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ORIGINAL PAPER

On the dynamics of the age structure, dependency, and consumption

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Abstract We examine the effects of population aging due to declining fertility and rising elderly life expectancy on consumption possibilities in the presence of intergenerational transfers. Our analysis is based on a highly tractable continuous-time overlapping generations model in which the population is divided into three groups (youth dependents, workers, and elderly dependents) and lifecourse transitions take place in a probabilistic fashion. We show that the consumption-maximizing response to greater longevity in highly developed countries is an increase in fertility. However, with larger transfer payments, the actual fertility response will likely be the opposite, leading to further population aging.

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1 Introduction

This paper considers how changes in fertility affect consumption through changes in the structure of dependency and how, via its effect on the dependency burden faced by working-age adults, the age structure in turn affects fertility. Highlighting the role of intergenerational transfers, we develop a highly tractable continuous-time overlapping generations model that allows us to graphically analyze of the relationships between economic dependency and the age structure. The model also clarifies a potentially important feedback loop between the two.

Our interest in the mutual dependency of fertility and a society's age structure is primarily motivated by considering the future prospects of some of the world's most developed countries. Countries such as Italy and Japan are now coming to the end of a decades-long period during which low fertility produced a transitorily low level of population dependency. Over the next 20 years, rapid population aging will drastically reduce consumption possibilities (and even more drastically affect government budgets). The effect of this consumption crunch on fertility—and thus on the age structure of the population even further down the road—is an issue that has received little attention from economists.

The effect of population aging on consumption is a widely studied topic. For example, Cutler et al. (1990) and Elmendorf and Sheiner (2000) discuss the effect of population aging in the United States on feasible and optimal paths of consumption. Given that a large share of resource transfers to the elderly is channeled through the state, population aging will have a particularly dramatic effect on government finances (see Lee and Edwards 2001). On the other hand, Bloom et al. (2001) examine how a "demographic dividend" resulting from reductions in fertility—that is, a period of several decades in which the ratio of working-age adults to dependent children and elderly is unusually high—affects income per capita in developing countries.

In the above literature, the important chain of causality is from the demographic to the economic. The underlying demographic inputs, most notably changes over time in fertility, are either taken as exogenous or related to phenomena such as declining child mortality that are outside the economic model being examined. A separate strand of literature considers the impact of old-age dependency on fertility through the pension system (Zhang 1995; Cigno and Rosati 1996; Wigger 1999).

In this paper, we close the loop by concentrating on the interdependence of fertility and the population age structure arising from economic dependency in the presence of intergenerational transfers. We clarify the short- and long-term consumption effects of population aging that result from reduced fertility, which are potentially quite different and are contingent on the initial age structure. Drawing on related numerical work (Hock and Weil 2007), we argue that current fertility rates in the developed world are below those that would minimize long-run economic dependency. Our model makes clear that, in such a situation, reductions in consumption associated with greater elderly life expectancy might be offset by increases in the fertility rate. Going beyond previous work (our own and that of others), we describe a feedback loop whereby rising old-age dependency reduces the disposable income of the working population, resulting in lower fertility and further population aging.[1] Consequently, the privately optimal response to population aging is at odds with the consumption-maximizing (or tax-minimizing) response. In a further extension, we consider a political interaction between members of different age groups concerning the generosity of government transfers to the elderly. We show that a political bargaining process can amplify the feedback from population aging to fertility, resulting in even lower fertility and yet more aging.

In addition to identifying the feedback from population age structure to fertility via the channel of dependency, a second contribution of this paper is to construct an analytically tractable dynamic model of population age structure. Specifically, we build a continuous-time overlapping generations model in which population is divided into three groups (young, working-age, and old) and transitions between groups take place in a probabilistic fashion as in Blanchard (1985). Within this model, fertility may be taken as exogenous or made an endogenous function of the population age structure.[2] Our model is simple enough to be analyzed graphically and yet captures the dynamic adjustment of age structure, fertility, and consumption to a variety of external shocks. Our focus is on is population aging due to increased life expectancy, but the basic framework can be extended to applications well beyond those pursued here.

The rest of the paper is structured as follows. In Section 2, we describe our model of the population age structure and develop the basic dynamic equations used in the subsequent analysis. Section 3 presents results from the model under the assumption that the fertility rate is exogenous. In Section 4, we allow

[1]The mechanism that we examine here shares the feature of the model by Easterlin (1987) that reductions in living standards trigger compensating reductions in fertility. However, we focus on the fiscal effect of transfers to the elderly on after-tax earnings rather than on the effect of cohort size on the pre-tax wage of young workers—in principle, the two mechanisms could easily co-exist. The model by Cerda (2005) of social security crises is similar in spirit to ours, but our model permits a broader understanding of the interrelatedness of consumption possibilities and age structure.

[2]Gertler (1999) extends the model of Blanchard (1985), allowing for two age groups (working age and retired). Grafenhofer et al. (2007) present a model of probabilistic aging involving eight age groups, which they use to quantitatively assess the effects of population aging on savings, consumption, and the tax rate. Both models, however, take fertility as exogenous. Bommier and Lee (2003) develop a model of an economy with a continuous age distribution. They derive some interesting aggregate steady-state results, but make limited progress in analyzing aggregate dynamics.

fertility to be an endogenous function of income and then analyze the dynamics of the complete system, comparing the actual and consumption-maximizing responses of the economy to an exogenous shock to old-age mortality. We also extend the model to allow for a political interaction between the elderly and working-age adults that determines the magnitude of government transfers to the elderly. Finally, Section 5 concludes.

2 A dynamic model of the age structure

Economists have long recognized the need to incorporate age heterogeneity into macroeconomic analysis. Samuelson (1958) and Diamond (1965) developed economic-demographic models in which agents progressed through a discrete set of ages. Overlapping generations (OLG) models of this type have been used extensively in the economic growth literature. OLG growth models typically assume a two- or three-period lifecourse, which allows for a relatively clean analysis of fertility and old-age dependency. However, periods within such models represent long spans of time in the real world (e.g., 20 to 30 years). More important, most models do not allow for variability in the time spent in each phase of life.

We develop here a somewhat stylized continuous-time model that borrows both from the traditional OLG framework and from the "model of perpetual youth" by Blanchard (1985). Given our interest in economic dependency, we model the lifecycle as a progression through a series of stages of economic life rather than conceptualizing it as a series of ages. Most individuals follow a pattern whereby they are first dependent on their parents, then work for some amount of time, and then retire. Accordingly, we divide the population into three groups: $A_{\mathrm{Y}}(t)$ is the stock of young people who have never worked at time t; $A_{\mathrm{M}}(t)$ is the stock of people in their working years; and $A_{\mathrm{O}}(t)$ is the stock of people who once worked but are retired by time t.

