Practice Test 1   Name_____

NOTE: You must show enough of your work so that the grader can follow what you did. If it is possible to find an exact answer by taking an algebraic approach, you will not receive full credit for an approximation. Calculators are not permitted.

1. For the following, determine points of discontinuity, stating the type of discontinuity (removable, jump, infinite, or none of these) and whether the function is left- or right-continuous.

   a)
$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} & \text{if } x \neq 2 \\ -1 & \text{if } x = 2 \end{cases}$$

   b)
$$f(x) = \frac{x+1}{x^2 - 1}$$

   c)
$$f(x) = \frac{1}{1 - e^x}$$

2. Find the value of the constant (a, b, or c) that makes the following functions continuous.

a)

$$f(x) = \begin{cases} x^2 - c & \text{if } x < 5 \\ 4x + 2c & \text{if } x \geq 5 \end{cases}$$

b)

$$f(x) = \begin{cases} 2x^3 - 2c & \text{if } x < 1 \\ 4x^2 + 2c & \text{if } x \geq 1 \end{cases}$$
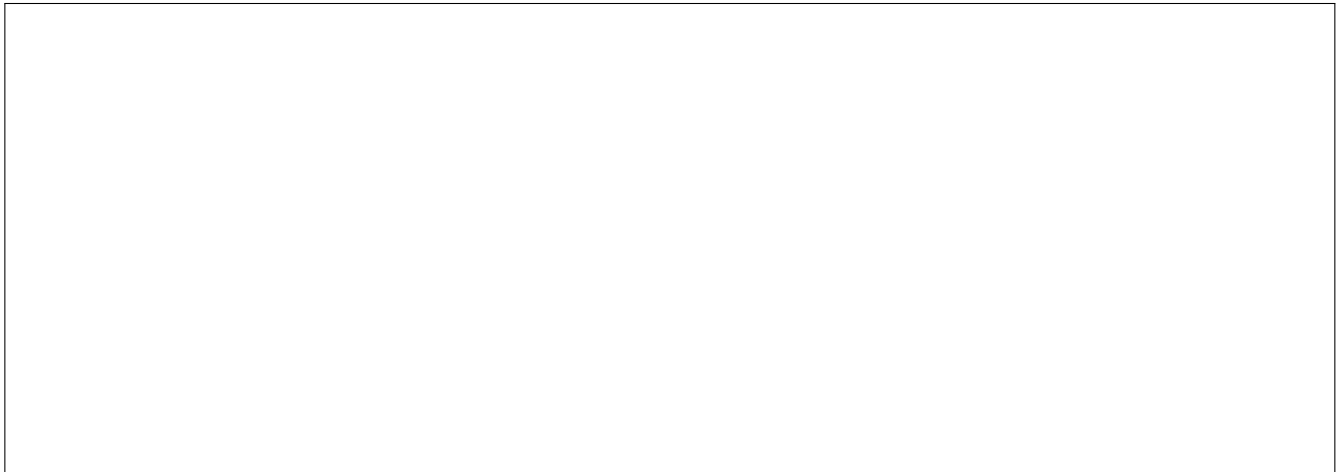
c)

$$f(x) = \begin{cases} x^{-1} & \text{if } x < -1 \\ ax + b & \text{if } -1 \leq x \leq \frac{1}{2} \\ x^{-1} & \text{if } x > \frac{1}{2} \end{cases}$$

