

# Mortality change, the uncertainty effect, and retirement

Sebnem Kalemli-Ozcan · David N. Weil

Published online: 27 February 2010  
© Springer Science+Business Media, LLC 2010

**Abstract** We examine the role of declining mortality in explaining the rise of retirement over the course of the twentieth century. We construct a model in which individuals make labor/leisure choices over their lifetimes subject to uncertainty about their dates of death. In an environment with high mortality, an individual who saves for retirement faces a high risk of dying before he can enjoy his planned leisure. In this case, the optimal plan is for people to work until they die. As mortality falls, however, it becomes optimal to plan, and save for, retirement. We analyze our model using two mathematical formulations of the survival function as well as data on actual changes in the US life table over the last century, and show that this “uncertainty effect” of declining mortality would have more than outweighed the “horizon effect” by which rising life expectancy would have led to later retirement.

**Keywords** Life cycle model · Retirement · Annuities

**JEL Classification** E21 · I12 · J11 · J26

## 1 Introduction

One of the most dramatic economic changes in the last 100 years has been the rise in retirement as an important stage of life. At the beginning of the twentieth century, retirement was a rarity. Many people didn’t live into old age, and most of those who did continued to work until shortly before death. By the end of the century, the vast majority of workers could expect to experience a prolonged period of healthy leisure after their working years were over.

---

S. Kalemli-Ozcan  
University of Houston, Houston, TX, USA

S. Kalemli-Ozcan · D. N. Weil  
NBER, Cambridge, MA, USA

D. N. Weil (✉)  
Brown University, Providence, RI, USA

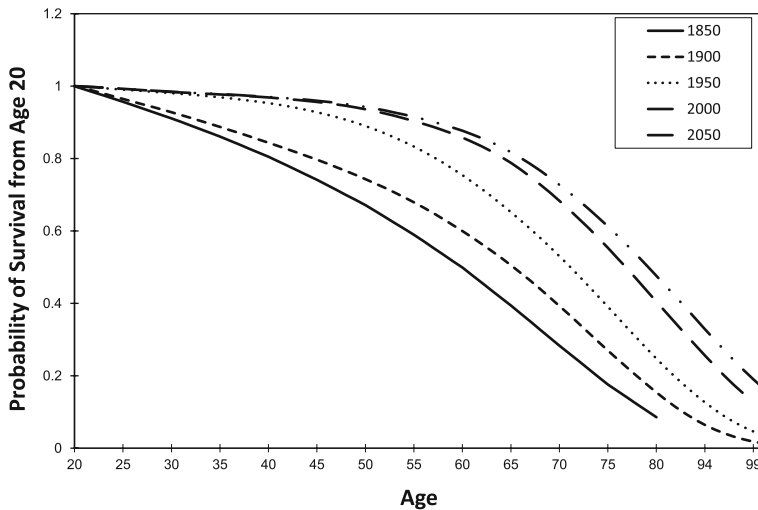
Needless to say, the economic repercussions of this change in the life cycle pattern of labor supply have been enormous. Since a significant fraction of consumption during retirement is publicly funded, the growth in retirement has strained government budgets—a phenomenon which will soon be exacerbated by population aging (Weil 2008). Meanwhile, private funding of anticipated retirements has led to the accumulation of vast pools of capital. Exactly what fraction of current capital accumulation can be attributed to life cycle savings is an issue of contentious debate. Modigliani's (1988) estimate is that as much as 80% of wealth can be attributed to life cycle saving, while Kotlikoff and Summers (1988) estimate that it is only 20%. According to Lee (2001), the fraction of wealth attributable to life cycle saving in the US doubled between 1900 and 1990.

Explaining the rise in retirement has been a major endeavor for economists in the last several decades. Three prominent explanations can be discerned. The first is that public pension programs, such as Social Security in the United States, have been instrumental in pushing workers out of the labor force, particularly through high implicit rates of taxation on wage income earned at older ages (Gruber and Wise 1998). The second explanation is that rising lifetime income has led workers to optimally choose a larger period of leisure at the end of life (Costa 1998). The final explanation is that changes in the technology of production have lowered the productivity of older workers, leading employers to seek to get rid of them (Graebner 1980; Sala-i-Martin 1996). None of these explanations, either taken separately or as a group, has been completely successful in accounting for the rise in retirement. This paper proposes a new explanation, which we call the “uncertainty effect,” for the rise in retirement. Our explanation is not meant to be a complete substitute for the factors discussed above, but rather an addition to the set of usual suspects that must be considered in explaining the rise of retirement.

In our model, the driving force behind the change in the life-cycle pattern of labor supply is a change in the pattern of mortality. Individuals make labor/leisure choices over their lifetimes subject to uncertainty about their dates of death. In an environment in which uncertainty surrounding the date of death is high, an individual who saved up for retirement would face a high risk of dying before he could enjoy his planned leisure. In this case, the optimal plan is for individuals to work until they die. As uncertainty regarding the date of death falls, individuals will find it optimal to plan, and save for, retirement. As we show below, a salient feature of changes in mortality that have occurred in developed world over the last century is that as life expectancy has risen, uncertainty regarding the date of death has fallen. The effect of falling mortality on labor supply complements several other effects of falling mortality, most notably on human capital investment and fertility, that have been analyzed in recent literature.<sup>1</sup>

The rest of our paper is structured as follows. In Sect. 2 two, we briefly examine historical data on mortality and retirement, and also discuss at greater length some of the other explanations for the rise of retirement. Sect. 3 presents and solves our model of endogenous retirement. The different parts of that section use three different formulations of the survival function: exponential, for mathematical simplicity; a realistic formulation based on the work of Boucekkine et al. (2002), which we can solve analytically; and a numerical analysis based on actual data from US life tables. Our key result is that in all these cases, increases in life expectancy can lead declines, in some cases discontinuous, in the age of retirement. Sect. 4 looks at the paths of consumption and asset accumulation implied by the retirement behavior

<sup>1</sup> See Meltzer (1992), Ehrlich and Lui (1991), Eckstein et al. (1999), Kalemli-Ozcan (2003), and Hazan (forthcoming).



**Fig. 1** Male survival probability in the US

studied in Sect. 3. Sect. 5 looks at the interaction between our new channel and one of the existing explanations for the rise in retirement, specifically rising income. Sect. 6 concludes.

## 2 Historical data

### 2.1 Mortality

Falling mortality has been one of the most significant aspects of the process of economic growth over the last several centuries. Male life expectancy at birth in the United States rose from 40.2 in 1850 to 47.8 in 1900 and 75.2 in 2005. The most significant component of mortality decline has been the reduction in infant and child deaths. Obviously, this aspect of the mortality decline is not related to the issues of retirement saving that we discuss in this paper. Although the decline in adult mortality has not been quite as dramatic as that for children, it has nevertheless been a significant part of the story of economic growth over the last century or more. Figure 1 shows the number of survivors from a cohort of 20-year-olds who would be alive at different ages, using life tables from the United States for 1850, 1900, 1950, and 2000, along with a forecast for 2050. In 1850, the probability that a 20-year-old would reach age 65 was roughly 40%. By 2000, it was roughly 80%. The total number of expected remaining years of life for a 20-year-old male rose from 38 to 55 over this period.<sup>2</sup>

<sup>2</sup> Sources: Haines (1998), Keyfitz and Flieger (1990). These are period rather than cohort life tables because the former are available for a much longer period of time. We examine male mortality because the full-time labor force was dominated by men in the age cohorts over which retirement rose to prominence. Further, prior to the rise of retirement, widows were generally supported by familial transfers rather than their own saving. For example, in 1940, only 18.4% of widows in the U.S. lived alone, while 58.7% lived with their adult children (McGarry and Schoeni 2000). Looking at joint survival probabilities paints a picture similar to what is seen in male survival. Using the 1850 male and female life tables and assuming independence in mortality, the probability that a couple consisting of a 20-year old man and a 20-year old woman would have at least one member survive to age 70 was .57; using the 2000 life table, the probability was .94.

## 2.2 Retirement

Paralleling the reduction in male adult mortality has been a massive increase in retirement. In 1930, the labor force participation rate for men aged 65 and over was 58%. By 1985, the rate had fallen to 16%, and has since remained roughly constant.<sup>3</sup> The trend in retirement prior to the Great Depression has been a subject of controversy among historians. [Ransom and Sutch \(1986, 1988\)](#) claim that the labor force participation rate for men over 60 was roughly constant between 1870 and 1930, while [Costa \(1998\)](#) and [Moen \(1994\)](#) argue that labor force participation for older men had been declining since the late nineteenth century.

Examining the labor force participation of the elderly does not tell the full story, of course, because, as shown above, a large fraction of the population never lived to this age. [Carter and Sutch \(1996\)](#) estimate that at the beginning of the twentieth century, a 55-year-old man had only a 21.5% probability of retiring before he died, excluding “death-bed retirement” associated with illness in the last weeks of life.

Another way to demonstrate the dramatic rise in retirement is to look at how the expected number of years that an individual would spend retired has changed over time. This measure incorporates changes in both mortality and labor force behavior. The expected period of retirement rose from 2.65 years (6.1% of expected adult life) for the cohort born in 1830 to 5.50 years (11.6% of expected adult life) for the cohort born in 1880 and 13.13 years (23.9% of expected adult life) for the cohort born in 1930 ([Lee 2001](#)). Similarly, Hazan (forthcoming) calculates that while life expectancy at age five rose from 52.5 years for men born in 1850 to 66.7 years for men born in 1930, expected years in the labor market only rose from 34.9 to 40.9 over the same period.<sup>4</sup>

Although we focus our analysis on data from the United States, the rise of retirement has been universal among developed countries. In the United Kingdom, for example, the labor force participation rate for men aged 65 and over fell from 73.6% in 1881 to 58.9% in 1921, 18.6% in 1973, and 8.9% in 2003. In Japan, the labor force participation rate for men aged 65–69 fell from 70% in 1960 to 47% in 2002. In France in the four decades following 1960, male life expectancy rose from 67.6 to 74.2 while the average age of retirement fell from 64.5 to 59.2.<sup>5</sup>

## 2.3 Explaining the rise in retirement

While we will argue that the decline in mortality is one explanation for the fall in labor force participation of the elderly, it would be foolish to argue that it is the only one. Since we cannot incorporate all of these effects into a single model, our approach will be to ask how large a change in retirement could plausibly be explained by the mortality effect alone, in a model where the other factors are not present. In the rest of this section, however, we briefly discuss three other channels.

The most obvious alternative explanation for the decline in labor force participation of the elderly is the growth of public pension programs such as Social Security in the United States. The evidence regarding how much of the rise in retirement is explained by public pension

<sup>3</sup> Such a change could theoretically be due to a changing age distribution of the population over 65 in the presence of constant age-specific participation rates. This is not the case, however: age specific participation rates have also fallen dramatically. See [Costa \(1998\)](#) and [Lumsdaine and Wise \(1994\)](#).