To focus on the dynamics of the age structure, we assume that output is produced solely by labor, which is supplied inelastically by people in their working years. The total pool of resources available for consumption is:

$$\Omega(t) = W(t) A_{\mathrm{M}}(t), \tag{1}$$

where $W(t)$ is the prevailing wage at time t. A system of transfers from the working supports the young and the elderly. As a result, we focus on the youth and old-age dependency ratios:

$$y(t) = \frac{A_{\mathrm{Y}}(t)}{A_{\mathrm{M}}(t)} \quad \text{and} \quad o(t) = \frac{A_{\mathrm{O}}(t)}{A_{\mathrm{M}}(t)}.$$

In the remainder of this section, we establish various properties of the equations governing the evolution of the dependency ratios, which provide the basis for our model's dynamical system.

2.1 Lifecycle transitions

Individuals progress monotonically through the age structure (A_{Y}, A_{M}, and A_{O}) and face a constant exit probability from each group. For the young and working, λ_{Y} and λ_{M} give the hazard of transition to work and to retirement, respectively. Among the elderly, λ_{O} is the probability of dying. All of the flows are determined by the transition parameters, $\{\lambda_{\mathrm{j}}\}$, except the flow of births into A_{Y}, which is given by $N(t)$. The following system of equations summarizes this model of the evolution of the age structure:

$$\dot{A}_{\mathrm{Y}}(t) = N(t) - \lambda_{\mathrm{Y}} A_{\mathrm{Y}}(t), \tag{2}$$

$$\dot{A}_{\mathrm{M}}(t) = \lambda_{\mathrm{Y}} A_{\mathrm{Y}}(t) - \lambda_{\mathrm{M}} A_{\mathrm{M}}(t), \tag{3}$$

$$\dot{A}_{\mathrm{O}}(t) = \lambda_{\mathrm{M}} A_{\mathrm{M}}(t) - \lambda_{\mathrm{O}} A_{\mathrm{O}}(t). \tag{4}$$

The parameters λ_{j} give the inverse of the average time spent in each age group, T_{j}.[3]

2.2 Dependency dynamics

The equation of motion for the old-age dependency ratio may be derived from Eqs. 3 and 4 as:

$$\dot{o}(t) = \lambda_{\mathrm{M}} - (\lambda_{\mathrm{O}} - \lambda_{\mathrm{M}})\, o(t) - \lambda_{\mathrm{Y}} y(t) o(t). \tag{5}$$

Thus, the dynamics of old-age dependency are determined entirely by the age structure and the transition parameters. Movement of the youth dependency ratio additionally depends on the flow of births, which is a function of the stock of fecund persons and the rate of fertility among the fecund. From Eqs. 2 and 3, the equation of motion for the youth dependency ratio may be written as:

$$\dot{y}(t) = \frac{N(t)}{A_{\mathrm{M}}(t)} - (\lambda_{\mathrm{Y}} - \lambda_{\mathrm{M}})\, y(t) - \lambda_{\mathrm{Y}} \left[y(t)\right]^2. \tag{6}$$

For simplicity, we assume that the fecund population is a fixed proportion, ϕ, of the workforce. Since all members of the workforce are homogeneous in our

[3] In our model, we ignore childhood and early-adult mortality, assuming that death occurs only among the retired. The model could easily be adapted to incorporate these other forms of mortality by changing the basic equations to:

$$\begin{aligned}
\dot{A}_{\mathrm{Y}}(t) &= N(t) - (\lambda_{Y} + \mu_{\mathrm{Y}}) A_{\mathrm{Y}}(t), \\
\dot{A}_{\mathrm{M}}(t) &= \lambda_{\mathrm{Y}} A_{\mathrm{Y}}(t) - (\lambda_{\mathrm{M}} + \mu_{\mathrm{M}}) A_{\mathrm{M}}(t), \\
\dot{A}_{\mathrm{O}}(t) &= \lambda_{\mathrm{M}} A_{\mathrm{M}}(t) - \mu_{\mathrm{O}} A_{\mathrm{O}}(t).
\end{aligned}$$

With this set-up, λ_j is the transition probability to the next group, and μ_j is the death rate in the group. Analysis of this model would parallel that in the main text, with qualitatively similar results.

model, an alternative interpretation of ϕ is to decompose it as $\phi = T_F/T_M$, where $T_F < T_M$ is the expected duration of fecundity for each member of M.[4] Let $n(t)$ represent the average fertility rate among the fecund. Under the above assumption, Eq. 6 reduces to:

$$\dot{y}(t) = \phi n(t) - (\lambda_Y - \lambda_M)\, y(t) - \lambda_Y \left[y(t)\right]^2. \tag{7}$$

We will refer to $\phi n(t)$ as the flow of births per worker.

The equation of motion for the youth dependency ratio (Eq. 7) does not depend explicitly on $o(t)$. Thus, if fertility is determined in a manner unrelated to the old-age dependency ratio, equilibrium analysis of the relative economic age structure is straightforward. This is the case in Section 3, where we take the fertility rate as given. In Section 4, we allow fertility to be endogenously determined. Given that fertility will be a function of after-tax income, transfers to the elderly through the pension system cause the old-age dependency ratio to play a role in the dynamics of the youth dependency ratio.

3 Economic dependency

In this section, we focus on how changes in the age structure attributable to fertility and mortality affect dependency and consumption in the presence of intergenerational transfers.[5] The traditional demographic measure of dependency is the support ratio, which is simply the ratio of workers to total population. However, consumption tends to vary substantially over the lifecycle (Cutler et al. 1990; Lee et al. 2008), implying that proportionate changes in youth and elderly dependency will have different effects on resource availability. Consequently, we summarize economic dependency using a consumption-weighted measure. Normalizing the consumption of workers to one, let ρ_Y and ρ_O denote the relative consumption of the youth and elderly. The extent of economic dependency is given by:

$$e(t) = \rho_Y y(t) + \rho_O o(t). \tag{8}$$

Given our assumptions, the aggregate resource constraint in Eq. 1 implies that the consumption of the working is $c_M(t) = \eta(t) \mathrm{W}(t)$, where

$$\eta(t) = \frac{1}{1 + e(t)} \tag{9}$$

is an aggregate consumption index affecting all age groups proportionately.

[4] An additional set of equations, consistent with those governing the economic age structure, could be specified to allow ϕ to vary over time. However, such a specification results in a model that is substantially more complicated to analyze while adding very little qualitatively to our analysis.

[5] A companion paper (Hock and Weil 2007) considers the additional effects of changes in the retirement age.

Increases in both youth and elderly dependency reduce per-capita resources in the economy, but their effects on η are scaled by the relative consumption requirements of the young and old. Below, we consider the impact of changes in the fertility rate on economic dependency and consumption. We then consider the consequences of population aging due to increased life expectancy among the elderly. First, however, we characterize the basic equilibrium of the model.

3.1 Equilibrium with exogenous fertility

With a fixed fertility rate, the model produces a dynamical system that converges to a stable equilibrium age structure determined by the underling demographic parameters. It is convenient to express the flow of births as $\phi n = G/T_M$, where G is the gross reproductive rate (GRR).[6] Holding G constant, solving Eq. 7 yields one positive root for the equilibrium youth dependency ratio. We express the positive root in terms of the expected time spent in each age group (T_j) rather than as the exit parameters (λ_j):

$$\bar{y} = -\frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right) + \sqrt{\left(\frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right) + \frac{T_Y}{T_M}G\right)}, \tag{10}$$

which corresponds to a vertical line in (y,o)-space.

