Modeling Diversity: Applications of Multilevel Analysis to School Psychology Research

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Much research in the field of School Psychology is conducted within educational institutions. Consequently, a large portion of the data school psychologists use is nested, i.e., students nested within classrooms, which are nested within schools, which are nested within counties, et cetera. Because of the nature of this type of data, it is frequently a good candidate for Multilevel Analysis. The purpose of the paper is to provide a non-technical introduction for school psychology students to the statistical method of Multilevel Analysis. Using an actual data set, this paper will give a cursory demonstration of why Ordinary Least Squares Regression can often be too restrictive for nested data, and how Multilevel Analysis can allow for a better accommodation of the diversity in some school psychology data.

What We Often Do: Ordinary Least Squares

Often in school psychological research, when we want to know how independent variables (IV; or predictors) predict a dependent variable (DV; or criterion), we use Ordinary Least Squares (OLS) Multiple Regression, i.e.,

$$Y_i = a + \sum_{h=1}^{p} b_h X_{hi} + e_i$$
, (1)

where *p* is the number of IVs used in the study and *i* is an individual in the study, ranging from 1 to *i*. (For a more in depth explanation of the derivation of the α and β coefficients, see Pedhauzer, 1997, Coehn, Coehn, Aiken & West, 2003, or most elementary calculus texts). Using equation (1), the researcher assumes that the regression coefficients (i.e., intercept, α , and slopes, β) are *invariant across all individuals*. Consequently, the *i*th individual's predicted DV value (Y_i) is simply the sum of: (a) a constant (α , controlled for X_{hi}); (b) IV affects (β_h); and (c) a random amount of error (ε_i , often called the *residual*).

Real Data

Using an actual data set, a OLS regression was run.¹ The data is made up of 3,236 students nested within 49 different schools. The IVs are scores on an IQ test (Ravens Matrices) and a social class ranking, which is used in this example as a proxy for SES. The DV is a standardized mathematics achievement test. Pertinent OLS output can be seen in Table 1.

Table 1

Unstandardized IVStandard Error В t-ratio p value Constant .54 8.46 .02 .68 34 .000. Ravens (IQ) .24 .05 4.7 SES .000 Adjusted R^2 R $R^{2}(Full Model)$.53 28 28

OLS Regression Output

From the OLS analysis, the key points are: (a) both SES and IQ help predict math achievement (i.e., have significant t-ratios, as indicated by the *p*-values), (b) IQ and SES explain 28% of the variance in math achievement, and (c) there is a small band of error surrounding the β coefficients.

One of the major assumptions in the OLS regression is that the observations are independent. Another is that the α (8.46) and β s (.68 & .24) are the same for *all* individuals in the data set; put another



way, the average math achievement score is appropriate for all students, and IQ and SES have the same affect for *every* student in *every* school.² Are these assumptions valid? Possibly, but probably not. Since the data are nested in nature (i.e., students are nested within schools) it is probably not valid to assume, *a priori*, that there is no variation. Like most things in statistics, these assumptions can be assessed empirically, using the *intra-class correlation* (ICC). Without going into much detail, the ICC (symbolized as the Greek letter rho, ρ) measures how much of the total variance is made up of the variance between the groups (in this example, the schools). The formula is:

$$r = \frac{t^2}{t^2 + s^2} , \qquad (2)$$

where *tau* (symbolized as τ^2) is the variance between the schools and *sigma* (symbolized as σ^2) is the total variance within each school [or, alternatively, the variance surrounding ε_i in equation (1)]. Obviously then, the sum of τ^2 and σ^2 is the total variance in the data.³ For our data, the total variance is approximately 58.4 and the between school variance is approximately 4.13, which produces a ρ of .075. This means that about 7.5% of the total variance in mathematics achievement scores is due to between school effects. Why is this important? If this effect is not taken into account, and an OLS regression is run, the type I error level (nominally at .05) is drastically increased (for a more in-depth explanation, see Kreft & De Leeuw, 1998).⁴

Therefore, it seems that the OLS assumption of independent observations does not hold up for this particular example (i.e., the scores on the math achievement test are somewhat dependent of the school the student is in). One way to solve this issue is to use Multilevel Analysis.