<sup>4</sup> Hazan also points out that because working hours per year declined over this period, the change in expected total working hours over the lifetime was negative.

<sup>5</sup> [Gruber and Wise \(1999\)](#), [Matthews et al. \(1982\)](#), [Yashiro and Oshi \(1999\)](#), [Pestieau \(2003\)](#), and [OECD \(2004\)](#).

programs is mixed, but generally suggests that such programs are not the dominant cause. Numerous studies find statistically significant effects of Social Security on the likelihood or retiring at certain specific ages. For example, the hazard of retiring at the “normal retirement age” of 65 was 20%, and the hazard of retiring at the first eligible age of 62 was 14%, in the United States in the 1990s. However, these studies generally imply a small impact of changes in Social Security benefits on average age of retirement.<sup>6</sup> Duval (2004), examining a panel of OECD countries, finds that changes in public pension programs as well as other social programs such as unemployment and disability insurance that are used as *de facto* early retirement schemes explain 31% of the decline in labor force participation over the last three decades. Similarly, Johnson (2001), using a panel of data from 13 countries, finds that old age insurance explains only 11% of the drop in labor force participation by men aged 60–64 over the period 1950–2000. By contrast, Gruber and Wise (1999), examining a cross section of OECD countries, argue that differences in public pension provisions are the dominant explanation for cross-country differences in labor force participation of men aged 55–64.

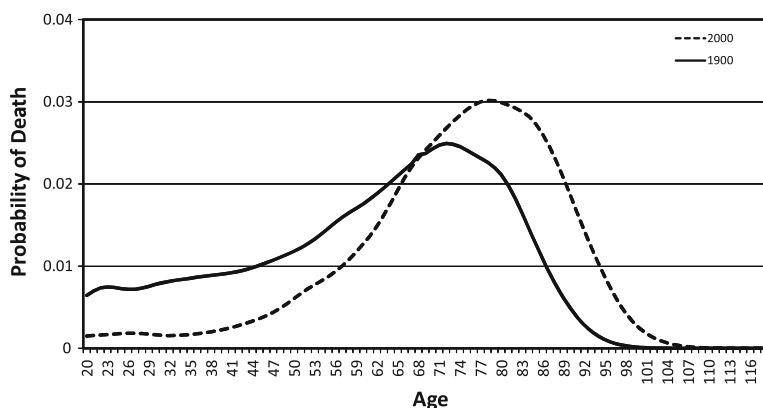
A second explanation for the rise in retirement focuses on the income effect of higher wages. In a taste-based model of retirement, the association of higher income per capita with longer retirement comes about because people in richer countries choose to spend a higher fraction of their income on retirement. When the wage increases, there is an income effect, which leads to the purchase of more of all normal goods, including leisure, and a substitution effect, which works to reduce leisure by raising its opportunity cost. For higher income to lead to longer retirement, not only must the income effect dominate, but there must also be some reason why this leisure must be taken at the end of life, rather than spread out evenly. The best argument for this phenomenon is that there is some kind of non-convexity in the enjoyment of leisure; a simple example would be that only people with full time leisure can move to Florida.<sup>7</sup>

A third explanation for the increase in retirement looks toward production technology. This explanation starts with the notion that older workers are less able to do certain tasks than younger ones. If the nature of production shifts toward those tasks over time, then the labor force participation of older people should go down. The argument that the nature of production has changed is presented in Graebner (1980) and Moen (1988). Moen argues that the decline of agriculture as a source of employment led to an increasing physical separation of the home and the workplace, making a gradual withdrawal from the labor force no longer possible.<sup>8</sup> Similarly, as factory labor and hourly wages replaced piece rate work, it became harder for an older employee to work at his own pace. Graebner argues that it is the nature of

<sup>6</sup> See Samwick (1998) and Coile and Gruber (2007) for a discussion.

<sup>7</sup> This is the argument of Costa (1998). Similarly, Fields and Mitchell (1984) argue that in the U.S. setting, retirement is well modeled as “a choice based on balancing the monetary gains from continued work versus leisure forgone,” rather than stemming from mandatory retirement regulations or ill health. Mandatory retirement covered only a minority of workers during the period at which they look, and they estimate that it was binding in the retirement decisions of only two or three percent of retiring workers. The Age Discrimination in Employment Act of 1978 and subsequent legislation, which have virtually eliminated mandatory retirement, have had little effect on actual retirement patterns.

<sup>8</sup> Long (1958) rejects the argument that urbanization was the cause in the decline of labor force participation for men over 65 over the period 1890–1950. The rate of labor force participation was higher in rural areas, but the decline was sharp in both areas: 5.4% per decade in rural areas, 4.4% per decade in urban areas. The rate of decline for the country as a whole was 5.4% per decade, indicating that there was some effect of movement from rural to urban areas.



**Fig. 2** Probability of death conditional on reaching 20

large scale, technologically advanced production to demand standardized workers.<sup>9</sup> On the other hand, the declining importance of physical strength in the production process (see Galor and Weil 1996) should have *ceteris paribus* raised the average age of retirement under the assumption that an individual's physical abilities decay more rapidly than mental abilities.

As an alternative to changes in productive technology, the cause of the increase in retirement could theoretically lie in changes to the technology of health and longevity. If the fraction of older people who are unable to perform relevant tasks increases, labor force participation rates should go down. The evidence, however, points in exactly the opposite direction: as retirement has increased the health of the elderly has improved (Fields and Mitchell 1984). In 1994, 89 percent of those between 65 and 74 reported "no disability whatsoever."<sup>10</sup> While it is theoretically possible that this improvement in the health of the elderly was *caused* by the same increase in retirement that we are trying to explain, the evidence, once again, goes in the other direction. Snyder and Evans (2006) find that an exogenous and unexpected reduction in Social Security benefits to the so-called notch generation (those born between 1917 and 1921) in the United States actually *lowered* mortality. Snyder and Evans attribute this effect to the observed higher labor force participation rate for those who received lower benefits.

### 3 Model

The explanation for rising retirement considered here looks to the very reduction in mortality discussed above. Specifically, we focus on the change in the uncertainty regarding mortality. This point can be seen most clearly by looking at Fig. 2. The figure shows the probability of dying at different ages, conditional on having reached age 20, using the cross-sectional male life tables for 1900 and 2000. The fact that the mean age of death goes up (from 61.2 to 73.9) is hardly surprising. What is more interesting is that the standard deviation of the

<sup>9</sup> A different mechanism by which technological change can lead to retirement is by making old workers obsolete, either because older workers are slower at learning new productive techniques, or because it is not worthwhile for older workers to learn new techniques, since they will have less time in the labor force in which to employ them. Graebner (1980) documents the widespread notion that retirement resulted because it was easier to train new workers than to retrain old ones. Both of these arguments suggest that retirement should be related not to the level of technology, but to its rate of change.

<sup>10</sup> For more on this point see Costa (1998).

age of death falls, from 18.1 to 14.2. And of course the coefficient of variation in the age of death (the standard deviation divided by the mean) falls by even more, from .30 to .19. By 2050, the mean age of death is expected to rise further, to 76.5, while the standard deviation will also rise, to 15.4. The coefficient of variation will remain almost exactly constant.

How should declining mortality affect the retirement age? Two different forces are at work. First, and most intuitively, longer life would be expected to increase the number of years during which an individual plans to work, simply because longer life means that there will be more years of consumption which need to be paid for. We call this channel the “horizon effect” of increased life expectancy. But a second effect arises from the fact that increases in life expectancy may reduce the uncertainty surrounding whether a person will live into old age. Where mortality is high, individuals are unlikely to live into old age, and so any savings for a planned retirement would likely be wasted. The optimal program in such circumstances would be to plan continue working in old age, should such an eventuality arise. In this case, a reduction in mortality, by making survival into old age more likely, can reduce the planned age of retirement. We call this second channel the “uncertainty effect.”

Whether the uncertainty effect or the horizon effect of increased life expectancy is more important depends on the manner in which life expectancy increases, as well as on the nature of the individual’s optimization problem. We set up the general problem in Sect. 3.1, and also show that if life expectancy rises in the context of certainty (or when there is a full set of annuities available), retirement age rises as well. We then consider three different forms of the survival function, in which we can model increases in life expenditure. We begin in Sect. 3.2 by using an exponential model of survival, which is unrealistic but mathematically simple. In Sect. 3.2 we apply a more realistic two-parameter model of survival introduced by Boucekkine et al. (2002). Finally, in Sect. 3.3 we use actual data on changes in adult survival in the United States.

### 3.1 Setup of the problem

Consider an individual who receives utility from both consumption and leisure. We assume that there are only two possible levels of labor supply: a person may either be fully employed or retired. We also assume that once a person retires, he may not re-enter the labor force.<sup>11</sup> Let  $\gamma$  represent the increment to utility from leisure due to retirement (that is, the difference between utility from leisure before retirement and after). For convenience, utility from leisure and consumption are taken to be separable. The instantaneous utility function is<sup>12</sup>

$$\begin{aligned} U &= \ln(c) && \text{if working} \\ &= \ln(c) + \gamma && \text{if retired.} \end{aligned} \quad (1)$$

An important feature of this utility function is that marginal utility of retirement leisure does not decline with the length of retirement—that is, there are no decreasing returns to retirement.

The assumption that utility from consumption is given by the log function is made so that changes in wages will not, by themselves, affect the optimal age of retirement. In general,

<sup>11</sup> The question of why retirement usually takes the form of a sudden and complete withdrawal from the labor force is a difficult one, and we do not address it here.

<sup>12</sup> To be clear, we are starting with more general utility function of the form  $U = \ln(c) + f(n)$ , where  $f(n)$ , the subutility from leisure, has the standard properties including a positive first derivative and negative second derivative. Our assumption is that  $n$  can take only two values,  $n_w$  for people who are working and  $n_r$  for people who are retired. Thus  $\gamma = f(n_r) - f(n_w)$ .

changes in wages (holding interest rates and mortality constant) will have both income and substitution effects on the demand for end-of-life leisure. With log utility these just balance each other, and so the derivative of the optimal retirement age with respect to the wage is zero. If utility is more curved than the log function, then the income effect will dominate, and increases in wages will lead to a reduction in the optimal retirement age. We explore this effect further in Sect. 5.

Future utility is discounted at rate  $\theta$ . Thus we can write lifetime utility as,

$$U = \int_0^T e^{-\theta x} [\ln(c(x))] dx + \int_R^T e^{-\theta x} [\gamma] dx, \quad (2)$$

where  $R$  is the age of retirement,  $T$  is the age of death, and  $R \leq T$ . We take age zero to be the beginning of adulthood. We abstract from fertility and child-rearing costs.

