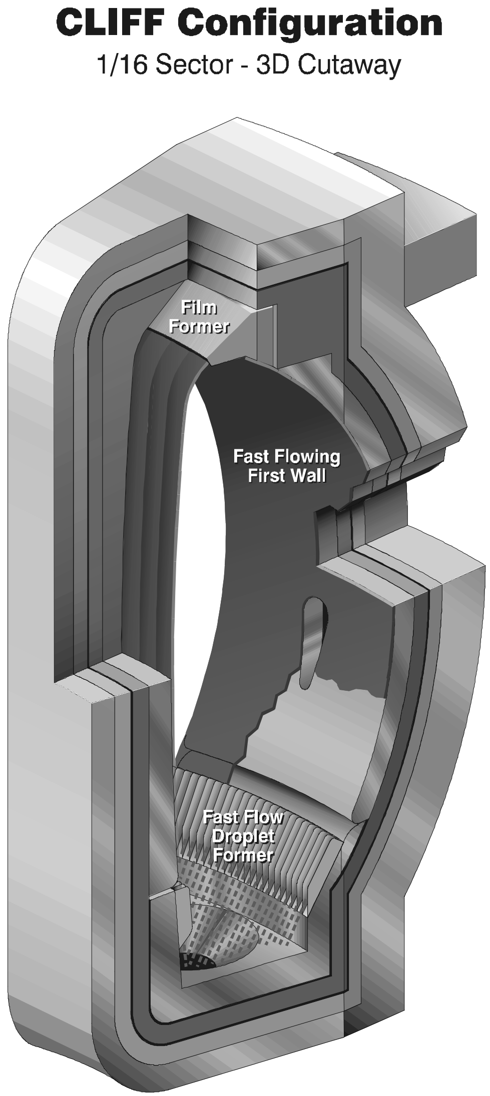


Free surface turbulent Flibe flow along the reactor first wall in the APEX design

Motivation and objectives of the study

The research is motivated by the ongoing **APEX** study (**A**dvanced **P**ower **EX**traction). In APEX, fast moving liquid layers flowing down the nuclear reactor first wall / blanket are used for the heat removal.

Apart from liquid metals, low-conductivity fluids such as molten salt Flibe ($(LiF)_n \bullet (BeF_2)$) are being considered. Unlike liquid metals, Flibe flows do not experience significant MHD forces but remain turbulent because Flibe electrical conductivity is about 30 times greater than that of seawater but 10^4 times less than that of liquid metals. Under a reactor strong magnetic field, turbulence pulsations in Flibe will be partially suppressed with an accompanying reduction in heat transfer. These effects are under consideration in the present study, where the **"K-ε" model of turbulence** is adjusted and then applied to the analysis of MHD turbulent flows.

Turbulence model - "K" and "ε" equations

We will start from applying Reynolds averaging to Navier-Stokes-Maxwell equations. Assuming low Re_m and using conventional closure approximations, one can derive the following equations for K and ε:

$$\frac{\partial K}{\partial t} + \left\langle v_j \right\rangle \frac{\partial K}{\partial x_j} = \underbrace{\nu_t \left(\frac{\partial v_i}{\partial x_j} \right)^2}_{\text{Production}} + \underbrace{\frac{\partial}{\partial x_j} [(\nu + \frac{\nu_t}{\sigma_K}) \frac{\partial K}{\partial x_j}]}_{\text{Diffusion}} - \underbrace{\varepsilon - \varepsilon_{em}}_{\text{Dissipation}};$$

$$\frac{\partial \varepsilon}{\partial t} + \left\langle v_j \right\rangle \frac{\partial \varepsilon}{\partial x_j} = C_1 \frac{\varepsilon}{K} \nu_t \left(\frac{\partial v_i}{\partial x_j} \right)^2 + \frac{\partial}{\partial x_j} [(\nu + \frac{\nu_t}{\sigma_\varepsilon}) \frac{\partial \varepsilon}{\partial x_j}] - C_2 \frac{\varepsilon}{K} \varepsilon - \frac{\varepsilon}{K} \varepsilon_{em}.$$

The first three terms on the RHS of the equations are standard, while the fourth one, with ε_{em}, stands for the Joule dissipation.

