Practice Test 1 **Name** Solutions

NOTE: You must show enough of your work so that the grader can follow what you did. If it is possible to find an exact answer by taking an algebraic approach, you will not receive full credit for an approximation. Calculators are not permitted.

1. Find the domain of the following function using interval notation:

   a) $f(x) = \sqrt{4-3x}$

   Note that for a square root function, we need only set the inside of the root greater than or equal to zero and solve.

   $$4 - 3x \geq 0 \Rightarrow -3x \geq -4 \Rightarrow x \leq \frac{4}{3}$$

   So we find the domain is $\boxed{(-\infty, 4/3]}$

   b) $f(x) = \dfrac{9}{x-6}$

   Note that for a rational function, we need only find when the denominator is equal to zero and remove these points from the real numbers to find the domain.

   $$x - 6 = 0 \Rightarrow x = 6$$

   So we find the domain is $\boxed{(-\infty, 6) \cup (6, \infty)}$

   c) $f(x) = \dfrac{2x+1}{\sqrt{5-x}}$

   Note that for a square root function, we usually only set the inside of the root greater than or equal to zero and solve, however since the square root is in the denominator, we will also take out when it is equal to zero.

   $$5 - x > 0 \Rightarrow\Rightarrow x < 5$$

   So we find the domain is $\boxed{(-\infty, 5)}$

   d) $y = \dfrac{x^2 - 9x}{x^2 - 81}$

   Note that for a rational function, we need only find when the denominator is equal to zero and remove these points from the real numbers to find the domain.

   $$x^2 - 81 = 0 \Rightarrow (x-9)(x+9) = 0 \Rightarrow x = -9,\ 9$$

   So we find the domain is $\boxed{(-\infty, -9) \cup (-9, 9) \cup (9, \infty)}$

2. Find the average rate of change of $g(x) = 2x^2 - 9$ on the interval $[4, b]$ and simplify.

$$\text{Average rate of change} = \frac{g(b) - g(4)}{b - 4} = \frac{(2b^2 - 9) - (2 \cdot 4^2 - 9)}{b - 4}$$

$$= \frac{2b^2 - 9 - 23}{b - 4} = \frac{2b^2 - 32}{b - 4} = \frac{2(b^2 - 16)}{b - 4} = \frac{2(b-4)(b+4)}{b - 4} = \boxed{2(b+4)}$$

3. Find the average rate of change of $f(x) = \dfrac{1}{t+4}$ on the interval $[9, 9+h]$ and simplify.

$$\text{Average Rate of Change} = \frac{f(9+h) - f(9)}{9 + h - 9}$$

$$= \frac{1}{h}\left(\frac{1}{9+h+4} - \frac{1}{9+4}\right)$$

$$= \frac{1}{h}\left(\frac{1}{13+h} - \frac{1}{13}\right)$$

$$= \frac{1}{h}\left(\frac{13}{13(13+h)} - \frac{13+h}{13(13+h)}\right)$$

$$= \frac{1}{h}\left(\frac{13 - 13 - h}{13(13+h)}\right)$$

$$= \frac{1}{h}\left(\frac{-h}{13(13+h)}\right)$$

$$= \boxed{\frac{-1}{13(13+h)}}$$

4. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = 5 - x^2$ and simplify.

First we find $f(x+h)$,

$$f(x+h) = 5-(x+h)^2 = 5-(x^2+2xh+h^2) = 5-x^2-2xh-h^2$$

Now we substitute in to find,

$$\frac{f(x+h)-f(x)}{h} = \frac{(5-x^2-2xh-h^2)-(5-x^2)}{h} = \frac{5-x^2-2xh-h^2-5+x^2}{h}$$

$$= \frac{-2xh-h^2}{h} = \frac{h(-2x-h)}{h} = \boxed{-2x-h}$$

5. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = 2x^2 - 3x$ and simplify.

First we find $f(x+h)$,

$$f(x+h) = 2(x+h)^2 - 3(x+h) = 2(x^2+2xh+h^2) - 3x - 3h = 2x^2 + 4xh + 2h^2 - 3x - 3h$$

Now we substitute in to find,

$$\frac{f(x+h)-f(x)}{h} = \frac{(2x^2+4xh+2h^2-3x-3h)-(2x^2-3x)}{h} = \frac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$$

$$= \frac{4xh+2h^2-3h}{h} = \frac{h(4x+2h-3)}{h} = \boxed{4x+2h-3}$$

6. For the functions $f(x) = \dfrac{1}{x-4}$ and $g(x) = \dfrac{2}{x} + 4$, find:

a) $(f \circ g)(x)$

$$\begin{aligned}(f \circ g)(x) = f(g(x)) &= f\left(\frac{2}{x}+4\right) \\ &= \frac{1}{\frac{2}{x}+4-4} \\ &= \frac{1}{\frac{2}{x}} \\ &= \boxed{\frac{x}{2}}\end{aligned}$$

b) $g(f(x))$

$$\begin{aligned}g(f(x)) &= g\left(\frac{1}{x-4}\right) \\ &= \frac{2}{\frac{1}{x-4}} + 4 \\ &= 2(x-4)+4 = 2x-8+4 \\ &= \boxed{2x-4}\end{aligned}$$