In the case where the date of death is uncertain, expected lifetime utility is given by

$$E(U) = \int_0^\infty e^{-\theta x} P(x) [\ln(c(x))] dx + \int_R^\infty e^{-\theta x} P(x) [\gamma] dx, \quad (3)$$

where  $P(x)$  is the probability of being alive at age  $x$ . Note that in this case,  $R$  is the planned age of retirement, but an individual who dies before age  $R$  will not experience any retirement at all. Since the only uncertainty that we admit to the model is about the date of death, and since this uncertainty is not resolved until it is too late to do anything about it, individuals will form time-consistent plans for consumption and retirement at the beginning of their lives.

Individuals who are working receive a wage of  $w$ . The real interest rate is  $r$ . We impose the condition that assets must be non-negative at all times. Although the exact justification for such a debt constraint is not relevant for our purposes, a reasonable assumption is that it results from capital market imperfections that prevent individuals from borrowing against their future labor income. Carroll (2001, Fig. 2) shows that non-collateralized debt is indeed very small relative to lifetime income. Solving our problem under the assumption that there is a small but non-zero limit on debt yields nearly identical results.

### 3.1.1 Optimal retirement with no uncertainty

We begin by considering the case where the date of death,  $T$ , is known with certainty. The individual's lifetime budget constraint is

$$\int_0^R e^{-rx} [w] dx = \int_0^T e^{-rx} [c(x)] dx. \quad (4)$$

The individual will maximize his utility subject to his budget constraint by choosing a path of lifetime consumption,  $c(x)$ , and an endogenous retirement age,  $R$ . The first order condition with respect to consumption implies,

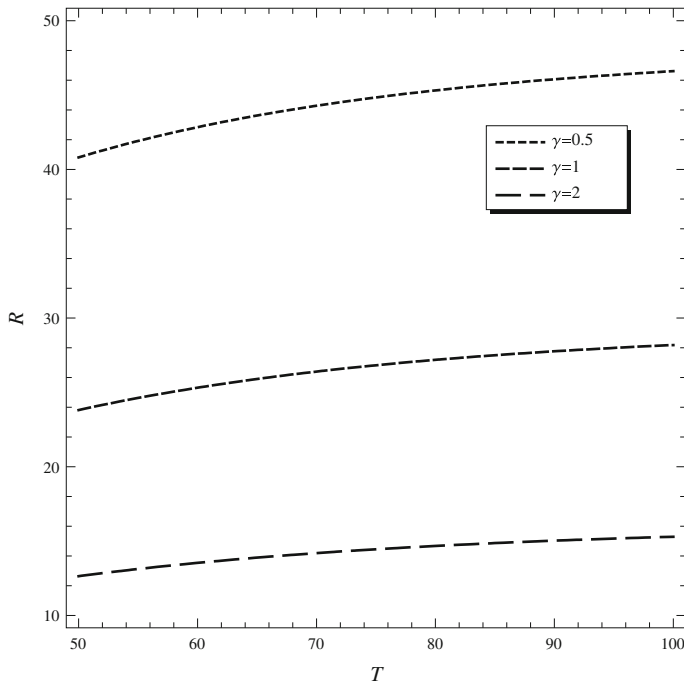
$$\dot{c}(x) = [r - \theta]c(x) \quad (5)$$

The equation above together with the budget constraint will give us the initial consumption,  $c_0$ ,

$$c_0 = \frac{w\theta}{r} \frac{(1 - e^{-rR})}{(1 - e^{-\theta T})}. \quad (6)$$

The first order condition with respect to the endogenous retirement age and Eq. 6 give us an implicit equation for the optimal retirement age,  $R$ ,





**Fig. 3** Optimal retirement under certainty

$$\gamma = \frac{r(1 - e^{-\theta T})}{\theta e^{(r-\theta)R}(1 - e^{-rR})} \quad (7)$$

The above equation will obviously only hold true if  $R \leq T$ . The alternative is that the individual is at a corner solution, where  $R = T$ .

The derivative of  $R$  with respect to  $T$  is

$$\frac{dR}{dT} = \frac{r}{\gamma e^{\theta(T-R)}[\theta + (r - \theta)e^{rR}]}. \quad (8)$$

It can be shown that this derivative is positive if optimum  $R < T$ . Thus in the case where there is no uncertainty, increases in life expectancy will lead to increases in the retirement age.<sup>13</sup> Figure 3 shows how retirement age rises with increases in life expectancy for different values of the utility of retirement,  $\gamma$ . The other parameters are  $r = .06$  and  $\theta = .03$ .

### 3.1.2 Optimal retirement in the presence of annuities

We begin by considering uncertain survival in the case of full annuities. Following Yaari (1965), we assume that the individual has access to “actuarial notes,” savings or borrowings which stay on the books until the consumer dies, in which case they are automatically canceled. Saving in the form of actuarial notes takes the form of conventional annuities. Borrowing in the form of actuarial notes is equivalent to conventional borrowing combined with term life insurance. The budget constraint for the individual is

<sup>13</sup> The derivative in Eq. 8 is positive since the necessary condition for the second order condition to maximization to hold is that  $\theta > (r - \theta)e^{rR}$ .

$$\int_0^R e^{-rx} P(x)[w]dx = \int_0^T e^{-rx} P(x)[c(x)]dx. \quad (9)$$

Because mortality raises the discount that individuals apply to the future and the annuity interest rate by the same amount, the growth rate of consumption is the same as in the case of no uncertainty, and so consumption at time zero is given by

$$c_0 = \frac{w \int_0^R e^{-rx} P(x)dx}{\int_0^\infty e^{-\theta x} P(x)dx}. \quad (10)$$

Along with the first order condition with respect to the endogenous retirement age, this expression gives us an implicit equation for the optimal retirement age,  $R$

$$\gamma = \frac{\int_0^\infty e^{-\theta x} P(x)dx}{e^{(r-\theta)R} \int_0^R e^{-rx} P(x)dx} \quad (11)$$

According to this equation, a change in  $P(x)$  will affect the optimal age of retirement only if it affects the ratio of the two integral terms. If the change in mortality raises the integral term in the numerator more than the integral term in the denominator (holding  $R$  constant), then the optimal retirement age will have to rise. For example, an increase in  $P(x)$  that affected survival after the planned date of retirement would raise the term in the numerator but not affect the term in the denominator, thus raising the retirement age.<sup>14</sup> An increase in survival concentrated in the pre-retirement years would lower the optimal retirement age.

### 3.2 Optimal retirement with exponential survival

We now turn to the case where the date of death is uncertain. Improvements in mortality take the form of changes in age-specific death probabilities. We begin by considering the case where individuals face a constant probability of death, so that the survival function is exponential. Although not realistic, this case allows for presentation of some of our basic results in a simplified setting. We then turn to consider a more realistic formulation of the survival function.

Let individuals have constant probability  $\rho$  of dying. The probability of being alive at age  $x$  is  $P(x) = e^{-\rho x}$ . Thus expected utility at the beginning of life is,<sup>15</sup>

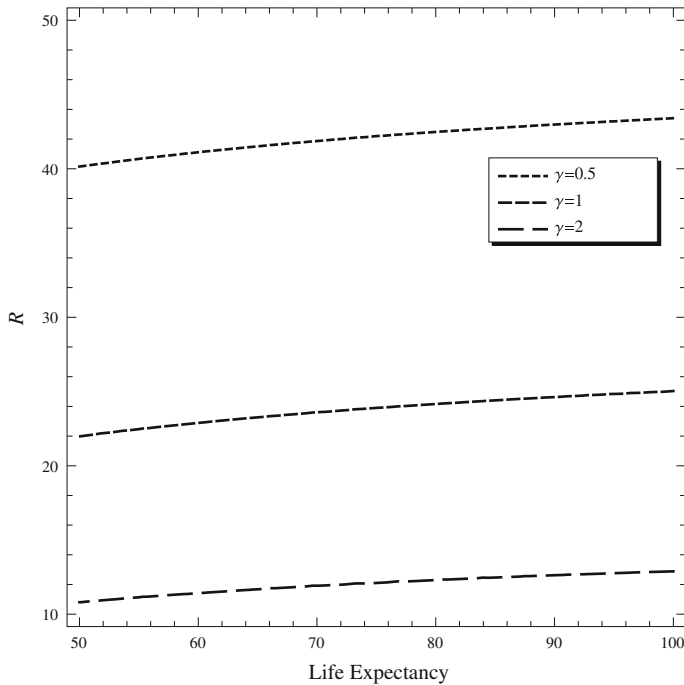
$$V = \int_0^\infty e^{-(\theta+\rho)x} [\ln(c(x))]dx + \int_R^\infty e^{-(\theta+\rho)x} [\gamma]dx.$$

#### 3.2.1 Exponential survival with annuities

As shown above, in the presence of annuities, the effect of change in life expectancy on optimal retirement depends on the particular change in the  $P(x)$  function. For the case of exponential survival, the first order condition (11) becomes

<sup>14</sup> Note that the case of no uncertainty considered above is simply a special case of this result where  $P(x) = 1$  for ages up to  $T$  and zero thereafter. If planned retirement is at some age before  $T$ , then an increase in lifespan will raise the integral term in the numerator and thus retirement age.

<sup>15</sup> Throughout this paper, we are concerned only with how mortality affects retirement and saving decisions, so we think of “birth” as being the beginning of working life. When we consider actual life tables, we begin our analysis assuming that the individual has already survived to age 20.



**Fig. 4** Optimal retirement with exponential survival

$$\gamma = \frac{r + \rho}{(\theta + \rho)e^{(r-\theta)R}(1 - e^{-(r+\rho)R})} \quad (12)$$

The assumption that  $r > \theta$  is sufficient to guarantee that  $\frac{dR}{d\rho} < 0$ . Thus with exponential survival in the presence of annuities, increases in life expectancy raise the retirement age, just as they do under certainty. Figure 4 shows the implied retirement age for different values of  $\gamma$  along with the same values of  $r$  and  $\theta$  used in Fig. 3. Retirement ages are slightly lower in the model with uncertainty and annuities than under certainty about the date of death, but the effect of reductions in mortality is quite similar. For example, for the parameters  $\gamma = 1$ ,  $r = .06$ , and  $\theta = .03$ , retirement age is 23.8 when lifespan is 50 and 28.2 when lifespan is 100 under certainty, vs. 22.0 when life expectancy is 50 and 25.0 when life expectancy is 100 in a model with annuities (all ages are measured from the beginning of adulthood). Thus both qualitatively and quantitatively, the model with uncertain mortality and full annuitization is quite similar to the model with no uncertainty, at least for this form of the mortality function.

### 3.2.2 Exponential survival with no annuities and no liquidity constraints

As mentioned above, we assume that assets must always be non-negative. This in turn means that individuals may die holding positive wealth (although not necessarily, since an individual may choose to work until he dies and hold no assets). Thus accidental bequests may be generated. We ignore these bequests, assuming in effect that they are thrown away. If individuals place positive value on bequests, but still value them less than their own consumption, then our main qualitative results will continue to hold.

