

## CRUNCH Seminars at Brown, Division of Applied Mathematics

Friday - July 2, 2021

### Learning Functional Priors and Posteriors from Data and Physics

Xuhui Meng, CRUNCH Group

**We develop a new Bayesian framework based on deep neural networks to be able to extrapolate in space-time using historical data and to quantify uncertainties arising from both noisy and gappy data in physical problems. Specifically, the proposed approach has two stages: (1) prior learning and (2) posterior estimation. At the first stage, we employ the physics-informed Generative Adversarial Networks (PI-GAN) to learn a functional prior either from a prescribed function distribution, e.g., Gaussian process, or from historical data and physics. At the second stage, we employ the Hamiltonian Monte Carlo (HMC) method to estimate the posterior in the latent space of PI-GANs. In addition, we use two different approaches to encode the physics: (1) automatic differentiation, used in the physics-informed neural networks (PINNs) for scenarios with explicitly known partial differential equations (PDEs), and (2) operator regression using the deep operator network (DeepONet) for PDE-agnostic scenarios. We then test the proposed method for (1) meta-learning for one-dimensional regression, and forward/inverse PDE problems (combined with PINNs); (2) PDE-agnostic physical problems (combined with DeepONet), e.g., fractional diffusion as well as saturated stochastic (100-dimensional) flows in heterogeneous porous media; and (3) spatial-temporal regression problems, i.e., inference of a marine riser displacement field using experimental data from the Norwegian Deepwater Programme (NDP). The results demonstrate that the proposed approach can provide accurate predictions as well as uncertainty quantification given very limited scattered and noisy data, since historical data could be available to provide informative priors. In summary, the proposed method is capable of learning flexible functional priors, e.g., both Gaussian and non-Gaussian process, and can be readily extended to big data problems by enabling mini-batch training using stochastic HMC or normalizing flows since the latent space is generally characterized as low dimensional..**