Based on Eq. 5, the equilibrium old-age dependency ratio associated with $\bar{y}$ is:

$$\bar{o} = Z_o(\bar{y}) = \frac{1}{(T_M/T_O - 1) + (T_M/T_Y)\,\bar{y}}. \tag{11}$$

This function is downward-sloping, convex with respect to the origin, and has a vertical intercept equal to $T_O/(T_M - T_O)$. The dynamics of the system imply that $(\bar{y},\bar{o})$ is a globally stable equilibrium as depicted in Fig. 1. Given the one-to-one relationship in Eq. 11, the function $Z_o(y)$ is a *locus of stable populations* when the demographic parameters are held constant.[7] We refer to the slope of this locus as the *marginal rate of demographic transformation* between the young and elderly.

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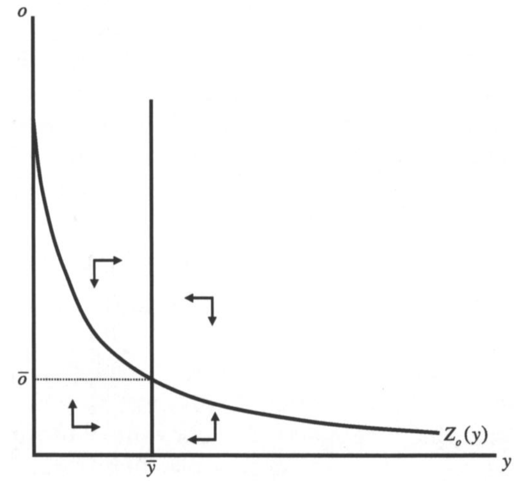
[6] The GRR is the number of children individuals can expect to have, assuming that they are subject to the currently prevailing rate of fertility for the entirety of their childbearing years. Given our assumptions regarding the stock of fecund persons, the GRR works out to $G = T_F n = \phi T_M n$.

[7] A stable population is one in which the relative size of the age groups is constant. When fertility is at the replacement rate ($G = 1$), the equilibrium dependency ratios are $\bar{y}^r = T_Y/T_M$ and $\bar{o}^r = T_O/T_M$, paralleling basic results from stable population theory.

Fig. 1 Global dynamics with constant demographic parameters



3.2 Fertility and consumption

Much of the 20th century was characterized by falling birth rates in the developed world. Our basic model can be used to analyze the transitional dynamics associated with such declines in fertility, along with the changes in steady-state consumption attributable to the associated shifts in age structure. After a fall in fertility, Eq. 11 implies that the equilibrium youth dependency ratio falls. As shown in Fig. 2, the decline in y maps to a higher equilibrium value of old-age dependency along the locus of stable populations.

Along the transition path, the declines in youth dependency are initially larger than the increases in old-age dependency, which reduces economic dependency. The result is an increase in the consumption index η, reflecting the "demographic dividend" discussed by Bloom et al. (2001). The standard argument is that the dividend is transient and eventually offset by population aging. However, a point that has received little attention in the literature is that the net effect on steady-state consumption is a priori ambiguous. This can be seen quite clearly in our model, which also demonstrates the existence of a threshold level of fertility below which further reductions ultimately reduce steady-state consumption. The inset of Fig. 2 shows a possible set of paths for y, o, and η after a decline in the fertility rate in the case where the new equilibrium has lower aggregate consumption than the original equilibrium.

Following Weil (1999), iso-dependency lines associated with a given level of economic dependency, e, are obtained from Eq. 8 as:

$$o = (e - \rho_Y y)/\rho_O \equiv I_e(y). \tag{12}$$

Overlaying iso-dependency lines along the locus of stable populations associates each equilibrium population age structure with a level of economic dependency. Iso-dependency lines closer to the origin in (y,o)-space are associated with a lower rate of economic dependency and a higher consumption index, η. Because $Z_o(y)$ is convex with respect to the origin, maximizing consumption yields a unique equilibrium pair $(\bar{y}^m, \bar{o}^m)$ as depicted in Fig. 3. At this point, the marginal rate of demographic transformation is equal to the marginal rate of substitution between consumption of the old and consumption of the young.

From Eqs. 10, 11, and 12, the consumption-maximizing point on the locus of stable populations is associated with an old-age dependency ratio of:

$$\bar{o}\left(G, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\left(\frac{\rho_Y}{\rho_O}\right)\frac{T_Y}{T_M}}. \tag{13}$$

Because o is a monotonically negative function of G, a unique fertility rate, G^m, that generates the equilibrium $(\bar{y}^m, \bar{o}^m)$, given values of the other parameters. At initial equilibria corresponding to higher fertility rates $G^h > G^m$, i.e., those in Region A of the $Z_o(y)$ locus in Fig. 3, small declines in the birth rate are associated with lower steady-state economic dependency. In this case, consumption will be higher at all future dates than in the initial steady state, as evident in Fig. 4.

For countries already in Region B of the locus of stable populations, further fertility reductions will result in a transient demographic dividend that

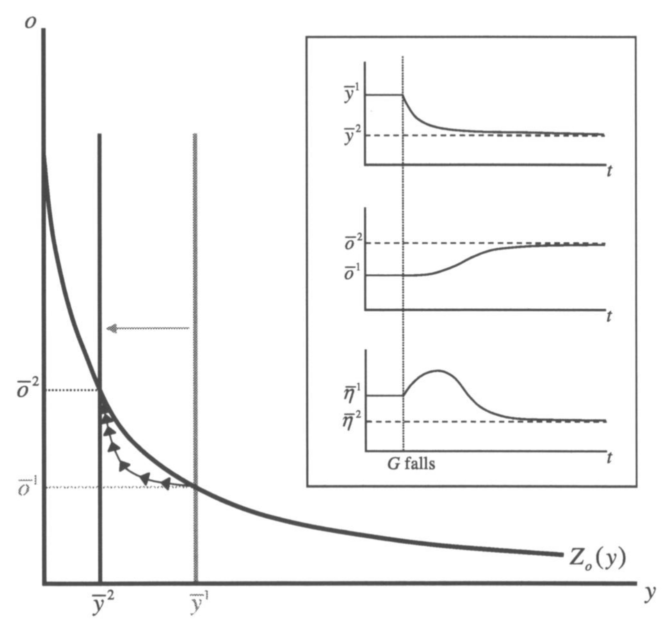


Fig. 2 The effects of a decline in fertility

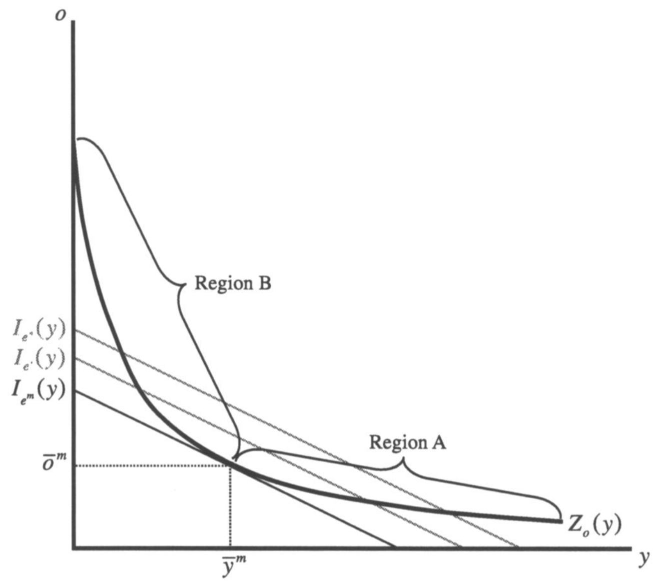


Fig. 3 Economic dependency and the locus of stable populations

completely dissipates due to population aging—higher elderly dependency eventually generates lower steady-state consumption. The numerical results of Hock and Weil (2007) suggest that the fertility rates observed in many