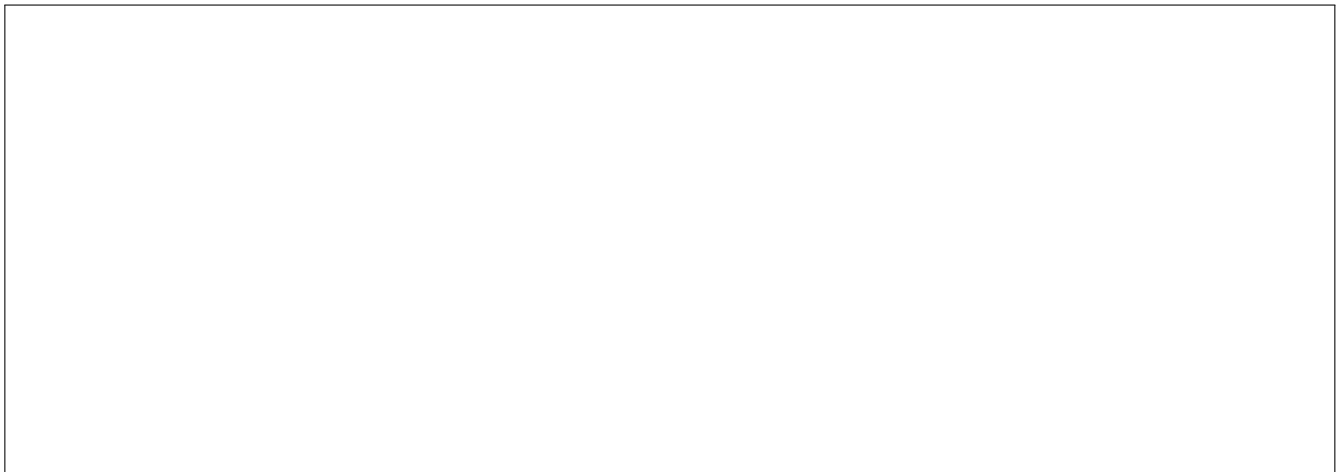3. Evaluate the limit if it exists. If not, determine whether the one-sided limits exist.

a)

$$\lim_{x \to 4} \frac{\sqrt{5-x}-1}{2-\sqrt{x}}$$

b)

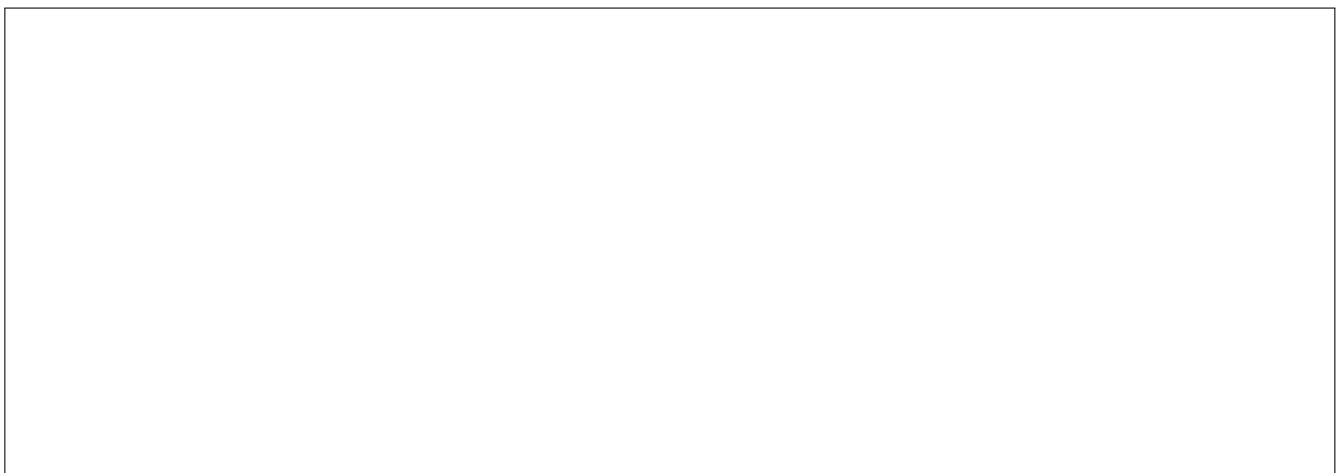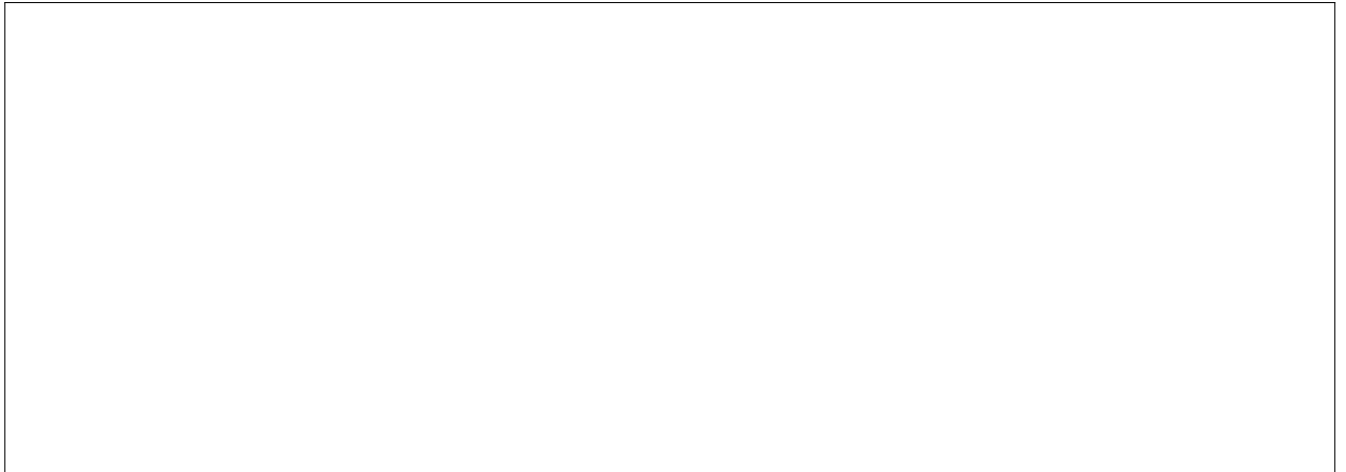$$\lim_{\theta \to \pi/2} \frac{\cot(\theta)}{\csc(\theta)}$$