Optimization problems with liquidity constraints are often difficult to solve using standard analytic techniques. In this section we make the model tractable by assuming that  $r > \rho + \theta$ . This condition guarantees that individuals will have rising consumption paths over the course of their lifetimes, and thus will hold positive assets at all times. In other words, we choose parameters such that the liquidity constraint never binds. In the next section, where liquidity constraints are considered explicitly, we can relax this constraint on parameters.

We can write the individual's lifetime budget constraint as,

$$\int_0^R e^{-rx} [w] dx = \int_0^\infty e^{-rx} [c(x)] dx. \quad (13)$$

The individual will maximize his expected utility subject to his budget constraint by choosing a path of lifetime consumption,  $c(x)$ , and an endogenous retirement age,  $R$ . The first order condition with respect to consumption implies,

$$\dot{c}(x) = [r - \theta - \rho]c(x) \quad (14)$$

The equation above together with the budget constraint will give us the initial consumption,  $c_0$ ,

$$c_0 = \frac{w(\theta + \rho)}{r} \left[ 1 - e^{-rR} \right]. \quad (15)$$

We can now re-write the individual's expected lifetime utility as a function of planned retirement age,  $R$ . Ignoring terms that do not contain  $R$ ,

$$V(R) = \frac{1}{\theta + \rho} \ln(1 - e^{-rR}) + \gamma \frac{e^{-(\theta + \rho)R}}{\theta + \rho} \quad (16)$$

Figure 5 shows how the  $V(R)$  function changes with life expectancy (the other parameters are as above). For high values of  $\rho$ , that is, when life expectancy is low, the  $V(R)$  function is upward sloping. In this case, the optimum is a corner solution of  $R = \infty$ ; in other words, the individual plans never to retire. For low values of  $\rho$ , in other words when life expectancy is high, there is a single interior solution.<sup>16</sup>

Figure 6 shows the relationship between retirement and life expectancy (which is just  $1/\rho$ ) for the same parameters used in Figs. 3 and 4. Notice that  $R$  is the *planned* age of retirement, but that many people will not live long enough to reach it. Thus there is no inconsistency in having the retirement age be greater than life expectancy. Figure 6 shows the key result: as life expectancy rises, the planned age of retirement falls.

Figures 3, 4, and 6 allow for an explicit analysis of the role of uncertainty in affecting retirement. Comparing either Fig. 3 or Fig. 4, on the one hand, to Fig. 6, on the other, shows

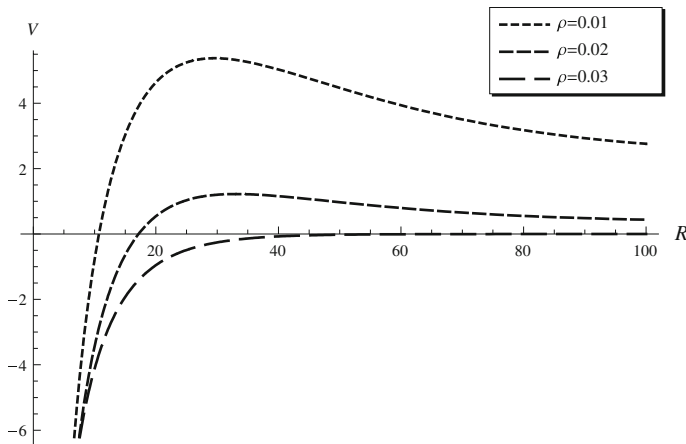
<sup>16</sup> This can be shown formally, as follows. From 16, the first order condition for optimal retirement age is

$$\frac{r}{\gamma(\theta + \rho)} = e^{(r - \theta - \rho)R} [(1 - e^{-rR})].$$

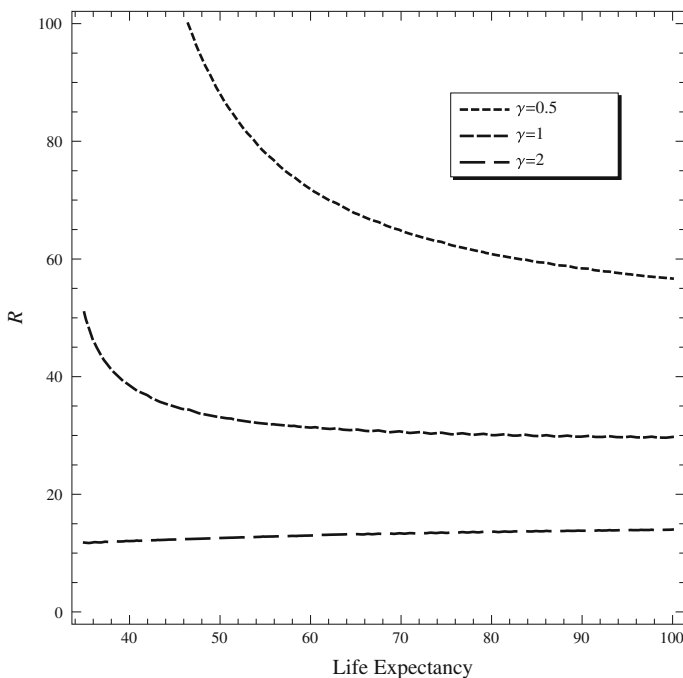
The effect of mortality on the optimal retirement age is given by the equation.

$$\frac{dR}{d\rho} = - \frac{-Re^{(r - \rho - \theta)R} + Re^{-(\rho + \theta)R} + r/(\gamma\theta + \rho)^2}{(r - \rho - \theta)e^{(r - \rho - \theta)R} + (\rho + \theta)e^{-(\rho + \theta)R}}.$$

It can be shown that this derivative is positive for large values of  $R$ . Thus, the initial effect of reductions in mortality (starting from a high level of  $\rho$ ) will be to reduce the age of retirement. However, once retirement age has fallen, it is possible that further improvements in mortality can raise the age of retirement. Specifically, retirement falls with life expectancy as long as  $R > \frac{1}{\theta + \rho}$ .



**Fig. 5**  $V(R)$  with exponential mortality and non-binding liquidity constraint



**Fig. 6** Optimal retirement with exponential mortality and non-binding liquidity constraint

the effect of uncertainty on optimal retirement age. (The fact that Figs. 3 and 4 are so similar shows that annuities and insurance can eliminate most of the effect of uncertainty on retirement age.) The rise in retirement age with life expectancy in either Fig. 3 or Fig. 4 is what we call the “horizon effect.” The fact that optimal retirement *falls* with life expectancy in Fig. 6 shows that the decline in retirement age due to falling uncertainty more than compensates for the rise due to longer horizons.

### 3.2.3 Exponential survival with liquidity constraints

Above we established one of our basic results under the assumption that  $r > \rho + \theta$ . If this condition does not hold, then the first order condition implies that individuals who hold positive assets will have falling consumption. It is thus possible that the liquidity constraint that assets be non-negative will be binding. Seemingly optimal paths that satisfy the infinite horizon budget constraint are not feasible because they involve negative assets at some point in time.

In this case, there are two possible types of optima. First, it may be optimal never to retire, in which case consumption is equal to the wage in all periods and assets are always zero. Second, it may be optimal to plan for a retirement, hold positive assets, and follow a declining path of consumption in accord with the first order condition. Life time utility in the first case is

$$U^{**} = \int_0^{\infty} e^{-(\theta+\rho)x} \ln w dx = \frac{\ln w}{\theta + \rho} \quad (17)$$

In the second case, we can solve for  $c_0$  from the budget constraint, taking into account the liquidity constraint by specifying  $c_0 \leq w$ . This yields

$$c_0 = \frac{\theta + \rho}{r} w (1 - e^{-rR}) \quad (18)$$

$$R \leq \frac{\ln(\theta + \rho) - \ln(\theta + \rho - r)}{r} \quad (19)$$

We denote the value of retirement age for which the second constraint holds with equality  $R^{**}$ . For  $R > R^{**}$ , initial consumption  $c_0$  would be greater than  $w$  and the consumption path therefore infeasible.

Life time utility is given by

$$V(R) = \int_0^{\infty} e^{-(\theta+\rho)x} \ln c(x) dx + \int_R^{\infty} e^{-(\theta+\rho)x} \gamma dx \quad (20)$$

$$= \frac{\ln(\frac{\theta+\rho}{r}(1 - e^{-rR})w)}{\theta + \rho} + \frac{r - \theta - \rho}{(\theta + \rho)^2} + \frac{\gamma e^{-(\theta+\rho)R}}{\theta + \rho} \quad (21)$$

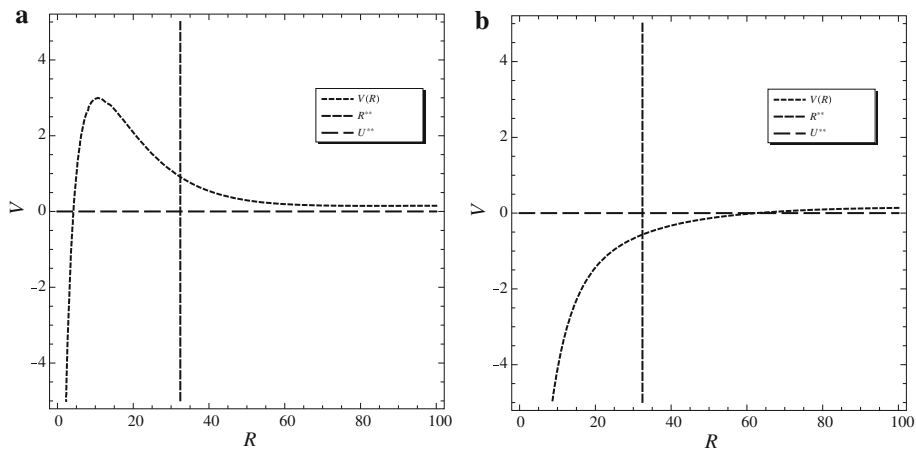
To determine the optimal retirement  $R^*$  satisfying (19), we examine the derivative of  $V(R)$ :

$$\frac{dV(R)}{dR} = \frac{r e^{-rR}}{(\theta + \rho)(1 - e^{-rR})} - \gamma e^{-(\theta+\rho)R} \quad (22)$$

For  $R \rightarrow 0$ ,  $\frac{dV(R)}{dR} > 0$ , while for  $R \rightarrow \infty$ ,  $\frac{dV(R)}{dR} \rightarrow 0$  from above. However, depending on parameter values,  $\frac{dV(R)}{dR}$  may become negative for intermediary values of  $R$ . Thus  $V(R)$  is either monotonically increasing or has an interior maximum followed by an interior minimum.