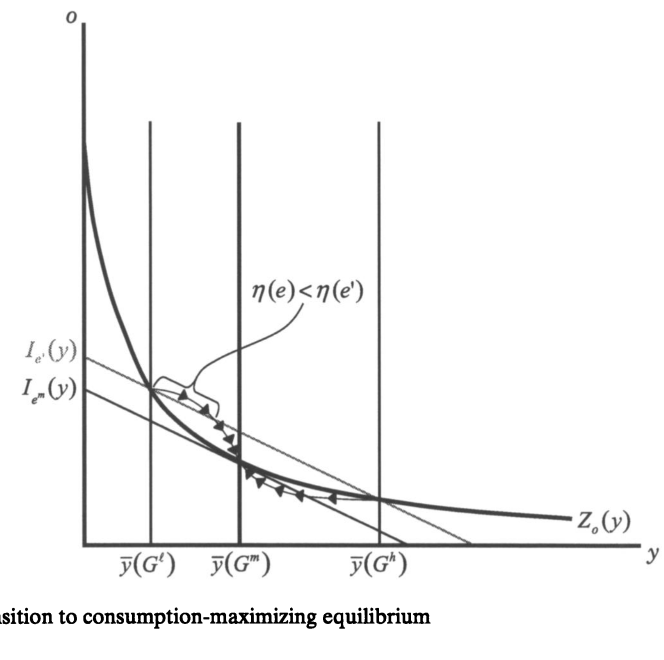


Fig. 4 Transition to consumption-maximizing equilibrium

highly developed countries are already below the consumption-maximizing rate.[8] Hock and Weil (2007) also indicate that increases in fertility rates in developed nations might be associated with steady-state consumption gains as large as 10 percent. However, reversing demographic momentum is also costly in the short run. As shown in Fig. 4, in a country with the same initial economic dependency as implied by G^h, an increase in the fertility rate from some lower value $G^\ell < G^m$ will initially lead to higher economic dependency because of the larger ratio of young to working. The gains in consumption come only later in the transition as the expansion of the workforce reduces the burden of elderly dependency.

3.3 Longevity and consumption

Recent population aging in the developed world is largely the product of the demographic momentum associated with falling fertility over the last half century. However, this process has been amplified by increases in life expectancy, which have become more and more concentrated among the elderly.[9] Holding the other parameters constant, our model generates relatively straightforward dynamics when elderly life expectancy rises. Specifically, the locus of stable populations—the $Z_o(y)$ curve in Fig. 1—shifts upward. The old-age dependency ratio increases monotonically toward the new equilibrium, driving down the aggregate consumption index along the way. With no change in the fertility rate, the youth dependency ratio remains unchanged. Below, we discuss how a multiplier effect might arise if the fertility decisions of the working are affected by the decline in per-capita resource availability associated with rising elderly dependency. Here, we focus on the implications for the consumption-maximizing rate of fertility.

Our model predicts that, ceteris paribus, higher life expectancy after retirement is associated with a higher consumption-maximizing fertility rate, G^m. Greater longevity increases the total cost of supporting the elderly, which raises the marginal rate of demographic transformation. While higher fertility increases equilibrium youth dependency, it translates to marginally larger reductions in old-age dependency on the locus of stable populations. Thus, whereas an increase in fertility from the initial consumption-maximizing rate

[8] The calculations in Hock and Weil (2007) use lifecycle consumption data from Japan and the United States as a benchmark and are derived from a simulated numerical analogue of the model presented here. Based on a range of mortality and retirement profiles that span what is observed in OECD member nations, consumption-maximizing fertility rates are generally well above replacement while actual fertility rates are substantially lower.

[9] In the United States, for example, life expectancy at age 65 has risen by approximately 3.5 years since 1950 (Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds 2005) and is projected to rise by an additional 4 years by 2050 (Lee and Carter 1992). Other highly developed countries have seen reductions in old-age mortality at least as large and can be expected to follow a similar trend in the future (Munnell et al. 2004).

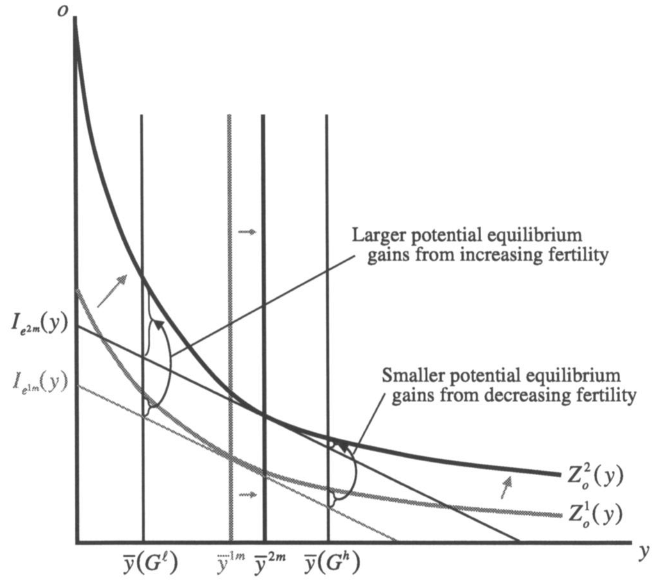


Fig. 5 Increased longevity and consumption possibilities

would lower consumption (through a higher value of y), this effect is more than offset by the eventual decrease in elderly dependency.

For countries with a fertility rate initially higher than G^m, such as G^h in Fig. 5, the consumption-maximizing rate draws closer to the existing fertility rate. As Fig. 5 demonstrates, not only does the best attainable level of consumption fall, so does the potential equilibrium gain from a reduced fertility rate.[10] For countries with a fertility rate lower than G^m (e.g., G^ℓ in Fig. 5), the consumption-maximizing fertility rate moves further away as old-age mortality declines. Although the best attainable level of consumption decreases, the potential equilibrium gains from a higher fertility rate are greater. Hence, the continued decline in childbearing rates exhibited in many highly developed countries is at odds with the fertility strategy that would maximize future equilibrium consumption. As we show below, the burden of old-age dependency itself potentially amplifies the discrepancy between actual and consumption-maximizing fertility.