Multilevel Analysis

Multilevel Analysis (MA) is a statistical method that allows for the incorporation of nesting (grouping) into the data analysis model (i.e., the data do not have to be independent observations). Using our example, MA can allow for fact that IQ and SES might have *different* prediction capabilities (i.e., regressions coefficients) depending on the school the student attends. Logically, this would make sense, as IQ would tend to predict differently for a student from a school with high average socio-economic status (SES) and high average student IQ, than it would for a student from a school with low average SES and low average IQ. In other words, MA allows for the IVs in the data to have diverse affects on the DV. *Random Slope*

A good first step to model the data's diversity is to let the intercept vary between groups. This means that some groups will start out with higher average scores on the DV, while other will start out with lower scores. From our example, it would be expected, based on previous research (Teddlie & Reynolds, 2000), that a school with high average SES would have higher average math scores than a school with lower SES.

A model that allows the intercept to vary across groups is:

$$Y_{ij} = \mathsf{b}_{0\,i} + e_i \,. \tag{3}$$

In this formula, a given school's (*j*) average math score (β_{0j}) is allowed to vary across schools. To allow for the schools to have different intercepts, the intercept term, β_{0j} , needs to be decomposed into a group average (sometime called the *grand mean*, symbolized as γ_{00}) and a group deviation, symbolized as u_{0j} :

$$b_{0\,i} = g_{00} + u_{0\,i} \,. \tag{4}$$

When equation (3) and (4) are combined, they lead to the model:

$$Y_{ij} = g_{00} + u_{0j} + e_i, \qquad (5)$$

where, again, γ_{00} is the grand mean for every student in every school and u_{0j} is the main effect for school *j* (i.e., how much school *j* deviates from the grand mean). When γ_{00} and u_{0j} combine, they form the average score on the math test for school *j*.

Because we are trying to predict *both* math achievement *and* the average math score for a given school, this is technically known as an *intercept-as-outcomes* model. To run an analysis using this model, we need a statistics program that can incorporate multilevel models.⁵ For the purposes of this paper, HLM 5



(Raudenbush, Bryk, Cheong, & Congdon, 2001) is used. Snijders and Bosker (1999, chapter 15) give a comprehensive listing of other available software.

After centering the IQ variable (i.e., standardizing it: x = 0, sd =1, see endthote 11 for further elaboration of why centering is needed) the analysis produced the following results:⁷

Table 2

Results from Unconditional, Intercept-as-Outcome Model

Fixed Effect	Coefficient	Standard Error	t-ratio	p value
Model for school				
means				
Intercept, γ_{00}	26.53	.32		
	Variance			
Random Effect	Component	df	χ^2	p value
School Mean, u_{0i}	4.13	48	285.99	.000
Within-School	54.34			
(level 1) variance, e_i				

From the results, we can see that the average school math score is 26.53, with a range of plausible values (i.e., 95% confidence interval) of (22.55, 30.51).

Explaining the Level-2 Variance

Now that we know the variance in schools' mean math scores (i.e., the level-2 variable's variance, which is 4.13), let's try to explain some of it. A plausible explanatory variable is school SES. Using the data in the current analysis, we do not have a school-level SES variable per se, but we can easily make one. Because we have an SES indicator for each student, we can average the SES status for each student in each school, and come up with a level-2 SES IV. For those who like equations, here is the equation for what we are doing:

for schools (1, 2, ... 49)
$$\overline{Y}_{.j} = \frac{\sum_{i=1}^{n} X_{ij}}{n}$$
, (6)

where *n* is the number of students (*i*) in school (*j*), and X_{ij} is the SES score for student *i* in school *j*. For those who do not like equations, this means we will be summing students' SES scores in a given school and dividing this sum by the total number of students in that school. This process will be iterated for *j* number of schools, which is 49 in our example.

Our new model to fit is then:

$$Y_{ii} = \mathbf{b}_{0i} + e_i, \tag{7}$$

which is the same as equation (3); our level-2 model has changed though, now being:

$$b_{0j} = g_{00} + g_{01} (\text{Mean SES}) + u_{0j}.$$
 (8)

The IV, *Mean SES*, is our amalgamated SES variable, grand-mean centered.⁸ Combining equations (7) and (8) gives the hierarchical model:

$$Y_{ij} = g_{00} + g_{01} (\text{Mean SES}) + u_{0j} + e_i.$$
(9)

Running the current model [equation (9)] in HLM 5 gives the following output:

Table 3

Fixed Effect	Coefficient	Standard Error	t-ratio	p value
Model for school				
means				
Intercept, γ_{00}	26.53	.32		
Mean SES, γ_{01}	.63	.32	1.98	.05
	Variance			
Random Effect	Component	df	χ^2	p value
School Mean, u_{0i}	3.78	47	257.44	.000
Within-School	54.34			
(level 1), e_i				