c)

$$\lim_{t \to 2} \frac{2^{2t}+2^{t}-20}{2^{t}-4}$$

4. Evaluate the limit.
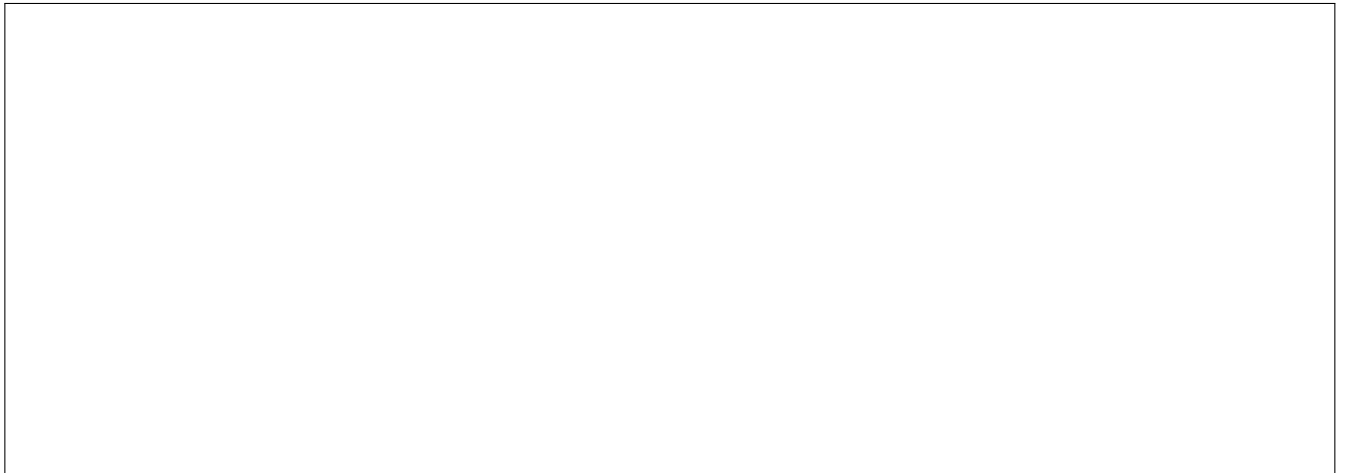
a)

$$\lim_{t \to 0} \frac{\csc(8t)}{\csc(4t)}$$

b)
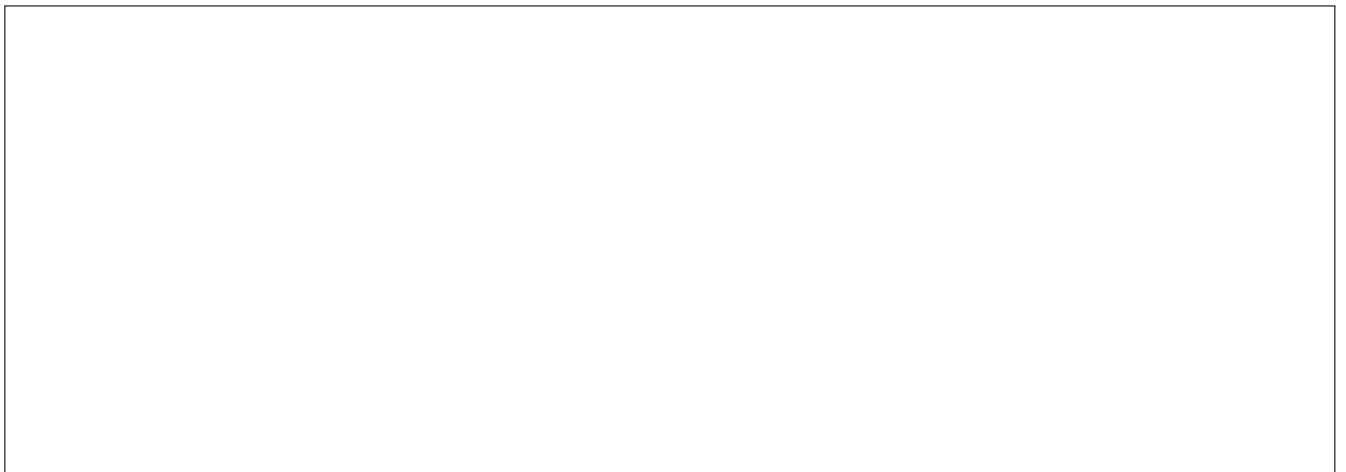
$$\lim_{\theta \to 0} \frac{\cos(2\theta) - \cos(\theta)}{\theta}$$