7. For the functions $h(x) = \dfrac{4}{(x+2)^2}$ find $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

Remember there are many answers to this question, but one of the simplest is,

$$f(x) = \frac{4}{x},\ g(x) = (x+2)^2$$

8. For the functions $h(x) = \sqrt{\dfrac{2x-1}{3x+4}}$ find $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$.

Remember there are many answers to this question, but one of the simplest is,

$$f(x) = \sqrt{x},\ g(x) = \frac{2x-1}{3x+4}$$

9. Describe how the graph of each given function is a transformation of the graph of the original function $f(x)$.

a) $f(-x)$

The graph is reflected horizontally, or across the y-axis.

b) $f(x) + 5$

The graph is shifted and five units up.

c) $f(x + 3)$

The graph is shifted three units left.

d) $4f(x)$

The graph is stretched vertically.

e) $f(2x)$

The graph is compressed horizontally.

10. Find the inverse of $f(x) = \dfrac{x}{x-2}$.

Interchange x and y, and solve to see,

$$x = \frac{y}{y-2} \Rightarrow xy - 2x = y \Rightarrow xy - y = 2x \Rightarrow y(x-1) = 2x \Rightarrow y = \frac{2x}{x-1}$$

So we find that $\boxed{f^{-1}(x) = \dfrac{2x}{x-1}}$.

11. Find the inverse of $f(x) = \dfrac{2x+3}{5x+4}$.

Interchange x and y, and solve to see,

$$x = \frac{2y+3}{5y+4} \Rightarrow 5xy + 4x = 2y + 3 \Rightarrow 5xy - 2y = -4x + 3 \Rightarrow y(5x-2) = -4x + 3 \Rightarrow y = \frac{-4x+3}{5x-2}$$

So we find that $\boxed{f^{-1}(x) = \dfrac{-4x+3}{5x-2}}$.

12. Verify that $f(x) = \dfrac{x}{2+x}$ and $g(x) = \dfrac{2x}{1-x}$ are inverses of each other.

We verify by computing either $f(g(x))$ or $g(f(x))$. Note, it is sufficient to compute only one of these.

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{2x}{1-x}\right) \\
&= \frac{\frac{2x}{1-x}}{2+\frac{2x}{1-x}} \\
&= \frac{\frac{2x}{1-x}}{\frac{2-2x+2x}{1-x}} = \frac{\frac{2x}{1-x}}{\frac{2}{1-x}} \\
&= \frac{2x}{1-x} \cdot \frac{1-x}{2} = x
\end{aligned}
$$

OR

$$
\begin{aligned}
g(f(x)) &= g\left(\frac{x}{2+x}\right) \\
&= \frac{2 \cdot \frac{x}{2+x}}{1-\frac{x}{2+x}} \\
&= \frac{\frac{2x}{2+x}}{\frac{2+x-x}{2+x}} = \frac{\frac{2x}{2+x}}{\frac{2}{2+x}} \\
&= \frac{2x}{2+x}\frac{2+x}{2} = x
\end{aligned}
$$

13. Verify that $f(x) = \dfrac{-x-a}{1-x}$, $x \neq 1$ is its own inverse.

We verify by computing $f(f(x))$.

$$
\begin{aligned}
f(f(x)) &= f\left(\frac{-x-a}{1-x}\right) \\
&= \frac{-\frac{-x-a}{1-x} - a}{1-\frac{-x-a}{1-x}} \\
&= \frac{\frac{x+a-a+ax}{1-x}}{\frac{1-x+x+a}{1-x}} = \frac{\frac{x+ax}{1-x}}{\frac{1+a}{1-x}} \\
&= \frac{x(1+a)}{1-x}\frac{1-x}{1+a} = x
\end{aligned}
$$

14. Write the equation in slope-intercept form for a line parallel to $f(x) = -5x - 3$ and passing through the point $(2, -12)$.

Note first that the slope of the given line, and thus the line we are looking for, is -5.

Next we use the point-slope equation and the given point, (2,-12), to find,

$$y - y_1 = m(x - x_1)$$
$$y - (-12) = -5(x - 2)$$
$$y + 12 = -5x + 10$$
$$\boxed{y = -5x - 2}$$

15. Write the equation in slope-intercept form for a line perpendicular to $f(x) = -2x + 4$ and passing through the point $(-4, -1)$.

Note first that the slope of the given line is -2, and thus the slope of the line we are looking for is $\frac{1}{2}$.

Next we use the point-slope equation and the given point, (-4,-1), to find,

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{1}{2}(x - (-4))$$
$$y + 1 = \frac{1}{2}(x + 4)$$
$$y + 1 = \frac{1}{2}x + 2$$
$$\boxed{y = \tfrac{1}{2}x + 1}$$