4 The feedback from aging to fertility

Thus far, we have taken the dramatic declines in fertility in the developed world as given. A variety of mechanisms have been proposed to explain this

[10]This can be established analytically since the marginal rate of demographic substitution rises for any value of y as T_O increases.

phenomenon, including a shrinking gender gap in wages (Galor and Weil 1996), labor market frictions (Adsera 2005), rigid gender norms (de Laat and Sevilla-Sanz 2011), and changing fertility preferences (Goldstein et al. 2003).[11] While shrinking family size norms may be a product of historical declines in fertility (Lutz et al. 2007), we consider an alternative mechanism whereby the age structure may interact with fertility. In pay-as-you-go pension systems, rising elderly dependency reduces the per-capita resources available to the working. Based on standard economic models of childbearing, this may translate into additional reductions in fertility, further population aging, and even lower disposable income.

We develop a basic extension to our model that allows for a dynamic interaction among old-age dependency, fertility, and the age structure. We retain the production structure described above, where workers provide labor inelastically in return for a wage of $W(t)$. Wages are subject to a proportional tax, τ, that funds public transfers to the old and the young. We assume that the elderly are supported entirely through public transfers in the form of a pension system while the young receive support in the form of both public and private expenditures.[12]

We also assume that the government budget is balanced period by period. In reality, of course, governments are able shift to expenditure burdens intertemporally, which, in developed countries, usually involves passing on debt to future generations. Constructing a theory of how governments choose the path of borrowing over time would take us far beyond the scope of the current paper. But, in the context of our model, slowing (or negative) population growth attributable to low fertility can significantly worsen the position of debtor governments by shifting the burden of payment to smaller future cohorts.[13] Thus, the negative effects of aging on fertility, which we examine in the context of balanced budgets, are likely to be exacerbated by pre-existing government debt.

The pension system replaces after-tax wages at a rate, β, while transfers to the young are a proportion, α, of the net pay of the working. Given the aggregate resource constraint (Eq. 1), the balanced-budget tax rate is:

$$\tau(t) = \frac{\alpha y(t) + \beta o(t)}{1 + \alpha y(t) + \beta o(t)}. \tag{14}$$

[11] Kohler et al. (2002) provide a more comprehensive review.

[12] Although somewhat stylized, this distinction mirrors the substantial difference in the pattern of transfers to the old and young observed in highly developed countries. For example, private transfers account for over 61% of the transfer-based consumption of persons under age 20 in the United States, but only 11% of total transfers to persons over age 65 (Mason et al. 2009). One consequence of this assumption is that we abstract from a self-interested old-age support motive for childbearing of the type highlighted by Cigno (1992).

[13] Today, Japan is the best example of a country where falling population size interacts in a toxic fashion with a high level of per-capita government debt.

Higher rates of youth and elderly dependency reduce after-tax income for workers and result in lower per-capita public transfers.

4.1 Privately optimal fertility

At each moment in time, fecund workers allocate their after-tax income to their own consumption and to bearing children. The price of goods and services is numeraire, and we assume that childrearing requires a fixed quantity of market-based childcare services. Consequently, the price of childbearing is $p_n(t) = \xi W(t)$.[14] Assuming a log utility function, fecund workers face the following optimization problem:

$$\max_{c(t),n(t)} \ln[c(t)] + \theta \ln[n(t)] \tag{15}$$

subject to

$$c(t) + p_n(t)n(t) = w(t), \tag{16}$$

where $c(t)$ is the consumption, $n(t)$ is the annual flow of births per fecund worker, and $w(t) = \left[1 - \tau(t)\right] \mathrm{W}(t)$ is the take-home wage of the working. Solving Eqs. 15 and 16 for the fertility rate, we have:

$$n^*(t) = \frac{\psi}{1 + \alpha y(t) + \beta o(t)} = \psi \left[1 - \tau(t)\right], \tag{17}$$

with $\psi = \theta/[\xi(1+\theta)]$. The flow of births per fecund worker satisfies some usual properties: fertility is a positive function of the relative preference for children (θ) and a negative function of childrearing costs (ξ). The fertility rate is also determined by the tax rate, which is, in turn, a function of the existing age structure of the population.

4.2 Socially optimal fertility

The indirect utility function of all agents in the economy is monotonically declining in the tax rate (holding α and β fixed) because consumption of

[14]Given the stochastic nature of aging in our model, parental payments are made upfront into a trust fund that pays for χ units of childcare services per period for the life of the child. To ensure that children always receive the requisite care, an actuarially neutral trust fund will set $\xi = \chi T_Y/(1 - gT_Y)$, where g is the expected rate of wage growth. For the path of trust fund payments to remain bounded, g must be less than $1/T_Y$. Taking 25 years as an upper limit on the average length of youth dependency, g must be less than 4% over the long run, which is in line with observed growth rates.

the young and elderly is proportional to workers' after-tax income. As a result, steady-state social welfare is maximized when the tax rate is minimized. Similar to the analysis in Section 3, the fertility rate that yields the tax rate—minimizing age structure, n_τ^m, is the implicit solution to:

$$\bar{o}\left(n_\tau^m, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\left(\frac{\alpha}{\beta}\right)\frac{T_Y}{T_M}}. \tag{18}$$

However, the privately chosen fertility rate will generally differ from the tax-minimizing rate, depending on preferences and private cost of children. Given that transfers to the elderly represent such a large fraction of public expenditures, the calculations in Hock and Weil (2007) suggest that current fertility rates in most developed nations are well below those that would minimize taxes in equilibrium.

The privately chosen fertility rate also does not take into account the effects of adult fertility choices on the tax rate via the age structure, resulting in a collective action problem of sorts. We can compare privately optimal fertility decisions to the outcome chosen by a social planner who seeks to maximize the steady-state utility of fecund workers, incorporating the effects of fertility decisions on the dependency ratios into the optimization decision. Substituting the budget constraint (Eq. 16) into the objective function (Eq. 15) and taking into account the relationship between the tax rate and the age structure in Eq. 14, the social planner's problem is:

$$\max_n \ln\left[\frac{1}{1+\alpha\bar{y}(n)+\beta\bar{o}(n)} - \xi n\right] + \theta \ln[n]. \tag{19}$$

Rearranging the first-order condition to Eq. 18, we can write:

$$n^*\left(\hat{n}\right) = \hat{n} + \frac{1}{\xi(1+\theta)}\left\{\frac{\left[\alpha\bar{y}'\left(\hat{n}\right)+\beta\bar{o}'\left(\hat{n}\right)\right]\hat{n}}{\left[1+\alpha\bar{y}\left(\hat{n}\right)+\beta\bar{o}\left(\hat{n}\right)\right]^2}\right\}, \tag{20}$$

where $\hat{n}$ is the social planner's fertility rate and $n^*\left(\hat{n}\right)$ is the fertility rate that workers would chose, conditional on the steady-state $(\bar{y}\left(\hat{n}\right), \bar{o}\left(\hat{n}\right))$.