Figure 7 shows two different possible configurations of  $U^{**}$  (utility from never retiring), the  $V(R)$  function, and the constrained level of retirement,  $R^{**}$ . In panel A of the figure, retirement at the interior optimum of the  $V(R)$  function is feasible and utility from this plan is higher than the utility of never retiring. In panel B, the interior optimum of  $V(R)$  is infeasible and utility at the constrained optimum  $V(R^{**})$  is lower than the utility of never retiring,  $U^{**}$ .<sup>17</sup>

<sup>17</sup> Panel B also can represent the case in which  $V(R)$  is monotonically increasing. We have not been able to rule out a final case, in which the interior optimum of  $V(R)$  is infeasible, but utility at the constrained optimum



**Fig. 7**  $V(R)$  with exponential mortality and binding liquidity constraint **a** interior optimum **b** optimum at infinity

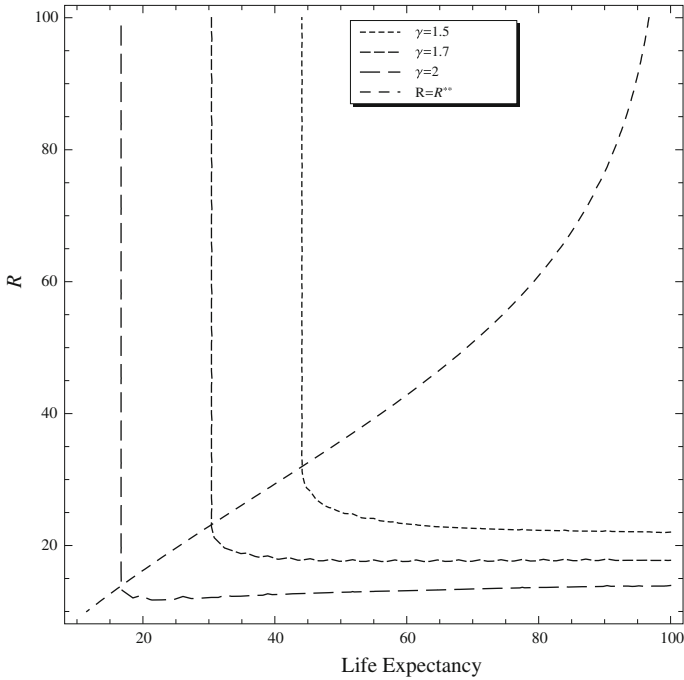
Figure 8 shows the effect of changing life expectancy  $1/\rho$  for different values of the utility from retirement,  $\gamma$  (the other parameters are  $r = .05$  and  $\theta = .04$ . These differ from the parameters used above in order to make the liquidity constraint bind.). As in the case where the liquidity constraint was not binding, when life expectancy is low enough, the optimal plan is never to retire. Increasing life expectancy from such a low level will lead to a reduction in the planned age of retirement followed, as life expectancy continues to increase, by a rise in retirement age. In contrast to the case without liquidity constraints, however, the response of retirement age to changes in life expectancy is highly nonlinear. For example when  $\gamma = 1.5$  the optimum moves from never planning to retire for life expectancy up to 44, to planning to retire at age 32 when life expectancy is just above 44 (all ages are from the beginning of adulthood). The reason for this behavior is found in the interplay of non-monotonicity of the  $V(R)$  function and liquidity constraint, as represented by the maximum non-constrained retirement age  $R^{**}$ . As will be seen below, this same phenomenon is present in even more extreme fashion when we examine realistic survival rates.

### 3.3 Realistic survival

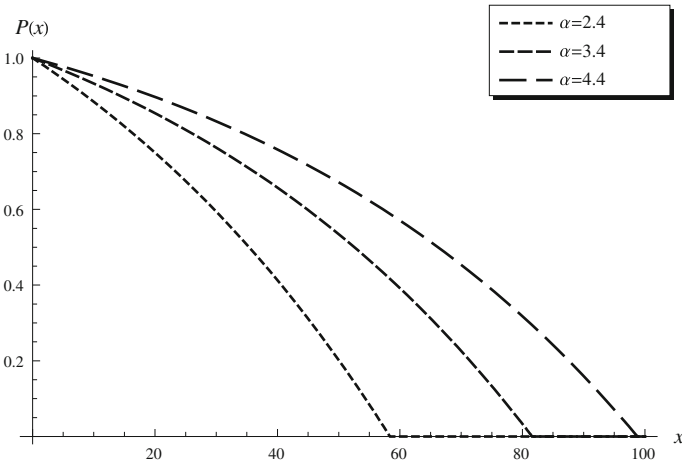
The analysis in the last section shows that for a simple specification of mortality, increases in life expectancy can actually cause a *decline* in the age of retirement. In other words, it is theoretically possible that the uncertainty effect will dominate the horizon effect. A natural question to ask is whether such an outcome is consistent with actual changes in mortality that have been experienced historically. We proceed in two stages. First, we consider a tractable but realistic model of survival in which we can derive results analytically. Following this, we look at actual data on survival in the United States and calculate optimal retirement ages numerically.

To approach this question analytically, we use a realistic survival function introduced by Boucek et al. (2002). The survival function (that is, the probability of being alive at age  $x$

$V(R^{**})$  exceeds the utility of never retiring. However, we have not been able to find any parameters that yield this case.



**Fig. 8** Optimal retirement with exponential mortality and binding liquidity constraint



**Fig. 9** Realistic survival function

is) is  $P(x) = \frac{e^{-\beta x} - \alpha}{1 - \alpha}$  where  $\beta < 0$  and  $\alpha > 1$ . This implies a life expectancy of  $\frac{1}{\beta} + \frac{\alpha \ln \alpha}{(1 - \alpha)\beta}$  and a maximum possible age of  $(P(x^{\max}) = 0)$  of  $-\frac{\ln \alpha}{\beta}$ . Figure 9 shows the survival function for  $\beta = -.015$  and different values of  $\alpha$ . As in previous figures, we take age zero to be the beginning of adulthood.

As in the previous section, we consider the possibilities that the individual plan to retire at some age  $R^*$  or alternatively plan to work until death. Unlike the previous section, however,



it turns out that there are no parameters under which an individual who chooses to work until death will always hold positive assets.<sup>18</sup> We thus proceed in two steps. First, we solve the model under the assumption that the liquidity constraint is not binding. We then consider the cases where the constraint is binding, and show how this alters the solution we derive.

### 3.3.1 Unconstrained solution

The optimal consumption path that maximizes  $E(U)$  for a given  $R$  is

$$c(x) = c_0 \frac{e^{-\beta x} - \alpha}{1 - \alpha} e^{(r-\theta)x} \quad (23)$$

Plugging this equation into the budget constraint we can solve for initial consumption

$$c_0 = \frac{(1 - \alpha)w}{rC} (1 - e^{-rR}) \quad (24)$$

where  $C := \frac{1-\alpha}{\theta+\beta} + \frac{\alpha(\frac{\theta}{\beta}-1)}{\theta}$ .

Substituting the optimal consumption path into the expected utility gives a function for life time utility in terms of  $R$  (leaving out terms without  $R$ ):

$$V(R) = \ln(1 - e^{-rR}) \int_0^T \frac{e^{-\beta x} - \alpha}{1 - \alpha} e^{-\theta x} dx + \gamma \int_R^T \frac{e^{-\beta x} - \alpha}{1 - \alpha} e^{-\theta x} dx \quad (25)$$

$$= \frac{\ln(1 - e^{-rR})}{1 - \alpha} C + \frac{\gamma}{1 - \alpha} \left( \frac{e^{-(\beta+\theta)R} - \alpha^{\frac{\theta+\beta}{\beta}}}{\theta + \beta} + \frac{\alpha(\alpha^{\frac{\theta}{\beta}} - e^{-\theta R})}{\theta} \right) \quad (26)$$

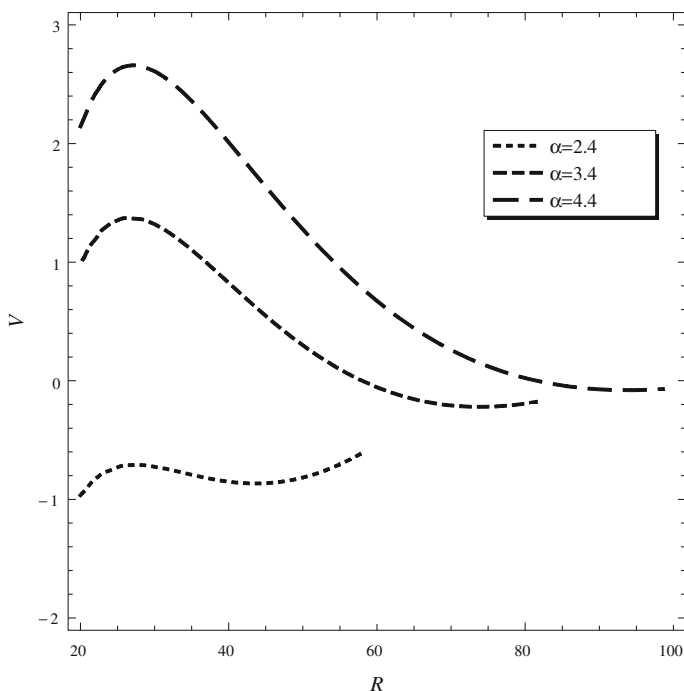
Figure 10 plots  $V(R)$  for different values of  $\alpha$  (in each case, the function is plotted only up to  $R = x^{\max}$ ). The other parameters are set as above:  $\beta = -.015$ ,  $r = .06$ ,  $\theta = .03$ ,  $\gamma = 1$ . For  $\alpha \geq 2.22$  (life expectancy = 30 years,  $x^{\max} = 50$  years),  $V(R)$  has a local maximum followed by a local minimum, after which the function increases monotonically. Label the local maximum  $R^*$ . If  $V(R^*) \geq U^{**}$  it is optimal to retire at age  $R^*$ , otherwise it is optimal to never retire. Graphically, one can see that never retiring is optimal for values of  $\alpha$  below a certain threshold ( $\hat{\alpha} \approx 2.45$ , implying life expectancy  $\approx 35$  years,  $x^{\max} \approx 61$  years).

### 3.3.2 Constrained solution

As mentioned above, under realistic survival it is always the case that an individual who chooses to work until death will hold zero assets at some age—in other words, the liquidity constraint is always binding at some point in time. In particular, if one looks at the paths of assets implied by the unconstrained  $V(R)$  function in the previous section, in the case where an individual never retires, assets are negative in the period immediately before the maximum possible age. We thus define  $\tilde{V}(R)$  as the utility resulting from retirement at age  $R$  taking into account the liquidity constraint.  $\tilde{V}(R)$  will be equal to  $V(R)$  up to some critical retirement age  $R^{**}$ , and then will lie below it.

We were not able to derive an analytic expression for  $\tilde{V}(R)$ . Instead, we solve for the function by brute force. For each possible retirement age, we first check whether the path of assets implied by the unconstrained solution is ever negative. If this is not the case, then  $\tilde{V}(R) = V(R)$ . If it is the case, we solve for utility along a constrained path which involves

<sup>18</sup> The reason is because under the realistic survival function the hazard of death rises toward infinity as age approaches the maximum possible life span.