5. Evaluate the following limits.
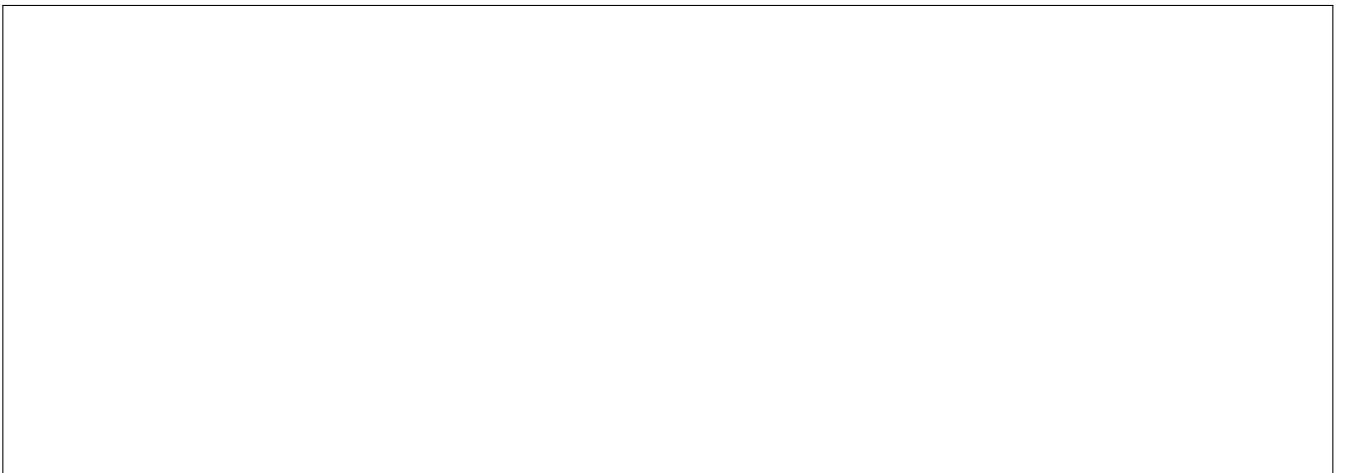
a)

$$\lim_{x \to \infty} \frac{9x^2 - 2}{6 - 29x}$$
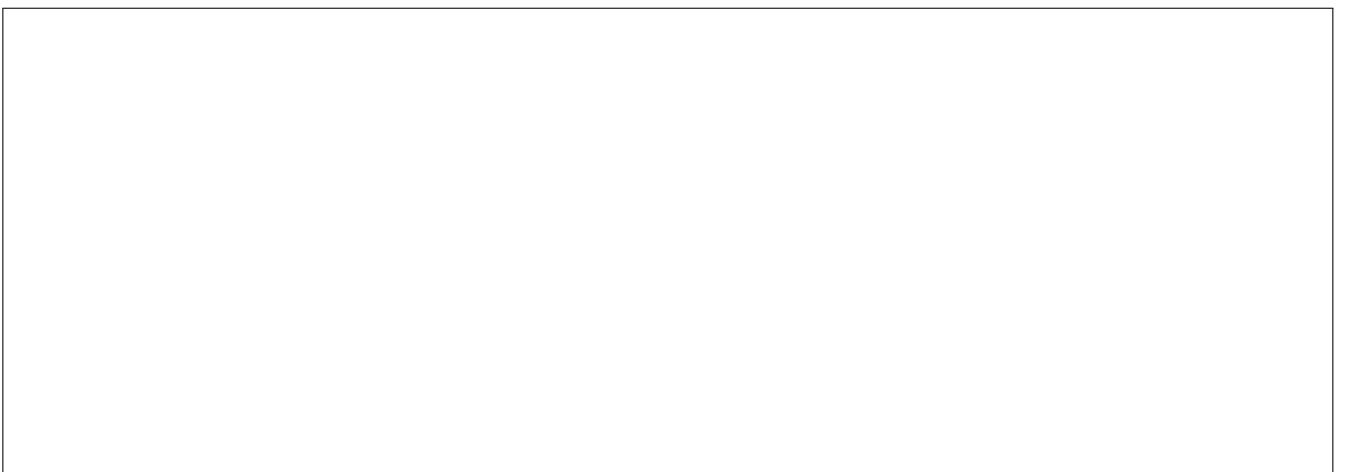
b)

$$\lim_{x \to \infty} \frac{5x - 9}{4x^3 + 2x + 7}$$