Using the curvature of the locus of stable populations in Eq. 11, the term in braces on the right-hand side of Eq. 20 has the same sign as:

$$\alpha\bar{y}'\left(\hat{n}\right) + \beta\bar{o}'\left(\hat{n}\right) = \bar{y}'\left(\hat{n}\right)\left\{\alpha - \beta\left[\bar{o}\left(\hat{n}\right)\right]^2\left(\frac{T_M}{T_Y}\right)\right\}.$$

Given that $y'(\cdot)$ is positive, the privately chosen fertility rate (conditional on the social planner's preferred age structure) is lower than $\hat{n}$ when:

$$\alpha - \beta\left[\bar{o}\left(\hat{n}\right)\right]^2\left(\frac{T_M}{T_Y}\right) < 0 \tag{21}$$

and is otherwise higher. Consider first the extreme case with no public expenditures on the young. When $\alpha = 0$, the left-hand side of Eq. 21 is unambiguously negative. Workers choose too low a fertility rate relative to the social planner's rate because they fail to account for the future increased burden of aging induced by their fertility decisions. In the opposite case, with no transfers to the elderly ($\beta = 0$), workers choose too high a fertility rate because they do not account for the externality imposed on other workers via publicly funded child care.

With public transfers to both the elderly and the young, the privately chosen rate of fertility coincides with the social planner's rate when Eq. 21 holds with equality. Based on Eq. 18, this convergence occurs if and only if individuals choose the tax-minimizing fertility rate. More generally, when privately chosen fertility differs from the tax-minimizing rate (n_τ^m), the social planner, in essence, balances the indirect utility gain from moving toward the tax-minimizing rate against the direct loss of utility from moving away from the private equilibrium. In particular, it can be shown that $\hat{n}$ lies in the interval between n_τ^m and $\bar{n}^*$, where $\bar{n}^* = n^*(n^*)$ is the steady-state rate of fertility chosen by private actors.[15]

Given the calculations described above, the more likely scenario is that $\bar{n}^* < \hat{n} < n_\tau^m$. In this case, the privately chosen fertility rate would not only be lower than the tax-minimizing rate, it would also be lower than the fertility rate associated with the highest possible level of equilibrium utility among fecund workers. This suggests a long-term gain from pro-natal policies for all agents in the economy. Child subsidies accompanied by additional lump-sum taxes concentrated on the fecund provide a theoretically attractive tool to induce private decision makers to internalize the effects of their fertility decisions on the aggregate tax rate.[16] While any intervention seeking to raise the fertility rate would lead to a temporarily higher tax burden (see the discussion surrounding Fig. 4), we show below that the dynamic response of agents to population aging increases the long-run gap between actual and tax-minimizing fertility. Thus, if they are to be implemented, pro-family policies should be introduced sooner rather than later.

4.3 Dynamics with endogenous fertility

The analysis of Section 3 can easily be augmented to analyze the effects of population aging attributable to increases in elderly life expectancy. The basic mechanism is that changes in elderly dependency, o, affect the tax rate

[15] A proof is available on request.

[16] In the case where all expenditures on children are privately funded, as in van Groezen et al. (2003), subsidies would be pegged to the child's expected lifetime contribution to the pension system. In our model, child subsidies would be determined by the expected reduction in the tax burden associated with childbearing.

and therefore disposable income. Fecund workers respond by reducing their childbearing, which results in an indirect link between the youth and elderly dependency ratios. Putting the privately optimal flow of births per fecund worker, n^*, into the equation of motion of the youth dependency ratio (Eq. 7), the locus of (stable) equilibrium values of y in (y,o)-space is given by:

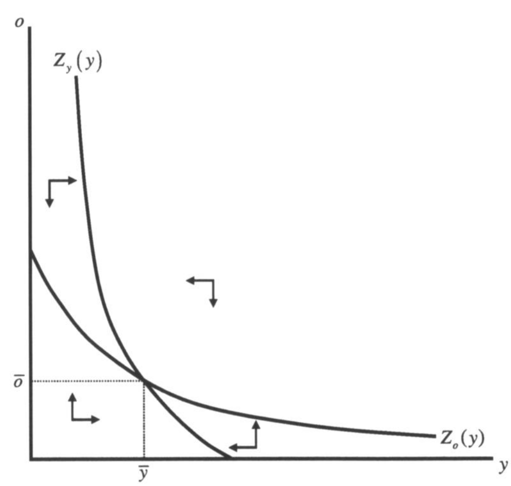
$$o|_{\dot{y}=0} = \frac{1}{\beta}\left[\frac{\phi\psi T_Y}{(1 - T_Y/T_M)\,y + y^2} - (1 + \alpha y)\right] \equiv Z_y(y).$$

The locus is downward-sloping and convex with respect to the origin, asymptotes to infinity as y approaches zero, and eventually crosses the horizontal axis. A single crossing in the positive orthant between the $Z_y(y)$ and $Z_o(y)$ loci implies a globally stable equilibrium in the population dependency ratios, as shown in Fig. 6.

The transition associated with an increase in longevity is depicted in Fig. 7. Lower rates of old-age mortality translate directly into increases in elderly dependency due to longer life expectancy after retirement. If the fertility rate were to remain unchanged, old-age dependency would rise from $\bar{o}^1$ to $\tilde{o}$, and the youth dependency ratio would remain constant at $\bar{y}^1$. However, increases in old-age dependency result in a higher tax rate, given the government's balanced-budget constraint. Because workers will have less disposable income, the rate of childbirth among the fecund will fall.

We showed in Section 3 that declines in fertility lead, ceteris paribus, to increases in old-age dependency. As a result, there is a multiplier effect associated with declines in old-age mortality: increased old-age dependency reduces fertility, which further increases old-age dependency. However, as the fertility rate falls, so does the youth dependency ratio. The consequent reductions in public expenditures on the young acts as a brake on the feedback

Fig. 6 Global dynamics with endogenous fertility



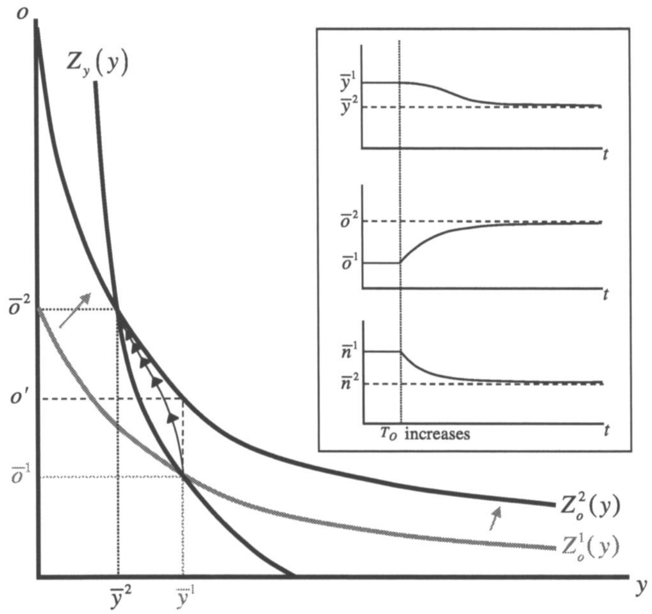


Fig. 7 Endogenous fertility: feedback from increased longevity to the age structure

cycle among rising old-age dependency, rising taxes, and falling fertility.[17] Nonetheless, the net result is a transition to an equilibrium with an old-age dependency ratio of $\bar{o}^2$ that is even higher than $\tilde{o}$. The ratio of young to working falls from $\bar{y}^1$ to $\bar{y}^2$ as a consequence of the fertility response of workers.