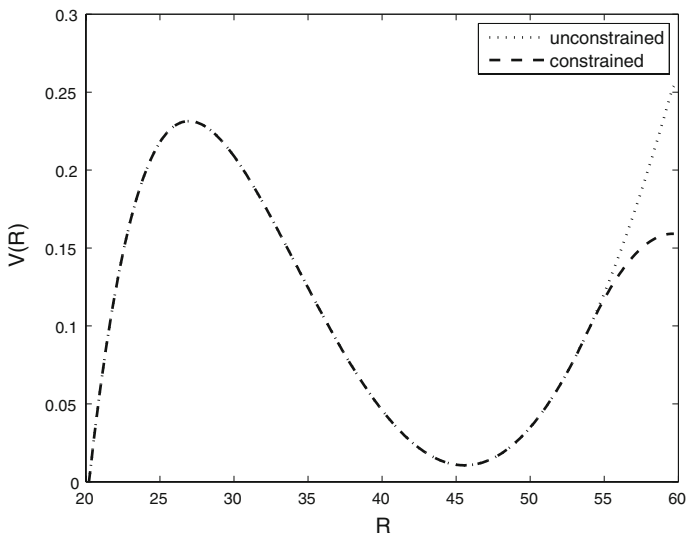
**Fig. 10**  $V(R)$  with realistic survival

holding positive assets and working up until some age  $x^*$ , then setting income equal to consumption between ages  $x^*$  and  $x^{**}$ , then building up assets again between age  $x^{**}$  and retirement.<sup>19</sup> Figure 11 shows an example of how the  $\tilde{V}(R)$  and  $V(R)$  functions compare for a particular set of parameters ( $\alpha = 2.45$  and all other parameters as above). The case is chosen in particular to show how it is possible for  $V(x^{\max}) > V(R^*)$  but  $\tilde{V}(x^{\max}) < \tilde{V}(R^*)$ . In other words, using the unconstrained  $V(R)$  function, never retiring is optimal, while recognizing the liquidity constraint, retiring at an interior optimum  $R^*$  is optimal.

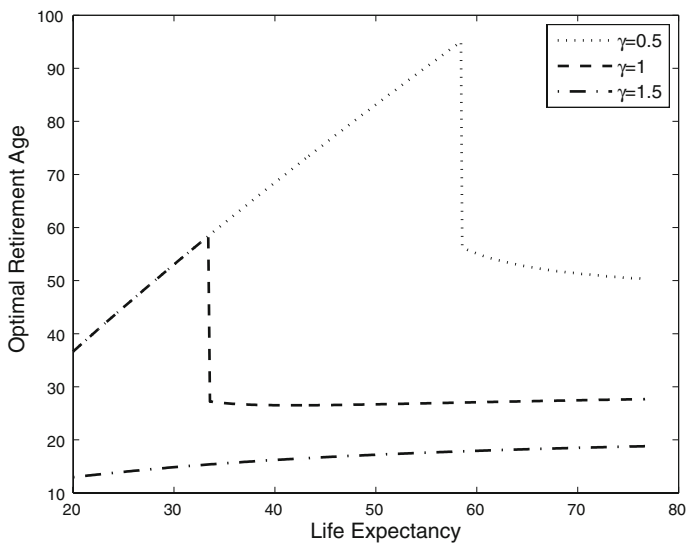
Despite this example, the general properties of the  $\tilde{V}(R)$  are quite similar to those of the  $V(R)$  function (and the set of parameters for which optimal retirement implied by the two differ is quite small). Specifically, there is an interior optimum and a corner solution of  $R = x^{\max}$ . Small changes in the parameters, such as the shape of the survival function or the utility from retirement, can lead to a shift in the utility maximizing age of retirement from a corner solution to an interior optimum. Figure 12 shows how the optimal age of retirement changes with life expectancy (as the parameter  $\alpha$  varies) for different values of the utility from retirement,  $\gamma$ .

The important implication of Fig. 12 is that because of the structure of the  $V(R)$  function, small changes in the parameters will lead to large changes in expected retirement age. The pattern of the  $V(R)$  function in which the two possible global optima are an interior optimum and a corner solution results from the dichotomy between the two strategies for optimal consumption mentioned above: to plan for a retirement, with the risk that one might die early and thus have wasted all of the money saved up; or to plan to pay for consumption in the

<sup>19</sup> The *Matlab* code is available on request.



**Fig. 11**  $V(R)$  and  $V\sim(R)$



**Fig. 12** Optimal retirement with realistic survival

event that one lives into old age by working. There is no way within the model to “convexify” between these two strategies.

Taken literally, this result would imply that there should be a sudden shift in behavior from never retiring to planning for a large retirement. Obviously this is not what is observed in the data, where the growth of retirement has been rapid (in historical terms) but hardly instantaneous. We do not think of this as a major failing of the model. In the real world, heterogeneity, institutions, learning, and a host of other factors would tend to cause retirement ages to adjust slowly, rather than jumping all at once, in response to a change in mortality.

### 3.4 Optimal retirement with actual survival rates

Although the realistic survival function shown in Fig. 9 are obviously a great improvement on the exponential model of survival considered in Sect. 3.2, it still is not a precise match to the actual survival schedules show in Fig. 1. Thus as a final exercise we examine the effect of actually observed mortality changes.

We use a discrete time version of our model to calculate optimal retirement ages for actual mortality data. Our procedure for finding the optimal retirement age is straightforward. We loop through possible retirement ages, and calculate optimal consumption and asset paths for each one subject to the constraint that assets are never negative.<sup>20</sup> We then calculate expected utility associated with each possible retirement age.

Figure 13 shows an example of our calculation of expected lifetime utility as a function of planned retirement age, in other words the  $V(R)$  function, using mortality data from 1900 through 2000.<sup>21</sup> We use the values of  $r = .06$ ,  $\theta = .03$ , as above. The value of  $\gamma$  is set such that, using 1980 mortality, the value of planning never to retire (the corner solution) is exactly equal to the utility from planning to retire at an interior optimum.

The figure shows the same phenomenon that was seen in the case of the realistic survival function analyzed in Sect. 3.3: for high mortality, the  $V(R)$  function is monotonically increasing, and the optimum plan is never to retire. As mortality falls (shown here as advancing calendar time), there emerges an interior optimum in the  $V(R)$  function, and for low enough mortality this becomes the global maximum in the function. For mortality in 1980, where by construction the two strategies produce equal utility, the interior optimum is at age 57 (for this calculation we actual age, not years since entering adulthood that we used previously.)

## 4 Paths of consumption and assets

In the previous section we showed that small changes in pattern of mortality can lead to large changes in the pattern of lifetime labor supply. This was true both examining the realistic formulation of the survival function in Sect. 3.3, and also when we examined actual changes

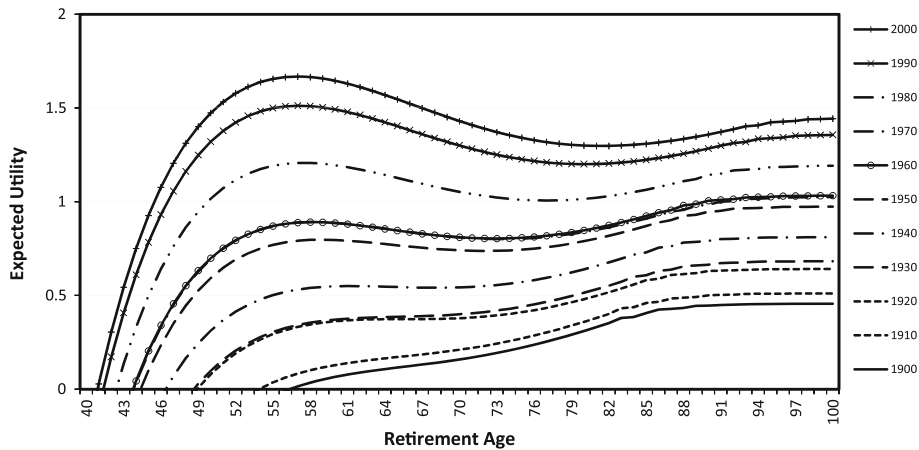
<sup>20</sup> Let  $P_t$  be the probability that an individual will be alive in period  $t$ , and let  $A_t$  be assets at the beginning of the period. The first-order conditions for consumption are

$$\begin{aligned} c_{t+1} &= c_t \frac{(1+r)}{(1+\theta)} \frac{P_{t+1}}{P_t} & \text{if } A_{t+1} > 0, \\ c_t &= w_t & \text{if } A_{t+1} = 0. \end{aligned}$$

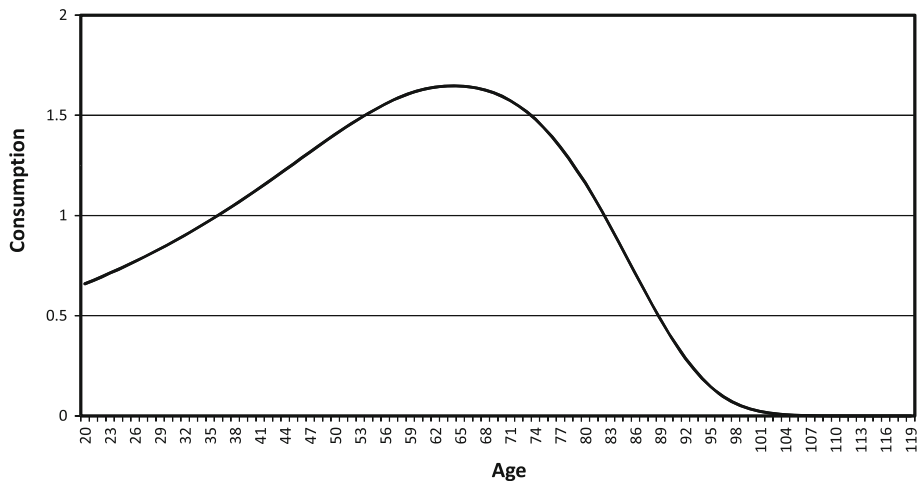
If assets are being carried into period  $t+1$ , then the usual first order condition for marginal utilities (adjusted by interest rate, time discount, and mortality probabilities) must hold. The only case in which the condition will not hold is if the individual would like to shift more consumption into period  $t$ , but is unable to do so because of the liquidity constraint. In this case he will consume all of his wages.

In the case of the problem being addressed here, we can take advantage of a special feature of mortality rates that always holds true in the data for ages beyond young adulthood: mortality rates are an increasing function of age. This delivers the result that, as long as assets are positive, the growth rate of consumption must be declining over time. The only time when consumption growth will not be declining is when assets are zero, in which case consumption is constant and equal to the wages. See Carroll (1997) for a similar dynamic programming algorithm.