c)

$$\lim_{x \to \infty} \frac{8x^2 + 7x^{1/3}}{\sqrt{16x^4 + 6}}$$

d)

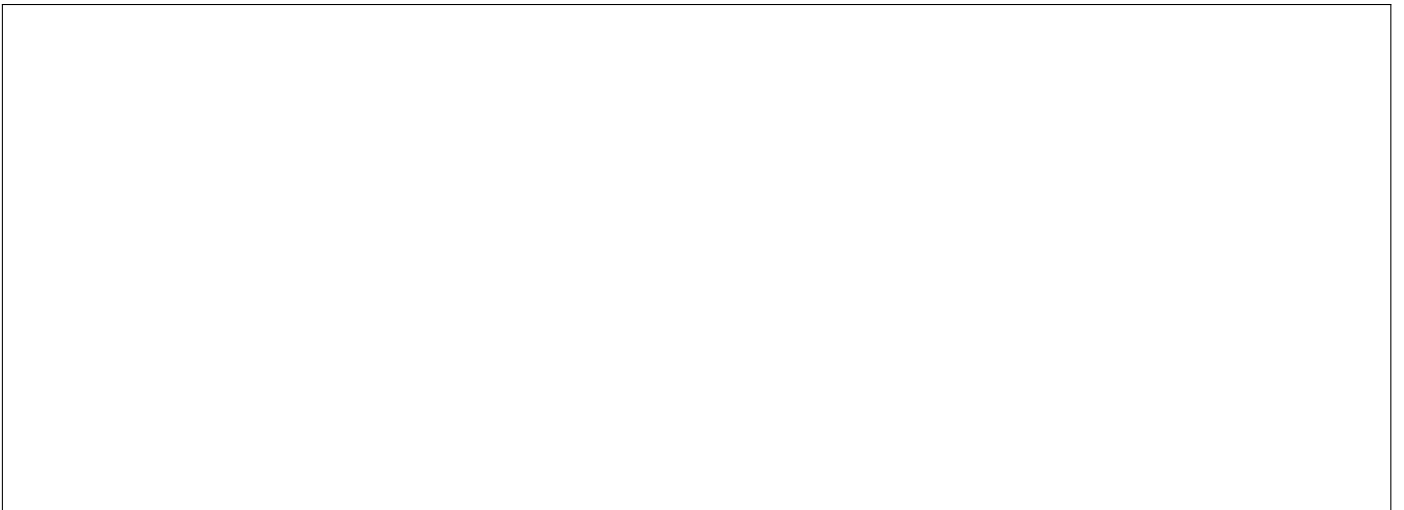$$\lim_{x \to \infty} \left(\sqrt{4x^4 + 9x} - 2x^2 \right)$$