As life expectancy among the elderly rises, the tax-minimizing fertility rate will increase per the mechanism discussed in Section 3.3. In highly developed countries where n^* is initially less than n_τ^m, the multiplier effect on longevity means a higher steady-state tax rate than would have resulted had there been no fertility adjustment ($\tilde{\tau}$ in Fig. 7). Thus, the privately optimal fertility response to a decrease in old-age mortality is at odds with the welfare-maximizing response. Workers reduce fertility, resulting in an older age structure, a higher tax burden, and lower utility among future workers and dependents.

4.4 Dynamics with endogenous fertility and endogenous pensions

Throughout the paper, we have assumed that a set of fixed support parameters govern public transfers from the working to the other age groups. In the

[17] Cerda (2005) describes a similar mechanism. However, in the absence of public transfers to the young in his model, low fertility may produce ever-rising payroll tax rates, eventually resulting in a social security melt-down.

model with endogenous fertility, fixed public support ratios (α and β) may be considered the result of a special case of a political bargaining process between the elderly and working. In this section, we consider how the dynamics of the model change if we extend the basic model to allow for a more general class of endogenous bargaining outcomes regarding the pension rate, β.[18] Incorporating richer bargaining dynamics suggests that an endogenous pension rate will exacerbate the consumption-reducing effects of an aging population over the long run.

We assume that the elderly and working compete in a political system that allocates resources based on a Tullock–Hirshleifer contest success function (CSF). Taking α as given, the total pool of resources to be split among the two groups is $wA_M + \beta w A_O$, where w continues to represent the after-tax wage and β is the replacement rate for old-age transfers. Following Hirshleifer (1989), a general form of the CSF to determine the share of resources allocated to the elderly is based on the relative population sizes as follows:

$$\frac{\beta w A_O}{wA_M + \beta w A_O} = \frac{(\pi A_O)^{\mu}}{(A_M)^{\mu} + (\pi A_O)^{\mu}}, \tag{22}$$

where π denotes the relative effectiveness of the elderly at bargaining and $\mu > 0$ represents the "decisiveness parameter." Rearranging Eq. 22, we have:

$$\beta = \pi^{\mu} o^{\mu-1}, \tag{23}$$

from which it is apparent that the constant value of β assumed in our earlier analysis is equivalent to $\mu = 1$. For $0 < \mu < 1$, the replacement rate would depend negatively on the relative size of the old-age population. Although there have been sharp downward adjustments in the replacement rate in response to a burgeoning elderly population, the relationship does not seem credible for a continuous bargaining process over the long run. As μ tends to infinity, the process becomes a majority-takes-all system of resource allocation, which also does not seem realistic. Hence, we focus on the case where $1 \leq \mu < \infty$; if the first inequality is strict, then there are increasing returns to minority voting blocs.

Extending the main dynamic model to allow for an arbitrary decisiveness parameter changes the zero-motion locus for y so that:

$$o|_{\dot{y}=0} = \left(\frac{1}{\pi}\left[\frac{\phi\psi T_Y}{(1 - T_Y/T_M)\, y + y^2} - (1 + \alpha y)\right]\right)^{\frac{1}{\mu}} \equiv \tilde{Z}_y(y). \tag{24}$$

This locus satisfies many of the same basic properties in the general case as in the special case of $\mu = 1$: it is monotonically decreasing, asymptotes to infinity

[18]For analytic simplicity, we continue to treat public support for the young, α, as fixed and exogenous, although the model could accommodate bargaining over both parameters.

as y approaches zero, and crosses the horizontal axis for a sufficiently high value of y.[19]

The decisiveness parameter affects the slope of $\tilde{Z}_y(y)$, which has important implications for the dynamics of fertility and the age structure as longevity increases. As is evident from Fig. 7, a flatter $\tilde{Z}_y(y)$ locus would amplify the population-aging multiplier effect arising from higher elderly life expectancy. Based on Eq. 24, it is straightforward to show that:

$$\operatorname{sgn}\left[\frac{\partial}{\partial\mu}\left|\tilde{Z}'_y(y)\right|\right] = \operatorname{sgn}\left[e^{-1} - \tilde{Z}_y(y)\right]. \tag{25}$$

A larger decisiveness parameter will result in a flatter slope of $\tilde{Z}_y(y)$ for low values of y, but a steeper slope for sufficiently high values of y. Denote the critical value of y at which the relationship changes as y^c.

Consider the differential impact of increased longevity on two countries that are initially identical in their age structure and fundamentals, differing only according to the political decisiveness parameter. If both countries still have a relatively young age structure, so that $y > y^c$, political bargaining may dampen the multiplier on longevity in the country whose pension rate depends positively on the size of the elderly voting bloc ($\mu > 1$) relative to the similar country with a fixed pension rate ($\mu = 1$). Nonetheless, the population will continue to age as life expectancy increases, and all countries will eventually reach a point after which $y < y^c$. Further increases in longevity will lead to a flatter $\tilde{Z}_y(y)$ locus in the country with $\mu > 1$, relative to the country with $\mu = 1$, implying a relatively stronger multiplier effect on old-age mortality. Thus, in the long run, a country whose pension rate depends positively on the relative size of the elderly voting bloc will experience accelerated population aging, as compared to an otherwise similar country with a fixed pension rate.[20]

For a sufficiently high decisiveness parameter, long-run transitions may not be completely smooth because multiple equilibria may appear in aging populations.[21] Figures 8 and 9 show how this process might unfold in response to increases in life expectancy that lead to upward shifts in the $Z_o(y)$ locus from an initially low level. In Fig. 8, an economy starting out in the neighborhood of an initial equilibrium $(\bar{y}^1, \bar{o}^1)$ will move toward $(\bar{y}_a^2, \bar{o}_a^2)$. Another equilibrium becomes supportable as the population ages because a high value of μ implies

[19] In general, the $\tilde{Z}_y(y)$ locus either stays below the horizontal axis or becomes undefined after the initial crossing; we ignore the trivial set of values of μ for which the locus would start to rise after meeting the horizontal axis.

[20] If the decisiveness parameter is less than one, then the multiplier effect on longevity will be amplified in the short run but dampened in the long run.

[21] It was not possible to analytically characterize the parameter configurations resulting in multiple equilibria, but numeric simulation yielded the configurations in Figs. 8 and 9 when using reasonable parameter values and a high value of μ.