The expression for ε_{em} has been derived here in the most general form as:

Turbulence model - Joule dissipation term

$$\varepsilon_{em} = D_I + D_{II} =$$

$$= \frac{\sigma}{\rho}[2(B_{01}^2 + B_{02}^2 + B_{03}^2)K - B_{01}^2\left\langle v_1^{'2}\right\rangle - B_{02}^2\left\langle v_2^{'2}\right\rangle - B_{03}^2\left\langle v_3^{'2}\right\rangle -$$

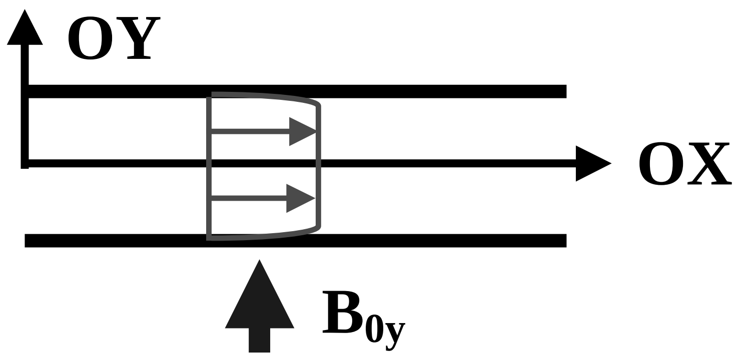
$$\underbrace{-2B_{01}B_{03}\left\langle v_1^{'}v_3^{'}\right\rangle - 2B_{01}B_{02}\left\langle v_1^{'}v_2^{'}\right\rangle - 2B_{02}B_{03}\left\langle v_2^{'}v_3^{'}\right\rangle -}_{D_I}$$

$$\underbrace{-B_{01}(\left\langle \frac{\partial\varphi^{'}}{\partial x_2} v_3^{'}\right\rangle - \left\langle \frac{\partial\varphi^{'}}{\partial x_3} v_2^{'}\right\rangle) - B_{02}(\left\langle \frac{\partial\varphi^{'}}{\partial x_3} v_1^{'}\right\rangle - \left\langle \frac{\partial\varphi^{'}}{\partial x1} v_3^{'}\right\rangle) - B_{03}(\left\langle \frac{\partial\varphi^{'}}{\partial x_1} v_2^{'}\right\rangle - \left\langle \frac{\partial\varphi^{'}}{\partial x_2} v_1^{'}\right\rangle)]}_{D_{II}} \; .$$

The expression derived includes terms with both velocity pulsations (D_I>0) and electric field fluctuations (D_{II}<0). In the general case, all 6 components of the Reynolds-stress tensor are present. We shall restrict our consideration to two particular cases of MHD channel flows with either a **wall-normal** or a **spanwise** magnetic field, which are important for fusion applications. In these cases only the diagonal components of the Reynolds-stress tensor enter the expression for ε_{em}.

Turbulence model - Joule dissipation, D_I

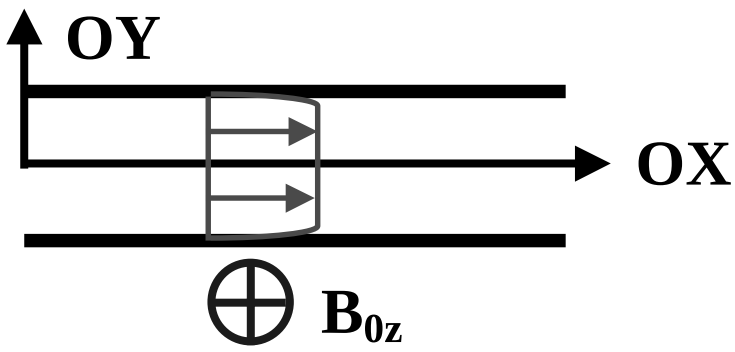
Wall-normal magnetic field:



$$D_I = \frac{\sigma}{\rho} B_{0y}^2 (\langle u'^2 \rangle + \langle w'^2 \rangle)$$

$$D_{II} = -\frac{\sigma}{\rho} B_{0y} (\left\langle \frac{\partial \varphi'}{\partial z} u' \right\rangle - \left\langle \frac{\partial \varphi'}{\partial x} w' \right\rangle)$$

Spanwise magnetic field:



$$D_I = \frac{\sigma}{\rho} B_{0z}^2 (\langle u'^2 \rangle + \langle v'^2 \rangle)$$

$$D_{II} = -\frac{\sigma}{\rho} B_{0z} (\left\langle \frac{\partial \varphi'}{\partial x} v' \right\rangle - \left\langle \frac{\partial \varphi'}{\partial y} u' \right\rangle)$$

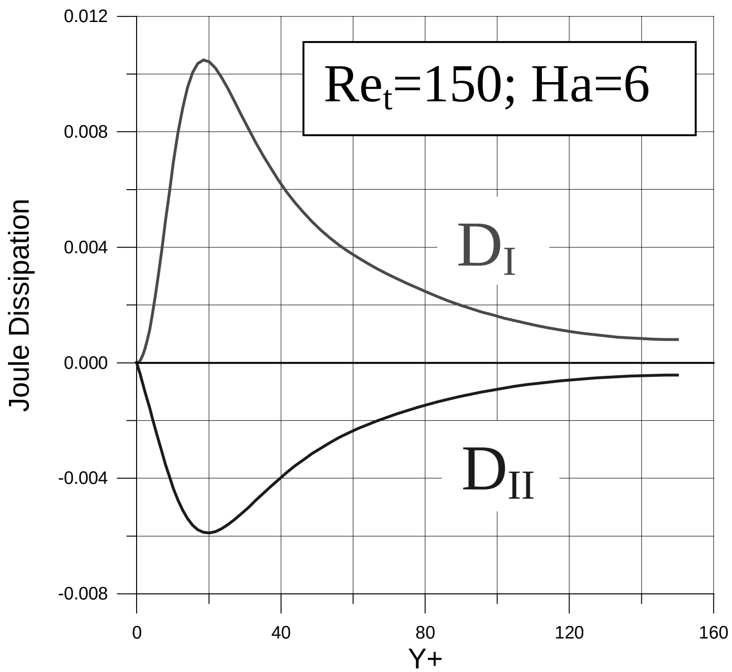
In both cases D_I contains $\langle u'^2 \rangle$, which gives the most contribution to the turbulence kinetic energy. It gives a ground to model D_I as

