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Local approximation of operators

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Many applications, such as system identification, classification of time series, direct and inverse problems in partial differential equations, and uncertainty quantification lead to the question of approximation of a non-linear operator between metric spaces X and . We study the problem of determining the degree of approximation of a such operators on a compact subset Kx C X using a finite amount of information. If F : Kx K>JJ , a well established strategy to approximate F(F) for some F E Kx is to encode F (respectively, F(F)) in terms of a finite number d (repectively m) of real numbers. Together with appropriate reconstruction algorithms (decoders), the problem reduces to the approximation of m functions on a compact subset of a high dimensional Euclidean space JRd, equivalently, the unit sphere §d embedded in JRd+i . The problem is challenging because d, m, as well as the complexity of the approximation on §d are all large, and it is necessary to estimate the accuracy keeping track of the inter-dependence of all the approximations involved. In this paper, we establish constructive methods to do this efficiently; i.e., with the constants involved in the estimates on the approximation on §d being O(d1 16). We study different smoothness classes for the operators, and also propose a method for approximation of F(F) using only information in a small neighborhood of F, resulting in an effective reduction in the number of parameters involved. To further mitigate the problem of large number of parameters, we propose prefabricated networks, resulting in a substantially smaller number of effective parameters. The problem is studied in both deterministic and probabilistic settings.