Results from Intercept-as-Outcome Model, with SES as Level-2 IV

It is noteworthy that the between schools variance has dropped from 4.13 (Table 3) to 3.78. Using the

formula $\frac{t_U - t_C}{t_U}$, where τ_u is variance between schools in the unconditional model (i.e., 4.13 in our

example) and τ_c is the variance between schools in the conditional model (i.e., 3.78 in our example), gives the proportion of variance between schools explained by the model with Mean SES in it. With our data, the value is 0.85, which means 8.5% of the true between-school variance in math achievement is accounted for

by Mean SES. Similarly, the conditional ICC (i.e., conditional on Mean SES) is $\frac{3.78}{3.78+54.34}$, or .065.⁹

When Mean IQ is used as the Level-2 explanatory IV [forming the level-2 variable using the same formula as in equation (6)], the HLM 5 results are as follows:

Table 4

Results from Intercept-as-Outcome Model, with IQ as Level-2 IV

Fixed Effect	Coefficient	Standard Error	t-ratio	p value
Model for school				
means				
Intercept, γ_{00}	26.53	.27		
Mean IQ, γ_{01}	.59	.12	4.95	.000
	Variance			
Random Effect	Component	df	χ^2	p value
School Mean, u_{0i}	2.61	47	205.83	.000
Within-School	54.34			
(level 1), e_i				

The amount of variance explained (above and beyond the unconditional model, i.e., the results in Table 2) is 36.8% and the conditional ICC is .046. Based on the explained variance and reduction in ICC, mean IQ is a better explanatory variable for the variance in school mean math score, and will be used instead of Mean SES.¹⁰



Explaining the Results

From the Mean-IQ as a level-2 predictor model, we can say that the average math achievement score for a school with an average group IQ is 26.53 (95% Confidence Interval: 23.4, 29.7). When the school's average IQ increases, the average math achievement also significantly increases. Additionally, we can say that IQ explains a significant amount of the between school variance (36.8%) in math achievement; but, there is still significant variance (i.e., 63.2%) left over between schools, even when accounting for school IQ. Last, the standard errors are more accurately estimated, and are larger than those given by the OLS estimation. *I've Seen this Before, Haven't I*?

For those familiar with Generalized Linear Models, equation (5) should look familiar because it is the formula for one-way, random-effects ANOVA. In the ANOVA conceptualization, β_{0j} is the coefficient for the randomly selected groups, and its variance, τ^2 , is the between group variance. Consequently, the variance of e_i , σ^2 , is then the within-groups variance. If a level-1 IV was also included in the model, then equation (5) would then look like the linear formula for random effects ANCOVA. *Hierarchical Linear Model (HLM)*

In the last section, we allowed each school to have its own average math achievement score, and we found that the average school IQ significantly predicted that average math score. Now we are going to divulge for a brief section and forget about what we found. To make things easier, we are going to focus first on an unconditional (i.e., no IVs included in the level-2 model), full hierarchical model by allowing both the intercept (β_{0j}) and the level-1 slope (β_{1j}) to vary (randomly) for each school. Specifically, the model is as follows:

$$Y_{ij} = b_{0j} + b_{1j} SES_{ij} + e_i,$$
(10)

where SES is the standardized SES score for student *i* in school *j*.¹¹ A student's math achievement score is now predicted by two parameters: the intercept (i.e., the performance of an average student in an average school), and the slope (i.e., the student's IQ's effect). Because both the intercept and slope have been centered, β_{0j} is school *j*'s mean math score. Allowing both the intercept and the slope to vary for a given school gives us the following equations:

$$b_{0j} = g_{00} + u_{0j}, \qquad (11a)$$

$$\mathsf{b}_{1\,i} = \mathsf{g}_{10} + u_{1\,i} \,, \tag{11b}$$

where

 γ_{00} is the average of the school math achievement scores across all the schools;

 γ_{10} is the average SES-math achievement regression slope across all the schools;

 u_{0j} is the unique increment to the intercept (average math achievement) associated

with school *j*; and

 u_{1j} is the unique increment to the slope associated with school *j*.

A caveat in having two randomly varying parameters is that not only is there between-group variance (τ_{00}) and within-group variance (τ_{11}) , but there is also covariance between the two parameters $(\tau_{10} \text{ and } \tau_{01})$. This issues involved in having a covariance term will not be explained more in this paper; for those interested, see Raudenbush and Bryk (2002).