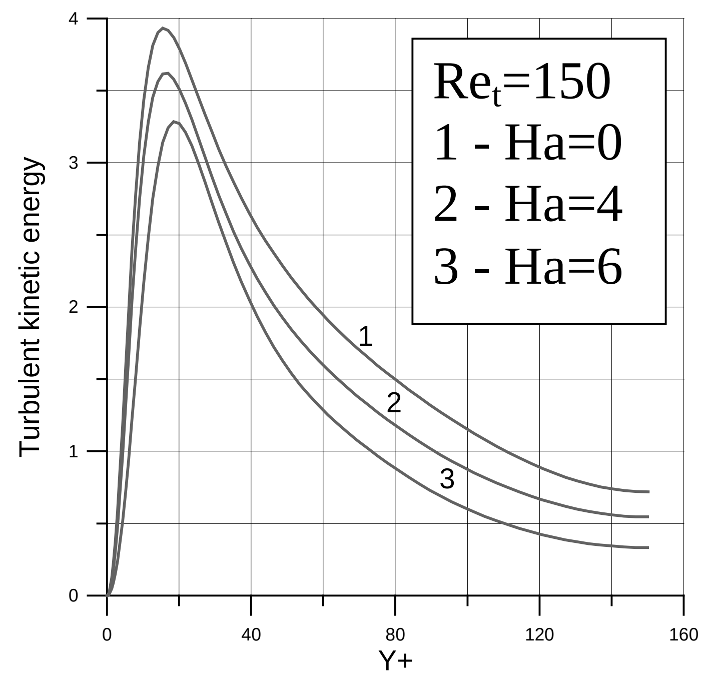
$$D_I = 2\frac{\sigma}{\rho} B_0^2 K$$

Turbulence model - Joule Dissipation, D_{II}

This Vg shows the distribution of D_{II} along with the distributions of D_I and K. The data are from the **Kasagi's DNS** (see : www.thtlab.t.u-tokyo.ac.jp/DNS/dns_database.html). Curves for D_I, D_{II} and K are similar in shape.

Wall-normal magnetic field





The distributions with the same features have been obtained by DNS by S.Satake (private communication) for a spanwise magnetic field (not shown here).

Modeling of the Joule dissipation term

In modeling the Joule dissipation term we took into account the following.

1. D_I and D_{II} distributions are similar in shape, so that $D_I = -\alpha D_{II}$ with α $(0<\alpha<1)$ depending on the flow parameters. See Kasagi's DNS database and DNS study by S.Satake.
2. The distribution of the resultant term, $D_I + D_{II}$, resembles in shape the distribution of the turbulent kinetic energy.
3. In channel flows with a weak transverse magnetic field, the turbulence structure resembles that in ordinary flows where streamwise vortices dominate. For such vortices $\varepsilon_{em} \approx D_I$. In the case of a stronger magnetic field, transition to a 2-D or a quasi 2-D state occurs, in which turbulent eddies are elongated in the field direction, so that $D_{II} \to -D_I$ and $\varepsilon_{em}=0$ in the limit.

All these give a ground to model the sink term in the K-equation and the destruction term in the ε-equation in the following form

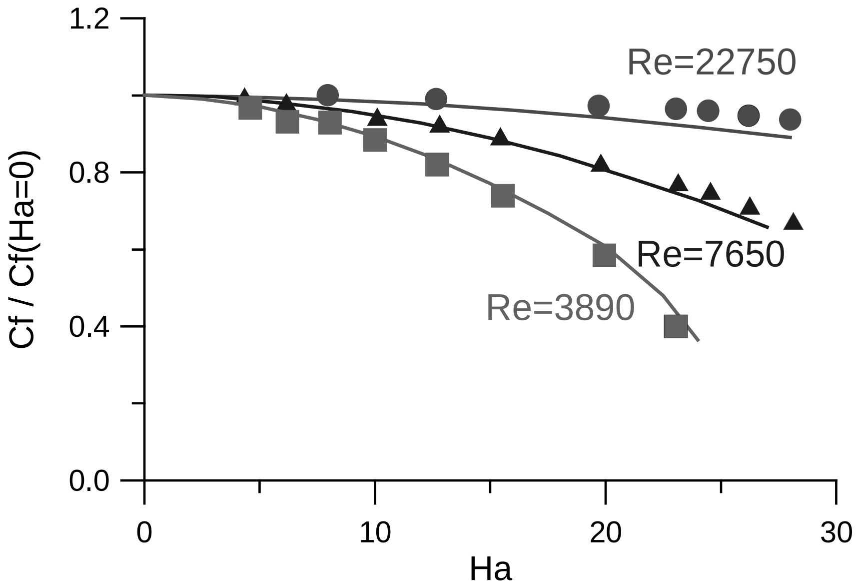
$$C_3 \frac{\sigma}{\rho} B_0^2 K, \ C_4 \frac{\sigma}{\rho} B_0^2 \varepsilon$$

with C_3 and C_4 decreasing as the interaction parameter, N, grows.