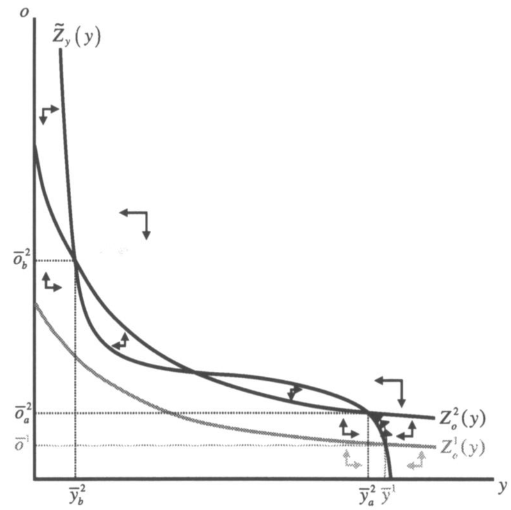


Fig. 8 Endogenous fertility and endogenous pensions: multiple equilibria

larger potential swings in the allocation of resources when relative group sizes change—a second, political multiplier effect. A large and exogenous increase in the relative number of elderly could result in a "melt-down" transition to $(\bar{y}_b^2, \bar{o}_b^2)$ involving drastic increases in pensions and taxes and similarly drastic reductions in fertility. Barring such a shock, $(\bar{y}_a^2, \bar{o}_a^2)$ represents a natural continuation of the demographic and political forces that generated the initial equilibrium.

As elderly life expectancy increases past a certain point, however, the unavoidable consequence of natural demographic and political forces is a

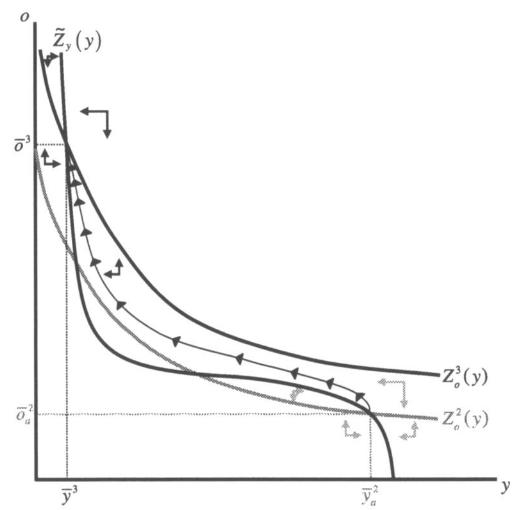


Fig. 9 Endogenous fertility and endogenous pensions: demographic melt-down

melt-down transition. As shown in Fig. 9, increased longevity will eventually shift the $Z_o(y)$ locus upward so that the bottom-right equilibrium disappears. This occurs because the elderly voting bloc becomes so large that it triggers the political multiplier. The surge in pensions paid to the elderly results in a rapid increase in taxes and a sharp slowdown in fertility, which, in turn, leads to further population aging. The system stabilizes at the upper-left equilibrium $(\bar{y}^3, \bar{o}^3)$, which is characterized by far fewer youth and a substantially larger elderly population.

We argued above that fertility rates in the developed world are already below the socially optimal rate. Developed countries face an even more difficult road ahead if a melt-down transition is still to occur. The working may seek to avoid a melt-down by reducing the wage replacement rate of the pension system, which may, for example, be achieved in the context of our model by working to increase their political effectiveness relative to the elderly (decreasing π). However, attempts to reform the pension system are better undertaken before the political multiplier triggers a transition to an economy with a substantially larger and more formidable elderly voting bloc.

5 Conclusion

In this paper, we have analyzed the dynamic evolution of a country's population age structure and fertility rate. Our particular focus was on the feedback from the population age structure to fertility via the channel of old-age dependency. In a country with a high rate of old-age dependency, working-age individuals will see a large fraction of their labor income redistributed to the elderly. In response, working-age individuals will reduce their fertility.

The dynamic nature of our analysis is particularly important because the short- and long-run effects of changes in fertility on dependency differ markedly. In the short run, a reduction in fertility unambiguously lowers a society's dependency burden by reducing the number of children relative to working-age adults. In the long run, reductions in fertility raise a country's old-age dependency ratio, potentially undoing the reduction in youth dependency. Our calculations in Hock and Weil (2007) indicate that developed nations with the lowest levels of fertility have already reached the point where further reductions in fertility raise rather than lower the long-run dependency burden.

We based our analysis on a continuous-time overlapping generations model that allows us to examine the dynamic evolution of the age structure and consumption dependency in response to changes in fertility (in the case where fertility is exogenous) or the joint evolution of fertility, age structure, and dependency in response to exogenous shocks such as old-age mortality. We have shown that, for countries already below the level of fertility that maximizes consumption in the steady state, the actual and optimal responses of fertility to changes in old-age mortality have opposite signs. In response to greater elderly longevity, such countries will effectively dig themselves into a deeper demographic hole by reducing fertility in order to maintain short-run consumption. We then extended the model to allow for a political interaction between members of the elderly and working-age generations in order to determine the magnitude of government transfers to the elderly. As the fraction of the population that is elderly increases, we expect the generosity of such transfers to expand, amplifying the feedback from population aging to lower fertility.

Our model is somewhat stylized in the interests of analytic tractability and ease of exposition. It could be extended along two major dimensions to improve its realism and forecast accuracy. First, it could be useful to expand the number of age groups beyond the three basic ones considered here. Moving beyond three age groups, however, would prevent us from examining the dynamics of the model analytically, forcing us to take a computational approach along the lines of Auerbach and Kotlikoff (1987), Grafenhofer et al. (2007), and Hock and Weil (2007). Second, the model could be extended to incorporate intertemporal optimization on the part of households, which would entail shifting consumption between periods in response to projected changes in demographics. Such intertemporal shifting, whether through investment in physical capital or purchase of foreign assets, could allow agents to save for their own old age or the country as a whole to save for periods of high old-age dependency. Although intertemporal reallocation would have little effect on the model's steady state, it would significantly influence the model's dynamics. As discussed in Cutler et al. (1990) and Elmendorf and Sheiner (2000), the ability of the economy as a whole to insulate itself against demographic shocks by amassing extra capital is limited by both depreciation and capital's declining marginal product. Attempts to smooth consumption in the face of demographic shocks may also spur a run-up of the price of capital assets when everyone is trying to save and a corresponding asset melt-down when everyone is trying to dissave.[22]

We suspect that the extensions discussed above would not alter the fundamental predictions of our model, which are relatively grim. Demographers have been concerned for decades about persistently below-replacement fertility in the developed world. Moreover, government interventions seeking to bolster the fertility rate have met with mixed success (Grant et al. 2004). Over the next several decades, as the effect of low fertility on consumption possibilities shifts from positive to negative, it will become increasingly difficult to muster support for pro-family policies in view of the associated increases in short-term youth dependency. In the absence of such efforts, our results suggest that the fiscal burden of elderly dependency will result in further declines in fertility, which will produce additional decreases in consumption over the long run.

[22] See Abel (2003) and Lim and Weil (2003).

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