6. Show that $\cos(x) = x$ has a solution in the interval $[0, 2]$.

7. For the following, use the limit definition to find $f'(a)$ and find an equation of the tangent line.
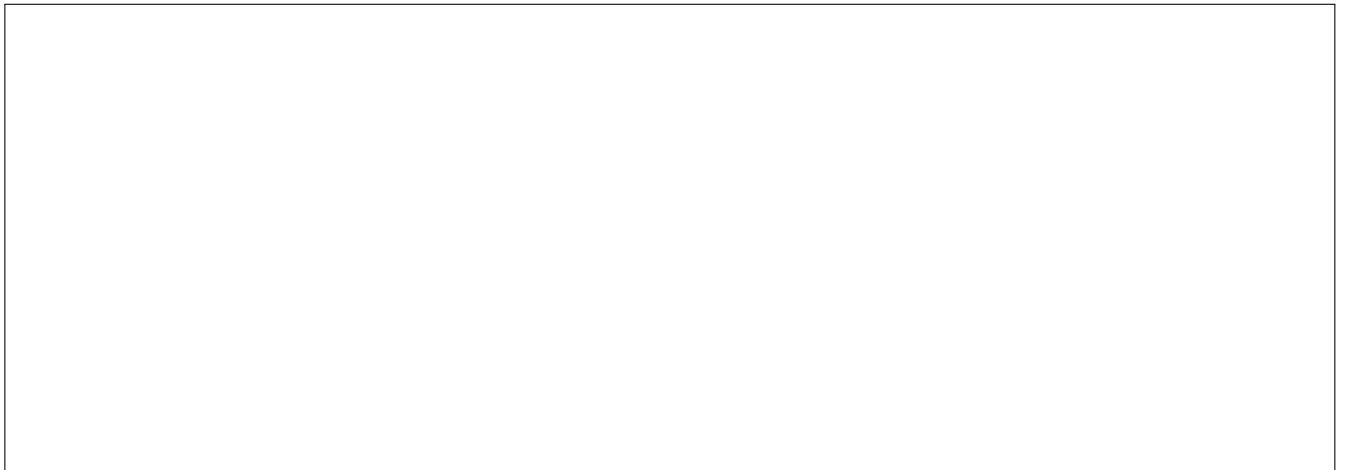$$f(x) = (2x)^{-1}, \quad a = 1$$