Modeling of the Joule dissipation term (cont.)

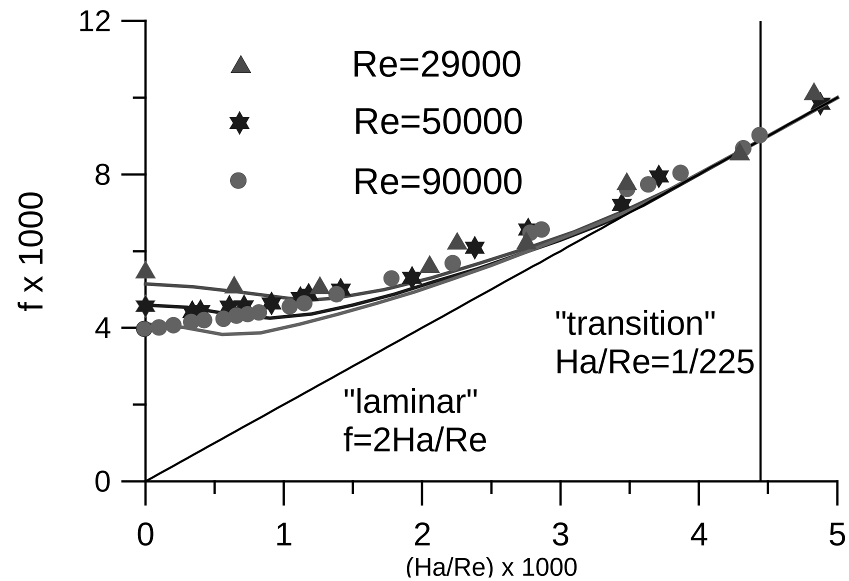
By analogy with H.-C.Ji and R.A.Gardner, "*Numerical analysis of turbulent pipe flow in a transverse magnetic field*" we introduced an exponential approximations for C_3 and C_4. By the computer optimization we have found

$$C_3 = 1.9\exp(-1.0N);\ C_4 = 1.9\exp(-2.0N)$$



<u>Spanwise magnetic field</u>

G.G.Branover, A.S.Vasil'ev, Yu.M.Gel'fgat, and E.V.Shcherbinin, "Turbulent flow in a plane perpendicular to a magnetic field," Magnetohydrodynamics **4**, 78 (1966).

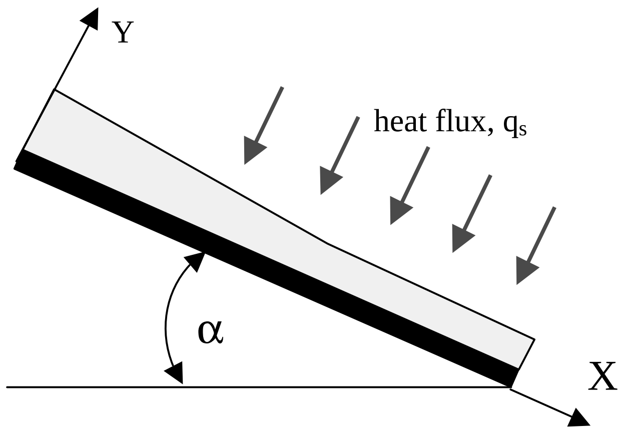


<u>Wall-normal magnetic field</u>

E.C.Brouillette and P.S.Lykoudis, "*Magneto-fluid mechanic channel flow. I. Experiment*," Phys. of Fluids **10**, 995 (1967)

Application to free surface flows - governing equations

Flow geometry - inclined chute with a large aspect ratio (2b>>h) with either a wall-normal (Case 1) or a spanwise magnetic field (Case 2)