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Technology Chair Rachelle R. Whittaker Indiana State University Analyzing the data in this unconditional HLM, we will be able to estimate the unconditional parameter variability in both intercepts and slopes. Combining equations (10) and (11) yields:

$$Y_{ij} = g_{00} + g_{10}SES_{ij} + u_{0j} + u_{1j}SES_{ij} + e_i,$$
(12)

which produces the following estimates when run in HLM 5:

Table 5Unconditional HLM

		Standard		
Fixed Effects	Coefficient	Error	t-ratio	p value
Average mean math achievement score, γ_{00}	26.48	.32		
Average SES- achievement slope, γ_{10}	.42	.09	4.68	.000
Random Effects	Variance Components	df	χ^2	p value
School average Math score, u_{0j}	3.94	48	273.17	.000
SES-achievement slope	.20	48	110.43	.000
Within-School (level 1), e_i	52.61			

From this output, we can see that, *on average*, SES is significantly related to math achievement (i.e., γ_{10} is significant). Additionally, because the variance components for both the intercept and slope are significant, we infer that significant differences exist among average math achievement between the schools, and that the relationship between SES and math achievement *within* each school varies *across* the population of schools. We can find 95% confidence intervals for the intercepts and slope, (22.54, 30.42) and (-.48, 1.32), respectively. Last, because we have a level-1 IV (SES), we can calculate the proportion reduction in variance

using the formula we used earlier, (i.e.,
$$\frac{t_U - t_C}{t_U}$$
), which yields ($\frac{54.34 - 52.61}{54.34}$), which in turn, equals

approximately 04. From this, we see that adding SES as a predictor of math achievement reduced the *within-school* variance by about 4%, and we can conclude that SES accounts for about 4% of the student-level variance in math achievement scores. Remembering that SES also explained about 8.5% of the between–school variance in math scores in the first intercept-as-outcomes model, it looks as if SES has a more cogent effect at the school level than at the student level, but, when it is the model by itself at both levels, it is significant both places.

Slopes- and Intercepts-as-Outcomes Model

Keeping equation (10) as the level-1 model, we know try to account for the between-school variance by adding IVs [Standardized IQ, (*SIQ*), and *SES*] to the level-2 equations. Specifically, we are going to fit the following models for the intercept and slope, respectively:

$$b_{0i} = g_{00} + g_{01}SIQ_i + u_{0i}$$
, and (13a)

$$b_{1j} = g_{10} + g_{11}SES_j + u_{1j}, \qquad (13b)$$

where u_{0j} and u_{1j} are the random effects of the intercept and slope, respectively, and have variances (τ_{00}) and (τ_{11}), respectively, and a covariance of τ_{10} .¹² Combining equation (10) and (13) yields the model:



$$Y_{ij} = g_{00} + g_{01}SIQ_j + u_{0j} + g_{10}SES_{ij} + g_{11}SES_jSES_{ij} + u_{1j}SES_{ij} + e_i.$$
(14)

Table 6 contains the output when using equation 14 to model our data in HLM 5.

Table 6

Intercepts-and Slopes-as-Outcomes Model

		Standard		
Fixed Effects	Coefficient	Error	t-Ratio	p value
Model for school math a	achievement			
means				
Intercept, γ_{00}	26.42	.29		
Mean IQ, γ_{01}	.56	.14	4.1	.000
Model for SES-math acl	hievement slopes			
Intercept, γ_{10}	.43	.09		
Mean SES	.21	.1	2.1	.000
	Variance			
Random Effects	Components	df	χ^2	p value
School mean, u_{0j}	2.8	47	214.1	.000
SES-achievement	.17	47	100.4	.000
Slope, u_{10}	.17	4/	100.4	.000
Within-School	52.6			
(level 1), e_i	52.0			

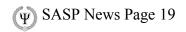
As we did earlier, we can assess the reduction in variance to see how much variance these level-2 predictors helps explain. For the intercept, we have the equation $\frac{3.94 - 2.75}{3.94}$, which equals .30, and for the slope we have the equation $\frac{.20 - .17}{.20}$, which equals .15. Consequently, from adding IQ to the level-2 intercept model we explain about 30% of the between-school variance in average math score; likewise,

adding average SES to the level-2 achievement-SES slope, we explain 15% of the between-school variance. All this is in addition to the 4% of the within-school variance explained by adding SES to the level-1 model. If we had more variables, this procedure could be iterated and more variance could be explained at both levels. For this didactic exercise though, we will end at this point and give a more thorough discussion of the findings.

