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Local Extreme Learning Machines (locELM):A Neural Network-Based Spectral Element-Like Method for Solving PDEs

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Deep neural network (DNN) based PDE solvers have witnessed a dramatic growth in the past few years. However, these solvers suffer from several drawbacks, which make them numerically less than satisfactory and computationally uncompetitive. The most prominent includes the general lack of convergence with a certain convergence rate, limited accuracy, and extremely high computational cost (very long time to train). Due to these limitations there seems to be a general sense that these DNN-based solvers, at least in their current state, cannot compete with traditional numerical methods, except perhaps for certain problems such as high-dimensional ones.

In this talk we introduce a neural network-based method (termed locELM) for solving linear and nonlinear PDEs that exhibits a disparate computational performance from the above DNN-based PDE solvers and in a sense overcomes the above limitations. This method combines the ideas of extreme learning machines, domain decomposition and local neural networks. The field solution on each sub-domain is represented by a local feed-forward neural network, and C^k continuity is imposed on the sub-domain boundaries. Each local neural network consists of a small number of hidden layers, whose coefficients are pre-set to random values and fixed through the computation, and the training parameters consist of the output-layer coefficients. The overall neural network is trained by a linear or nonlinear least squares computation, not by the back-propagation (or gradient descent) type algorithms. We also present a block time marching scheme together with locELM for long-time dynamic simulations of time-dependent PDEs. The current method exhibits a clear sense of convergence with respect to the degrees of freedom in the neural network. Its numerical errors typically decrease exponentially as the number of training parameters or the number of training data points increases, much like the traditional spectral or spectral element type methods. We compare the current locELM method with the physics informed neural network (PINN) method, the deep Galerkin method (DGM), and the traditional finite element method (FEM). The numerical errors and network training time of locELM is considerably smaller, typically by orders of maginitude, than those of DGM and PINN. The computational performance of locELM is on par with the FEM in terms of accuracy and computational cost, and it outperforms FEM as the problem size becomes large. A number of numerical examples will be presented to demonstrate these points.