$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + f_x + \frac{\partial}{\partial x}[(\nu+\nu_t)\frac{\partial U}{\partial x}] + \frac{\partial}{\partial y}[(\nu+\nu_t)\frac{\partial U}{\partial y}] + f_{em};$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + f_y + \frac{\partial}{\partial x}[(\nu+\nu_t)\frac{\partial V}{\partial x}] + \frac{\partial}{\partial y}[(\nu+\nu_t)\frac{\partial V}{\partial y}];$$

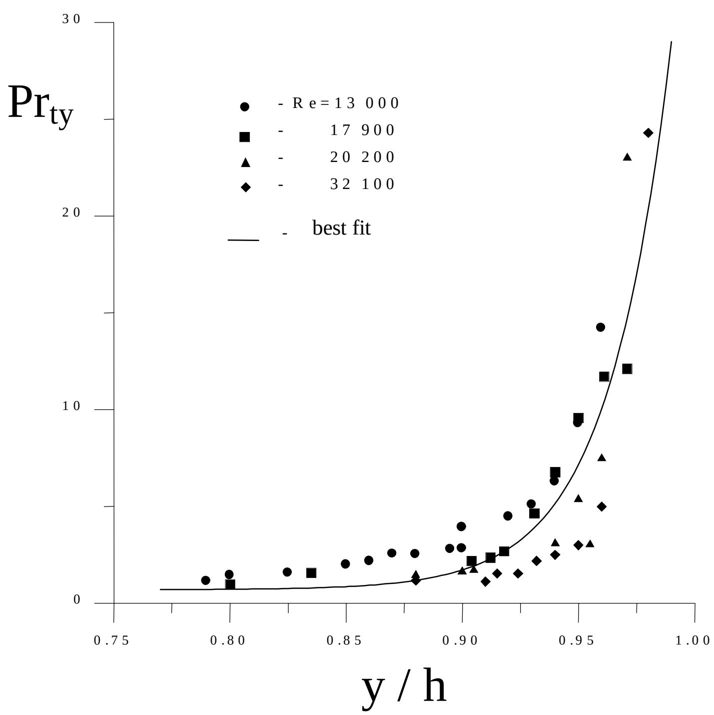
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0;$$

$$\rho C_p(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}) = \frac{\partial}{\partial x}[k(1+\frac{\nu_t}{\nu}\frac{Pr}{Pr_{tx}})\frac{\partial T}{\partial x}] + \frac{\partial}{\partial y}[k(1+\frac{\nu_t}{\nu}\frac{Pr}{Pr_{ty}})\frac{\partial T}{\partial y}].$$

$$\text{Case 1:}\quad f_{em} = -\sigma\rho^{-1}B_0^2(U - h^{-1}\int_0^h U dy); \qquad \text{Case 2:}\quad f_{em} = 0$$

Application to free surface flows - turbulent Prandtl number

Turbulent Prandtl number



Turbulent Prandtl number near the surface in a free surface water flow

The Reynolds Analogy is not valid in the near-surface area, because the turbulent transport from the surface is damped. In terms of the turbulent Prandtl number it means that Pr_{ty} grows as the distance from the surface decreases. In the present study, Pr_{ty} was calculated by using the eddy diffusivity for momentum obtained with the present model, while the eddy diffusivity for heat was taken from the experiments for water flows (see Ueda et al.).

There are two shortcomings:

- the experimental data have been obtained for a flow regime with Fr<1 (subcritical);
- MHD effects have not been included

Further evaluation of Pr_t is a subject of future studies.