8. Calculate the derivative of the following. You do not need to simplify your answer for b) and c).
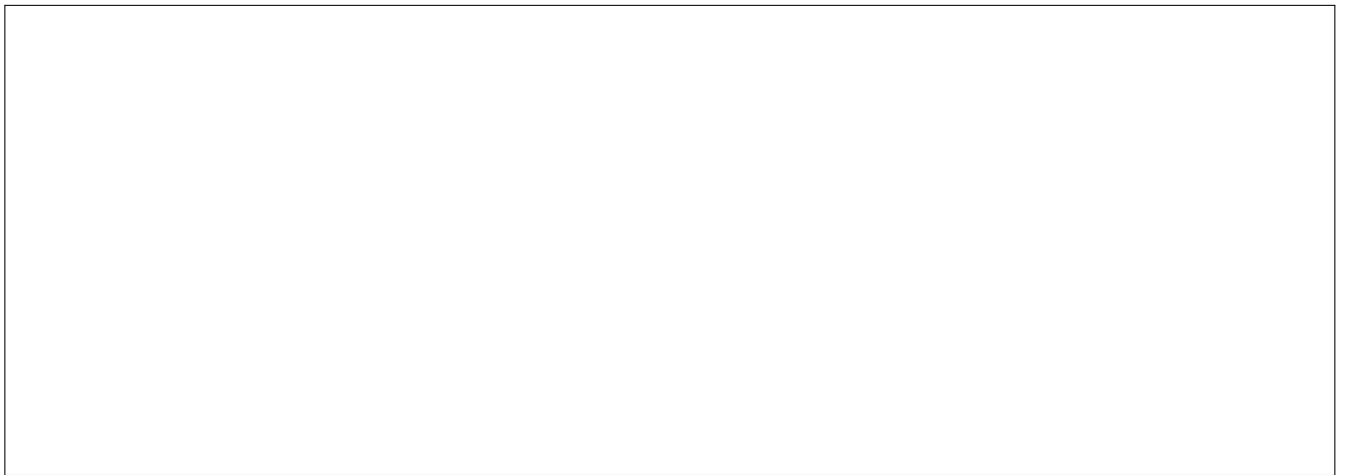
a)
$$f(x) = 6x^4 + 7x^{2/3}$$
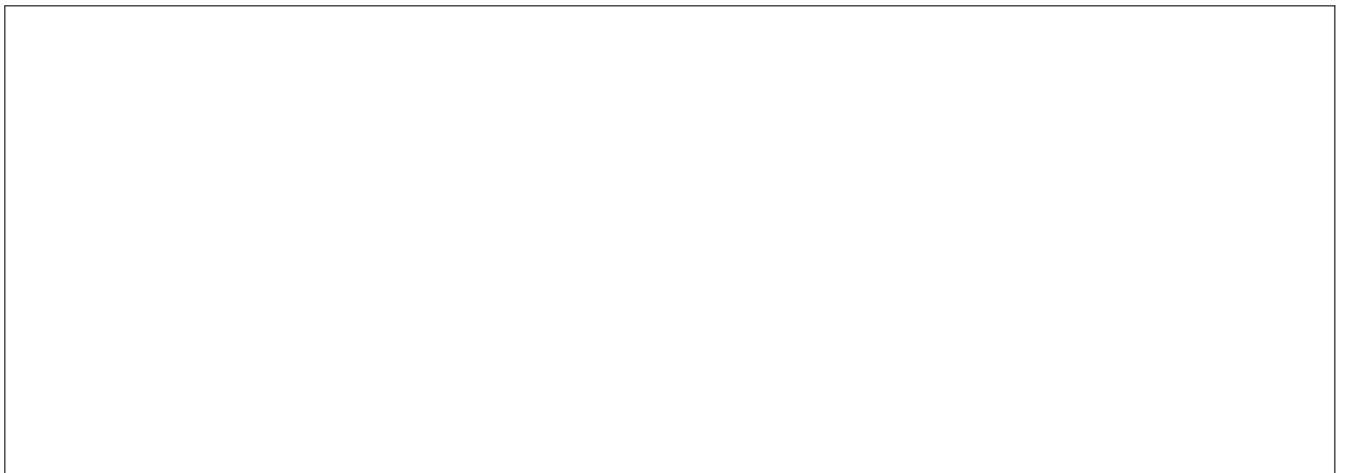
b)

$$f(x) = (4e^x - x^2)(x^3 + 1)$$

c)

$$f(x) = (x + 1)(2x - 2)(x^2 + x)$$

9. Calculate the derivative of the following. You do not need to simplify your answer.

a)

$$f(x) = \frac{x^4 + e^x}{x + 1}$$

b)

$$h(t) = \frac{t}{t^2 + 1}$$

10. Find the rate of change of the following.

a) The area of a circle with respect to the diameter.

b) Volume of a cube with respect to its side s when $s = 5$.