<sup>21</sup> As in Fig. 1, we use period rather than cohort life tables because the former are available for a longer time period.



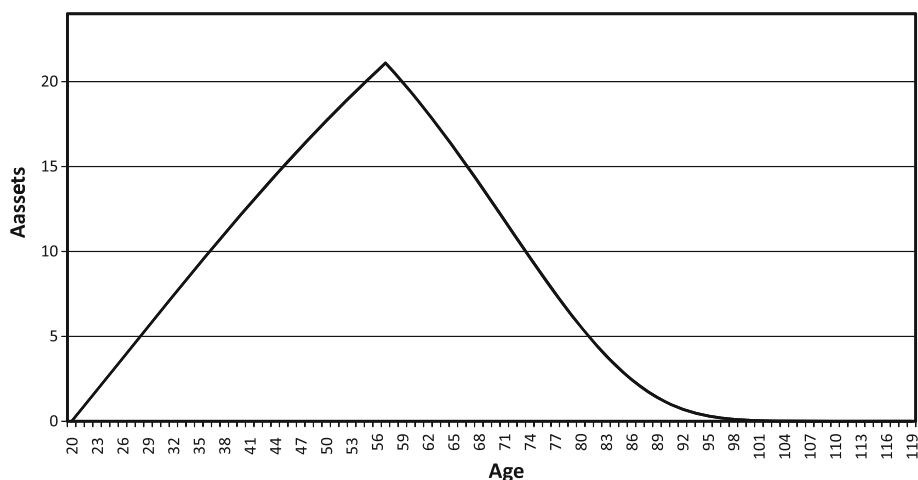
**Fig. 13** Expected utility as a function of retirement age



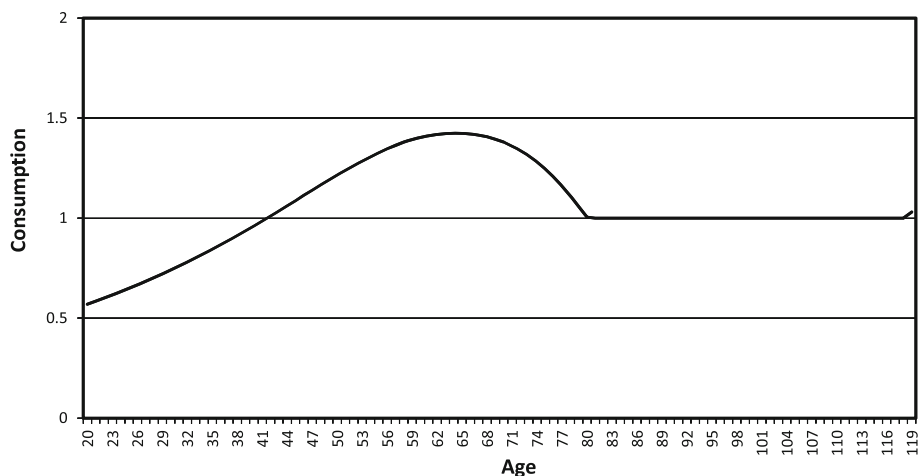
**Fig. 14** Optimal consumption path.  $R^* = 57$ , life table = 1980

in US mortality in Sect. 3.4. As those sections showed, for a range of mortality regimes, the difference in *utility* from choosing one strategy as opposed to the other are not great.

It is interesting to note, however, that these two strategies of lifetime labor supply do involve very different paths for lifetime consumption and asset accumulation. We illustrate this using the example of actual mortality rates for the United States, as shown in the previous section. We use these paths to illustrate the qualitative change in the lifetime pattern of asset holdings that takes place when retirement behavior changes. We do not think that the actual profiles in our simple model can be matched to data. Figures 14 and 15 show the paths of consumption and assets assuming 1980 mortality and a planned retirement age of 57. Figures 16 and 17 show the same paths when the individual plans never to retire. Recall from the previous section that we have chosen the value of retirement leisure,  $\gamma$  so that utility under these two strategies is equal.



**Fig. 15** Optimal asset path.  $R^* = 57$ , life table = 1980

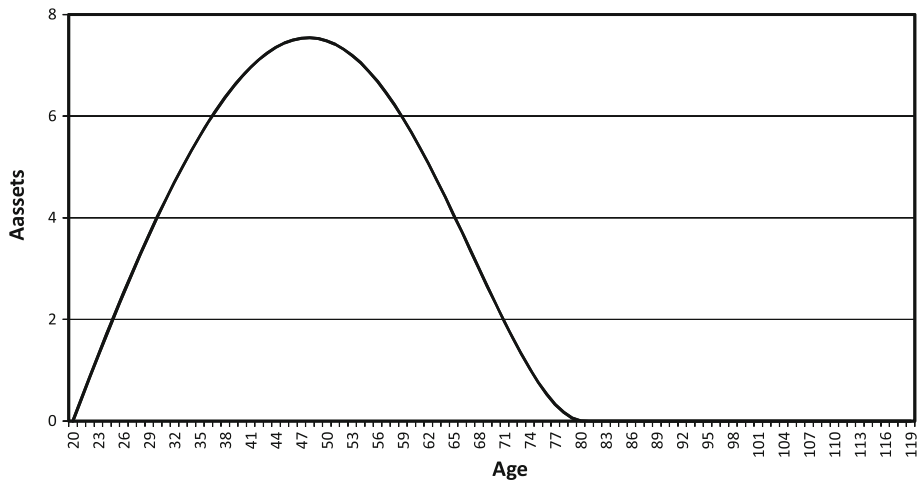


**Fig. 16** Optimal consumption path  $R^* = \infty$ , life table = 1980

The optimal consumption and asset paths when the expected retirement age is 57 have the standard life-cycle shape. The growth rate of consumption is highest at the beginning of life (age 20, in our problem), because the probability of death, which functions like the discount rate, is lowest.<sup>22</sup> Consumption growth falls over time, reaching zero at age 65, then becoming negative. Assets grow from the beginning of life until retirement at age 57, and fall monotonically thereafter.

In the case where the individual plans never to retire, the path for consumption (Fig. 16) has the same shape (although a higher level) as when the retirement age is 57, until the age of 83. After age 83, consumption becomes equal to the wage since the liquidity constraint is binding. The path of assets (Fig. 17) is different from the previous case in two ways: First, in the case where there is no retirement, assets reach zero late in life, while in the case of

<sup>22</sup> See Hurd (1990).



**Fig. 17** Optimal asset path  $R = \infty$ , life table = 1980

retirement, assets in old age are always positive. The second difference between the asset paths when the retirement age varies is in their size: When retirement is at age 57, total assets peak at roughly 20 times annual income; when the individual plans never to retire, assets peak at 9 times annual income. (In the case where there is no retirement, the accumulation of assets is due to the difference between the interest rate and the time discount rate.) Obviously, if this model of labor supply and savings were embedded in a general equilibrium model with capital accumulation and production, the change in retirement behavior would have significant implications for the wages and the interest rate.

## 5 Income vs uncertainty effects on retirement

In the model as set up in Sect. 3, we suppressed any income effects on retirement by using the log utility function. One natural extension of the model is to allow for income effects, and to see how these interact with uncertainty. Technically, this is simply a matter of replacing the log utility function with a Constant Relative Risk Aversion utility function. As long as the coefficient of relative risk aversion is greater than one, increases in income will *ceteris paribus* lower the retirement age.

The utility function is

$$\begin{aligned}
 U &= \frac{c(x)^{1-\sigma}}{1-\sigma} && \text{if working} \\
 &= \frac{c(x)^{1-\sigma}}{1-\sigma} + \gamma && \text{if retired.}
 \end{aligned}$$

where  $\sigma \neq 1$ .

Using the realistic survival function, the optimal consumption path that maximizes  $E[U]$  for a given  $R$  is

$$c(x) = c_0 \left( \frac{e^{-\beta x} - \alpha}{1 - \alpha} \right)^{\frac{1}{\sigma}} e^{\frac{r-\theta}{\sigma} x} \quad (27)$$

Plugging  $c(x)$  into the budget constraint we can solve for initial consumption:

$$c_0 = \frac{w(1 - e^{-rR})}{r} \int_0^T \left( \frac{e^{((1-\sigma)r-\theta)x} e^{\beta x} - \alpha}{1 - \alpha} \right)^{\frac{1}{\sigma}} dx \quad (28)$$

Expected utility as a function of the retirement age is given by:

$$V(R) = \left( \frac{w(1 - e^{-rR})}{r} \right)^{1-\sigma} \left( \int_0^T \left( \frac{e^{-\beta x} - \alpha}{1 - \alpha} \frac{e^{((1-\sigma)r-\theta)x}}{1 - \sigma} \right)^{\frac{1}{\sigma}} dx \right)^{\sigma} \quad (29)$$

$$+ \frac{\gamma}{1 - \alpha} \left( \frac{e^{-(\beta+\theta)R} - \alpha^{\frac{\theta+\beta}{\beta}}}{\theta + \beta} + \frac{\alpha(\alpha^{\frac{\theta}{\beta}} - e^{-\theta R})}{\theta} \right)$$

The  $V(R)$  function has the same general properties as in the case of realistic survival and log utility analyzed above: depending on parameters it can either be monotonically increasing or can have an interior maximum, which may be a global maximum.<sup>23</sup>

The three panels of Fig. 18 show optimal retirement as a function of life expectancy under realistic survival.<sup>24</sup> The general pattern of life expectancy's effect on retirement is the same as under logarithmic utility: for low life expectancy, the optimum is the corner solution of never retiring (or, as shown in the figure, retiring at the maximum possible age). As life expectancy rises, there is a discrete jump to the interior optimum of retiring well before the maximum possible age. After that, retirement age is relatively insensitive to life expectancy.

Within each panel, we show the optimal retirement age for three different levels of the wage,  $w = 1$ ,  $w = 1.2$ , and  $w = 1.4$ . The figure shows that, when the coefficient of relative risk aversion is greater than one, increases in the wage lower the age at which there is discrete jump to retirement and also lower the retirement age which constitutes the interior optimum. Further, the effects of wage and survival on the level of life expectancy at which the jump to retirement occurs are complementary: the higher the wage, the lower the level of life expectancy at which the jump occurs.

Comparing the panels with different values of  $\sigma$  shows that as risk aversion rises, the income effect increasingly comes to dominate the uncertainty effect as the major cause of retirement. However, using higher values of the coefficient of relative risk aversion also leads to a somewhat troubling conclusion: the model using these values of the risk aversion parameter generates dramatically falling age at retirement as income rises further. That is, unlike the uncertainty effect (which predicts a one-time rise in retirement, followed by rough constancy of the fraction of life spent retired), the income effect predicts that retirement will come to represent an ever larger fraction of life as income rises.