Application to free surface flows - boundary conditions

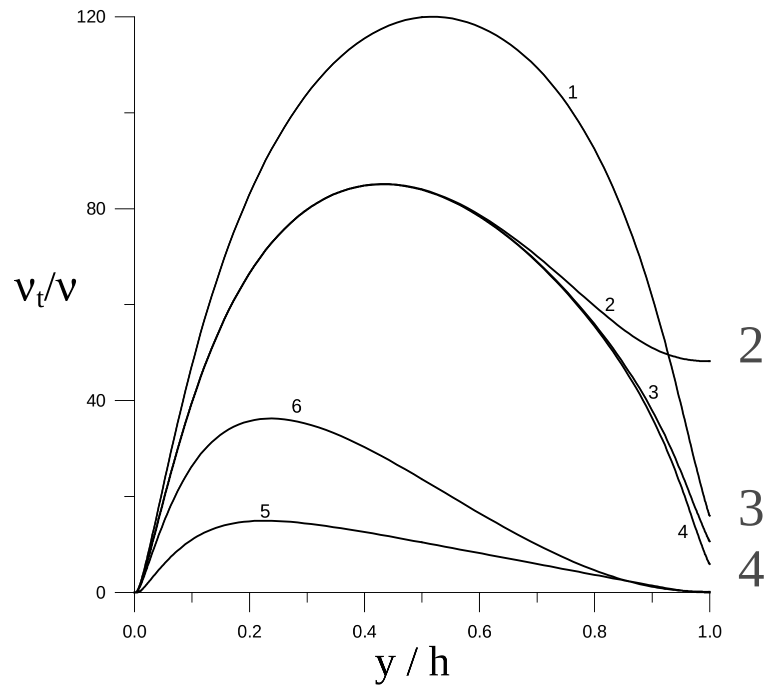
The Hossain and Rodi boundary conditions for "K" and "ε" are used for ordinary free surfaces. We modified these conditions by correcting the dissipation length scale at the surface:

$$\left(\frac{\partial K}{\partial y}\right)_s = 0; \quad \varepsilon_s = \frac{C_\nu^{3/4} K_s^{3/2}}{0.07 h \kappa} \frac{l_0}{l_1}.$$

l_0 is the dissipation length scale without a field with symmetry BCs.
l_1 is the same in the presence of a magnetic field.

The new free surface BCs give more physical results than the Symmetry BCs or the BCs by Hossain and Rodi.