Discussion

It is hoped that the previous cursory walking-through of Multilevel Analysis has shown the readers some of its unique and desirous capabilities. For instance, while the Ordinary Least Squares method of Multiple Regression can tell the researcher of the omnibus amount of variance explained in the dependent variable by independent variables, it does not have the capability of splitting it up into within and between group variance. Further, when data are nested, OLS methods are unreliable as they reject the null hypothesis at a level too liberal by most researchers' standards (which also makes the regression confidents have too-small standard errors). Multilevel Analysis, on the other hand, can combat these problems by allowing a splitting of within and between group variance as well as allowing data to be nested and still having appropriate alpha levels.

Specifically, in the example worked in this paper, it was shown that an Ordinary Least Squares analysis of the affects of IQ and SES on math achievement explained about 28% of the total variance. When the same data were put into a hierarchical model, it was shown that SES explains about 4% of the within-



schools variance on math scores, while IQ explains about 30% of the between-schools average score, and school-average SES explains about 15% of the individual affect of SES on math achievement. Put more simply, from the current data, it seems that students with higher SES perform better on the math achievement test, and part of this can be explained by the fact that richer schools tend to have, on the average, higher math achievement. Additionally, schools with higher average IQs tend to have students who have higher math achievement, when SES is controlled. This is not a complete explanation of the data (after all, there was a significant amount of variance still left over in both level-1 and level-2 variances), but it does show that math achievement is a complex phenomenon that needs *both* individual- and group- variable explanation. Using Multilevel Analysis, the diversity inherent in this data was allowed to come through a lot more clearly than when OLS was used.

Further Reading

For those who are interested in MA, here is a list of good introductory texts. Hox, J. (1995). *Applied multilevel analysis*. Amsterdam: T-T Publikaties. [Available free

at: http://www.fss.uu.nl/ms/jh/publist/amaboek.pdf]

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Footnotes

1. The data set is available freely on the Internet from the Centre for Multilevel Modelling [http://multilevel.ioe.ac.uk/intro/datasets.html]. Original data are from Mortimore, Sammons, Stoll, Lewis, & Ecob (1988). For the purpose of this paper, the SES variable was reordered so that a low value equals low SES and vice versa for high SES values. A caveat to using this data is that *it is for didactic purposes only*, i.e., no inferences about IQ, SES, or math achievement should be made from the results of the analyses.

2. There are more assumptions than these in regression analysis. For more information, see Coehn, Coehn, Aiken & West (2003); Pedhauzer (1997).

3. The intra-class correlation can be computed in SPSS (v. 10) under *Variance Components* in the *General Linear Model* option using math achievement as the DV and school number as the random factor.

4. Type-I error happens when one rejects the null-hypothesis when it is true and, therefore, should be retained.

5. Technically, SPSS v.10 and above can analyze intercepts-as-outcomes models. For more information on this, see chapter 15 of Snijders and Bosker (1999).

6. A free, although limited, version of HLM 5 is available on the Internet [http://www.ssicentral.com/other/hlmstu.htm]. The data used for this paper's analysis are purposely truncated so anyone can use the free version of HLM 5 to run their own analysis.

7. From this point on, instead of OLS, the analysis will use Full Maximum Likelihood. Raudenbush and Bryk (2002, chapters 3, 13, & 14) explain the formulae and theory behind this estimation method.

8. This is the only centering option available for level-2 variables in HLM 5. For further explanation on the differences in centering at their meaning at different levels, see Raudenbush and Byrk (2002, pp. 31-35).



9. The conditional intraclass correlation (ICC) is the amount of the total variance that is due to the between groups variance, *after* controlling for various IVs. In our example, 6.5% of the total variance is between schools, after controlling for the schools' SES level.

10. Mean SES looses most of its explanatory power when Mean IQ is also in the level-2 model. Consequently, for the sake of parsimony, only Mean IQ is kept.

11. When using IVs that have no real value at 0 (such as IQ and SES), it is important to transform them so that 0 has meaning. An easy way of doing this is to standardize the score, which means 0 is interpreted as a student with an average SES score. Another way, which the HLM 5 program will do for you, is to grandmean center the variable, which means person *i*'s score will be a deviation from the grand mean, or

 $(X_{ii} - \overline{X}_{..})$. Consequently, a score of 0 means an average SES status.

12. A model with both average SES and average IQ in both level-2 equations was fit, but the model presented (i.e., equation (14)] fit better. For the sake of brevity, only the better-fitting model is presented.

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