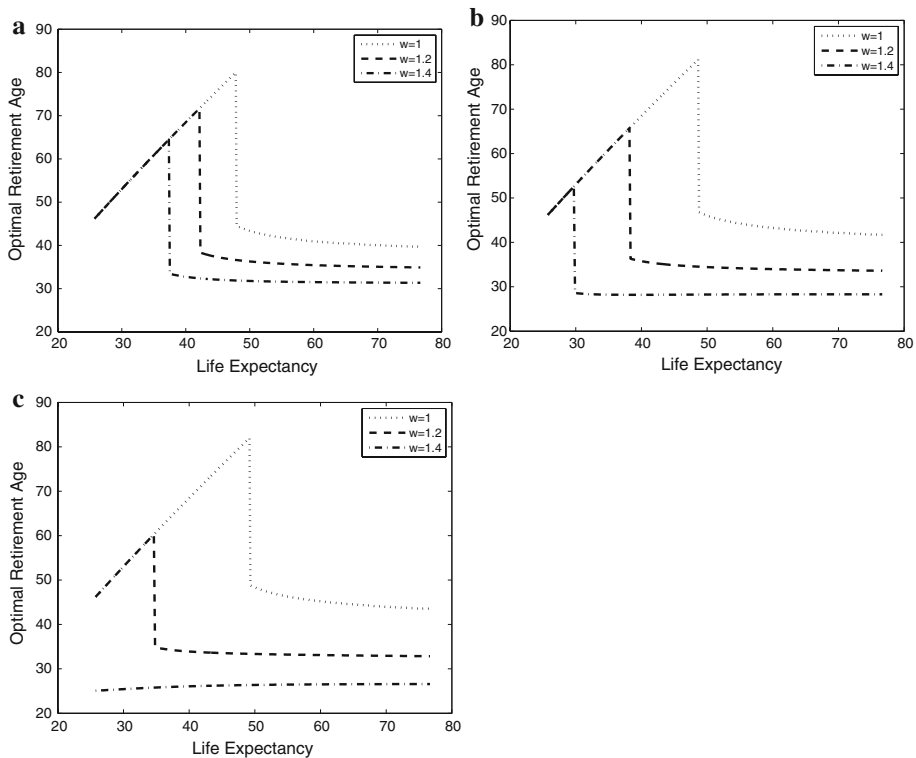
## 6 Conclusion

Our paper has shown how a reduction in mortality can lead to a shift in the life-cycle pattern of labor supply. High mortality leads to uncertainty about the age of death, and in this environment individuals will find it optimal to work until they die. As mortality falls, it becomes

<sup>23</sup> As in the case with logarithmic utility, the maximum of the  $V(R)$  function may violate the liquidity constraint. However, having found in Sect. 3.3 that the difference between the  $\tilde{V}(R)$  function that incorporates the liquidity constraint and the  $V(R)$  function that does not is very minor, we continue to work with the latter for mathematical simplicity.

<sup>24</sup> The values of the parameters are  $r = .06$ ,  $\theta = .03$ ,  $\beta = -.015$ , and  $\gamma = .7$ .





**Fig. 18** Optimal retirement with realistic survival, CRRA utility. **a**  $\sigma = 2$ ; **b**  $\sigma = 3$ ; **c**  $\sigma = 4$

optimal to plan for a period of leisure at the end of life — that is, for retirement. We show, using both a realistic mathematical formulation of the survival function as well as actual life table data for the United States over the last century, that this “uncertainty effect” can more than compensate for the more intuitive effect of higher life expectancy in raising the retirement age.

As we stressed in the introduction, we do not think that the uncertainty effect is the only explanation for the rise in retirement over the last century. Changes in government policy and productive technology, as well as the effect of higher income in raising the demand for leisure, have all played a role. Further, interactions among several of these channels are likely to be important. For example, we have shown that the uncertainty effect interacts with the income effect to produce a larger reduction in labor force participation than either channel separately. A project for future work in this area will be to apportion causality for the increase in retirement between the different channels we have discussed.

A second dimension along which the model can be extended is to examine how changes in labor supply feed back, via increased life cycle savings, into higher levels of income. That is, one could marry our model of endogenous retirement and savings with a growth model. In contrast to the standard Overlapping Generations model, in which the fraction of life spent working is fixed, our model suggests that the emergence of retirement, and thus of life cycle saving, may be one of the key steps in the process of modern economic growth.

**Acknowledgments** We thank Andrew Foster, Herschel Grossman, Chinhui Juhn, Robin Lumsdaine, Enrico Spolaore, and seminar participants at The Bank of Israel, Bar-Ilan University, Ben-Gurion University, Bilkent University, Brown University, Hebrew University, University of Houston, Koc University, University of Maryland, Rice University, University of Rochester, Tel Aviv University, and the Harvard School of Public Health Conference on Population Aging and Economic Growth for helpful comments. We are particularly grateful to Omar Licandro, who refereed the paper for the *Journal of Economic Growth*. His report, which was anonymous until after the paper was accepted, served as the basis for major improvements in our results and their presentation. Isabel Tecu provided superlative research assistance.

## References

- Boucekkine, R., Croix, D., & de la Licandro, O. (2002). Vintage human capital, demographic trends, and endogenous growth. *Journal of Economic Theory*, 104(2), 340–375.
- Carroll, C. (1997). Buffer-stock saving and the life cycle/permanent income hypothesis. *Quarterly Journal of Economics*, 112(1), 1–55.
- Carroll, C. (2001). A Theory of the consumption function, with and without liquidity constraints. *Journal of Economic Perspectives*, 15(3), 23–45.
- Carter, S., & Sutch, R. (1996). Myth of the industrial scrap heap: A revisionist view of turn-of-the-century American retirement. *Journal of Economic History*, 56(1), 5–38.
- Coile, C., & Gruber, J. (2007). Future social security entitlements and the retirement decision. *The Review of Economics and Statistics*, 89(2), 234–246.
- Costa, D. (1998). *The evolution of retirement: An American history, 1880–1990*. Chicago, IL: The University of Chicago Press.
- Danziger, S., Haveman, R., & Plotnick, R. (1981). How income transfer programs affect work, savings, and the income distribution: A critical review. *Journal of Economic Literature*, 19(3), 975–1028.
- Duval, R. (2004). The retirement effects of old-age pension and early retirement schemes in OECD countries. Working Paper, OECD.
- Eckstein, Z., Mira, P., & Wolpin, K. (1999). A quantitative analysis of Swedish fertility dynamics: 1751–1990. *Review of Economic Dynamics*, 2(1), 137–165.
- Ehrlich, I., & Lui, F. (1991). Intergenerational trade, longevity, intrafamily transfers and economic growth. *Journal of Political Economy*, 99(5), 1029–1059.
- Fields, G., & Mitchell, O. (1984). *Retirement, pensions, and social security*. Cambridge: MIT Press.
- Graebner, W. (1980). *A history of retirement: The meaning and function of an American Institution, 1885–1978*. New Haven, CT: Yale University Press.
- Gruber, J., & Wise, D. (1998). Social security and retirement: An international comparison. *American Economic Review*, 88(2), 158–163.
- Gruber, J., & Wise, D. (1999). Introduction and Summary. In J. Gruber & D. Wise (Eds.), *Social security and retirement around the world*. Chicago, IL: University of Chicago Press.
- Hurd, M. (1990). Research on the elderly: Economic status, retirement, and consumption and savings. *Journal of Economic Literature*, 28(June), 565–637.
- Haines, M. (1998). Estimated life tables for the United States, 1850–1910. *Historical Methods*, 31(4), 149–167.
- Hazan, M. (forthcoming). Longevity and lifetime labor supply: Evidence and implications. *Econometrica*.
- Johnson, R. (2001). The effect of old-age insurance on male retirement: Evidence from historical cross-country data. Working Paper, Federal Reserve Bank of Kansas City.
- Kalemli-Ozcan, S. (2003). A stochastic model of mortality, fertility and human capital investment. *Journal of Development Economics*, 70(1), 103–118.
- Keyfitz, N., & Flieger, W. (1990). *World population growth and aging*. Chicago, IL: The University of Chicago Press.
- Kotlikoff, L., & Summers, L. (1988). The contribution of intergenerational transfers to total wealth: A reply. In D. Kessler & A. Masson (Eds.), *Modeling the accumulation and distribution of wealth*. New York, NY: Oxford University Press.
- Lee, C. (2001). The expected length of male retirement in the United States, 1850–1990. *Journal of Population Economics*, 14(4), 641–650.
- Long, C. (1958). *The labor force under changing income and employment*. Princeton, NJ: Princeton University Press for NBER.

- Lumsdaine, R., & Wise, D. (1994). Aging and labor force participation: A review of trends and explanations. In Y. Noguchi & D. Wise (Eds.), *Aging in the United States and Japan: Economic trends*. Chicago, IL: University of Chicago Press.
- Matthews, R., Feinstein, C., & Odling-Smee, J. (1982). *British economic growth, 1856–1973*. Stanford, CA: Stanford University Press.
- Meltzer, D. (1992). Mortality decline, the demographic transition and economic growth. Ph.D. Dissertation, University of Chicago.
- Modigliani, F. (1986). The role of intergenerational transfers and life cycle saving in the accumulation of wealth. *Journal of Economic Perspectives*, 2(2), 15–40.
- Moen, J. (1988). Past and current trends in retirement: American men from 1860 to 1980. *Federal Reserve Bank of Atlanta Economic Review*, 73(July), 16–27.
- Moen, J. (1994). Rural nonfarm households: Leaving the farm and the retirement of older Men, 1860–1980. *Social Science History*, 18(1), 55–75.
- OECD. (2004). Labor Market Statistics Indicators online database.
- Pestieau, P. (2003). Are we retiring too early? In O. Castellino & E. Forno (Eds.), *Pension policy in an integrating Europe*. Northampton, MA: Edward Elgar.
- Ransom, R., & Sutch, R. (1986). The labor of older Americans: Retirement of men on and off the job, 1870–1937. *Journal of Economic History*, 46(1), 1–30.
- Ransom, R., & Sutch, R. (1988). The decline of retirement in the years before social security: US retirement patterns, 1870–1940. In R. Ricardo-Campbell & E. Lazear (Eds.), *Issues in contemporary retirement*. Stanford, CA: Stanford University Press.
- Sala-i-Martin, X. (1996). A positive theory of social security. *Journal of Economic Growth*, 1(2), 277–304.
- Samwick, A. (1998). New evidence on pensions, social security, and the timing of retirement. *Journal of Public Economics*, 70(2), 207–236.
- Weil, D. (2008). Population aging. In S. Durlauf & L. Blume (Eds.), *New Palgrave dictionary of economics* (2nd ed.). New York, NY: Palgrave MacMillan.
- Yashiro, N., & Oshi, T. (1999). Social security and retirement in Japan. In J. Gruber & D. Wise (Eds.), *Social security and retirement around the world*. Chicago: University of Chicago Press.