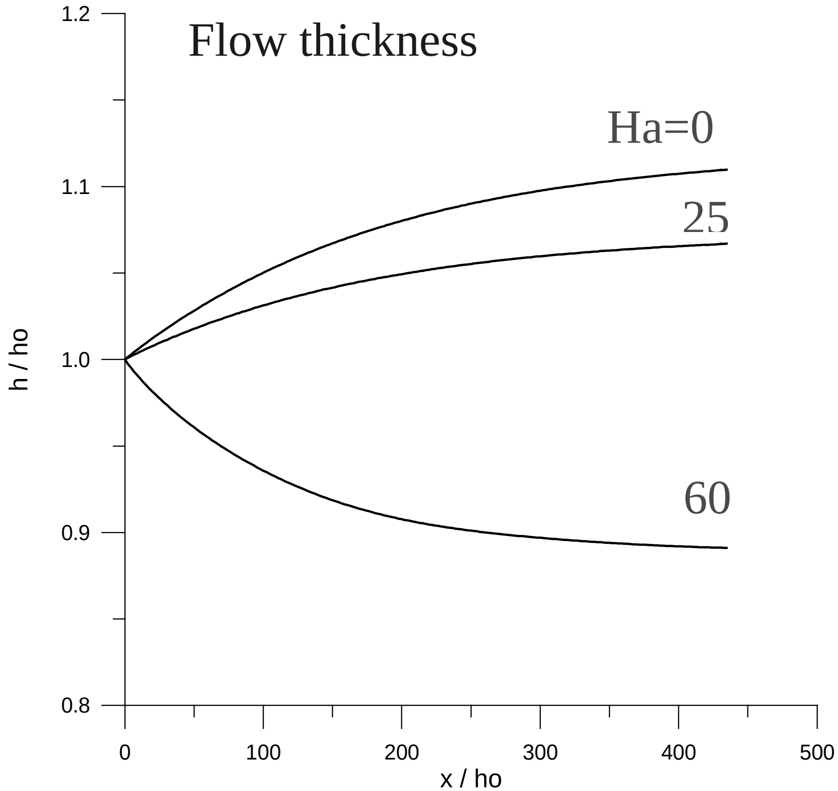
Eddy viscosity

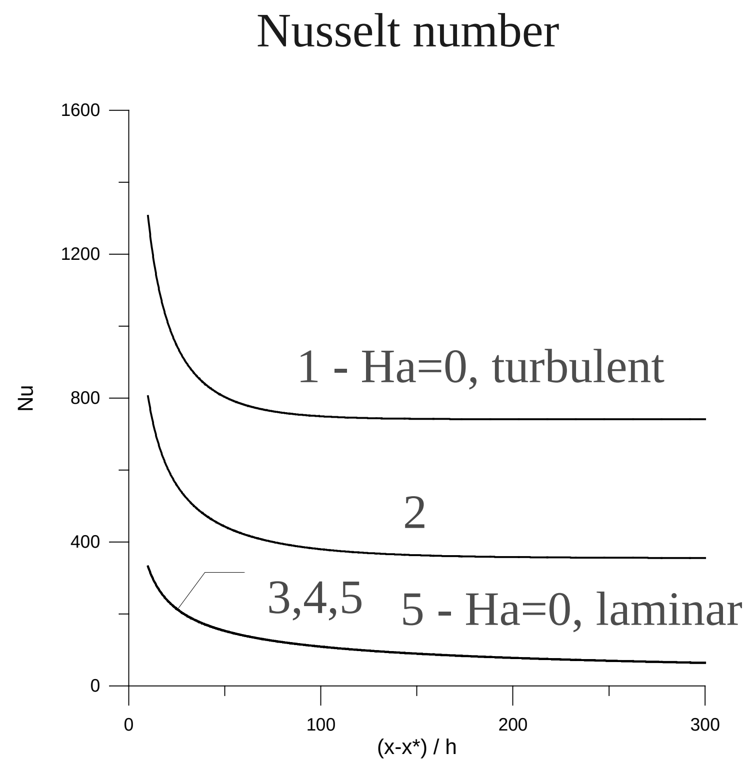


Re=30 000, Fr=0.8
2,3,4 - spanwise magnetic field, Ha=25

2 - symmetry BCs
3 - BCs by Hossain and Rodi
4 - modified Hossain and Rodi BCs

Application to free surface flows - results





Spanwise magnetic field: Re=30 250; Fr=44 350; α=45

Turbulence reduction by a magnetic field results in a smaller flow thickness

Turbulence reduction is accompanied by the heat transfer degradation.

2 - Ha=25, spanwise magnetic field
3 - Ha=60, spanwise magnetic field
4 - Ha=25, wall-normal magnetic field

Future studies

The results presented here reflect a preliminary stage of ongoing study. Although the closure coefficients C_3, C_4 approximated with the exponents gave a reasonable agreement with the experimental data, the model needs further improvements.

- First, the anisotropy in the turbulence structure associated with the Hartmann effect has not been introduced. As a result, the agreement with experimental data in Case 1 (wall-normal field) is slightly worse.
- Second, the model in its present form gives inaccurate predictions for the case of a streamwise magnetic flux. Also, its applicability to the multi-component magnetic field case must be verified.

All these shortcomings send us in search of better modeling for the Joule dissipation term. In our future studies we will pay attention to more accurate modeling of the part of the Joule dissipation term with the electric potential pulsations (D_{II}). Direct numerical simulations will accompany these studies.

Further evaluation of the turbulent Prandtl number with and without a magnetic field will be conducted based on the FLIHY[1] experimental data for free surface flows in both subcritical ($Fr<1$) and supercritical ($Fr>1$) regimes.

[1] FLIHY (FLIbe HYdrodynamics) experimental facilities are pending construction at UCLA