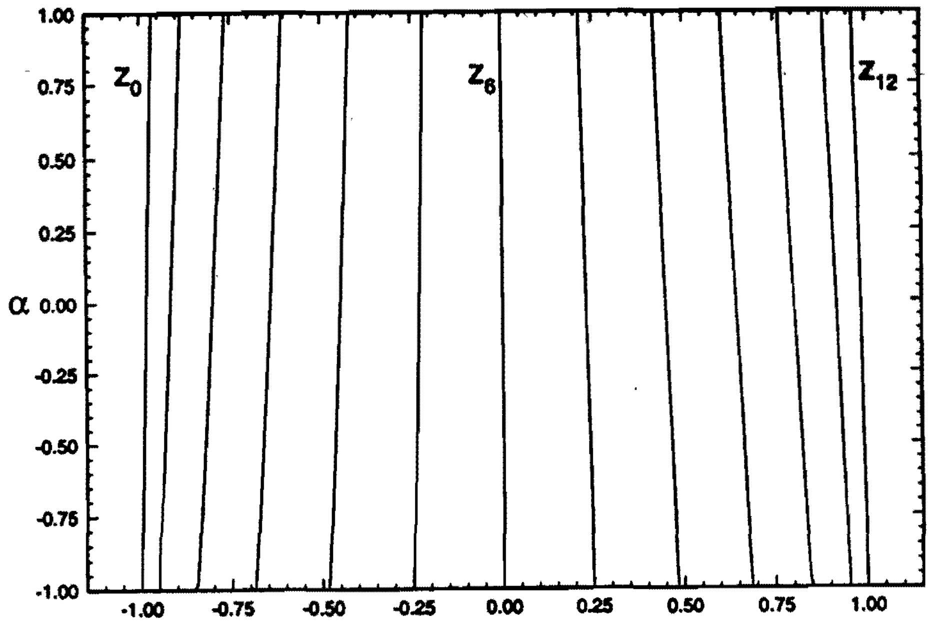
First-Derivatives

Gauss-Lobatto

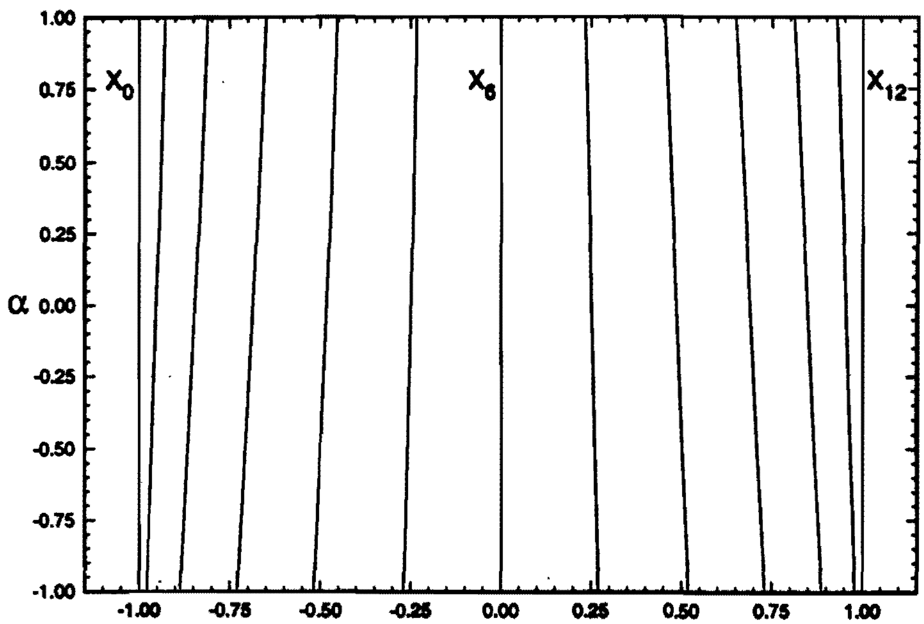
$$\mathbf{D}_{ij} = \begin{cases} \frac{\alpha - N(N+2\alpha+1)}{2(\alpha+2)} & i = j = 0 \\ -\frac{(-1)^N \Gamma(N+2\alpha+1)}{N!(\alpha+1)\Gamma(2\alpha+1)} \frac{1}{(1+x_j) P_N^{(\alpha)}(x_j)} & i = 0, j \in [1, N-1] \\ \frac{(-1)^N}{2} & i = 0, j = N \\ \frac{(-1)^N N!(\alpha+1)\Gamma(2\alpha+1)}{\Gamma(N+2\alpha+1)} \frac{P_N^{(\alpha)}(x_i)}{1+x_i} & i \in [1, N-1], j = 0 \\ \frac{1}{x_i - x_j} \frac{P_N^{(\alpha)}(x_i)}{P_N^{(\alpha)}(x_j)} & i \neq j, i, j \in [1, N-1] \\ \frac{\alpha x_i}{1-(x_i)^2} & i = j \in [1, N-1] \\ -\frac{(-1)^N}{2} & i = N, j = 0 \\ \frac{\Gamma(N+2\alpha+1)}{N!(\alpha+1)\Gamma(2\alpha+1)} \frac{1}{(1-x_j) P_N^{(\alpha)}(x_j)} & i = N, j \in [1, N-1] \\ -\frac{N!(\alpha+1)\Gamma(2\alpha+1)}{\Gamma(N+2\alpha+1)} \frac{P_N^{(\alpha)}(x_i)}{1-x_i} & i \in [1, N-1], j = N \\ \frac{-\alpha + N(N+2\alpha+1)}{2(\alpha+2)} & i = j = N \end{cases}$$

Gauss

$$\tilde{\mathbf{D}}_{ij} = \begin{cases} \frac{(\alpha+1) x_i}{1-(x_i)^2} & i = j \\ \frac{\left(P_{N+1}^{(\alpha)}\right)'(x_i)}{(x_i - x_j)\left(P_{N+1}^{(\alpha)}\right)'(x_j)} & i \neq j \end{cases}$$



. The Gauss collocation points, z_i, for $N = 12$ for the ultraspherical polynomial, $P_{12}^{(\alpha)}(x)$, as a function of α.



The Ultraspherical Gauss-Lobatto collocation points, x_i, for $N =$ 12 for the polynomial, $P_{12}^{(\alpha)}(x)$, as a function of α.