Exact Resuslts on Massless $\mathbb{Z}_3$-QCD via Anomaly Matching

Yuya Tanizaki

RIKEN BNL Research Center, Brookhaven National Laboratory

June 11, 2018 @ Paris

Collaborators: Yuta Kikuchi (RBRC), Tatsuhiro Misumi (Akita), Norisuke Sakai (Keio)

References: 1710.08923[hep-th], 1711.10487[hep-th]

Contents

Introduction: Confinement and chiral symmetry, $\mathbb{Z}_3$-QCD

Technique: New 't Hooft anomaly of massless 3-flavor QCD

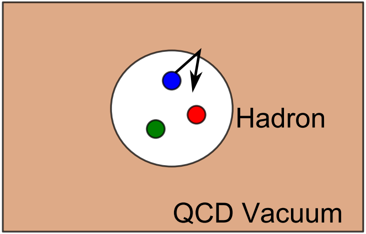
Application: Constraint on the phase diagram of massless $\mathbb{Z}_3$-QCD

Introduction: Confinement and chiral symmetry, $\mathbb{Z}_3$-QCD

Confinement $\Rightarrow$ Chiral symmetry breaking?

Heuristic argument (like the bag model) indicates that

$$\text{Confinement} \Rightarrow \text{Chiral symmetry breaking}$$



Can we make the EXACT statement related to this problem based on QCD?

Confinement and Center symmetry

Confinement/deconfinement is characterized by *center symmetry*.

$SU(N)$ gauge field a on a spacetime manifold M
= patches $\{U_i\}$, one-forms $a^{(i)}$ on U_i, and transition functions $g_{ij}\in SU(N)$ on $U_i\cap U_j$, satisfying

$$a^{(j)} = g_{ij}^{-1}a^{(i)}g_{ij} + g_{ij}^{-1}\mathrm{d}g_{ij},\ g_{ij}g_{jk}g_{ki} = 1.$$

Center symmetry is the symmetry to change transition functions by centers of $SU(N)$:

$$g_{ij} \mapsto g_{ij}\exp\left(\frac{2\pi\mathrm{i}}{N}n_{ij}\right) \equiv \omega^{n_{ij}}g_{ij}.$$

The order parameter is Wilson loop (or Polyakov loop on torus).

(This is one-form symmetry: Gaiotto, Kapustin, Seiberg, Willet (2014))

Center symmetry and fundamental quarks

Center symmetry is explicitly broken with fundamental quarks
$\Rightarrow$ Let's consider a nice setup ($\mathbb{Z}_{N_C}$-QCD (Kouno et al. 2012, ...)).

Prepare $N_F = N_C$ quarks, and put the boundary condition on the quark field as $(f = 1, \dots, N)$

$$q_f(\boldsymbol{x}, x_4 + L) = \omega^f q_f(\boldsymbol{x}, x_4).$$

This theory has the $(\mathbb{Z}_N)_{\mathrm{shift,center}}$ symmetry, defined by

$$\Phi = \mathrm{tr}_{\mathrm{c}}\left[P\exp\left(\mathrm{i}\int a\right)\right] \mapsto \omega\,\Phi,\ q_f \mapsto q_{f+1}.$$

$\mathbb{Z}_N$-QCD

Using operators of QCD, the partition function of $\mathbb{Z}_N$-QCD is

$$\mathcal{Z}_{\mathbb{Z}_N-\mathrm{QCD}} = \mathrm{tr}_{\mathcal{H}}\left[\mathrm{e}^{-L(\widehat{H}-\mu\widehat{Q})}\exp\left(\mathrm{i}\sum_{f=1}^{N}\frac{2\pi f}{N}\widehat{Q}_f\right)\right].$$

We will see that anomaly matching argument constrains the possible phases of $\mathcal{Z}_{\mathbb{Z}_N-\mathrm{QCD}}$:

(Discrete) chiral symmetry is broken in the center-symmetric phase, e.g.

$$T_{\mathrm{deconf}} \le T_{\mathrm{chiral}}$$

for massless $\mathbb{Z}_N$-QCD. (1710.08923, 1711.10487)

(Similar, related results for pure YM, Bifundamental QCD, adjoint QCD, etc.: Gaiotto, Kapustin, Komargodski, Seiberg (1703); Tanizaki, Kikuchi (1705); Komargodski, Sulejmanpasic, Unsal (1706); Shimizu, Yonekura (1706); ...)

Anomaly matching

't Hooft anomaly $\equiv$ Global symmetry that cannot be gauged.

Theorem

't Hooft anomaly is renormalization-group invariant. *('t Hooft, '80)*

Classic Example

Massless QCD has symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_V$ with an 't Hooft anomaly:

$$D^\mu J^a_\mu = 0, \quad D^\mu J^{5a}_\mu = \frac{1}{4\pi^2} \mathrm{tr}\left(T^a \left\{ F_V^2 + \frac{1}{3} F_A^2 + \cdots \right\} \right)$$

$\Rightarrow$ This is matched by ChSB and the WZW term for massless pions.

To match the anomaly, ground states must be *nontrivial*.

Technique: New 't Hooft anomaly of massless 3-flavor QCD

Symmetry of massless QCD

QCD Lagrangian:

$$S = \frac{1}{2g^2}\int \mathrm{tr}(G \wedge *G) + \int \sum_{f=1}^{N_F} \overline{q}_f \gamma_\mu D_\mu q.$$

Symmetry of massless QCD (i.e. Lagrangian + $\mathcal{D}\overline{q}\mathcal{D}q$):

$$\frac{SU(N_F)_L \times SU(N_F)_R \times U(1)_V \times (\mathbb{Z}_{2N_F})_A}{\mathbb{Z}_{N_C} \times (\mathbb{Z}_{N_F})_L \times (\mathbb{Z}_{N_F})_R \times \mathbb{Z}_2}$$

- For later purpose, I evade the double counting correctly (e.g. $U(N) = [SU(N) \times U(1)]/\mathbb{Z}_N$, $U(1)_L \times U(1)_R = [U(1)_V \times U(1)_A]/\mathbb{Z}_2$)
- Due to the gauge invariance, symmetry must be divided by $\mathbb{Z}_{N_C} \subset SU(N_C)$.

Further comments on chiral symmetry

One way to think about $(\mathbb{Z}_{2N_F})_A$ is that it is anomaly-free subgroup of $U(1)_A$.

For later application, let us see another perspective:

$$(\mathbb{Z}_{2N_F})_A \subset SU(N_F)_L \times SU(N_F)_R \times U(1)_V.$$

Indeed, the generator of $(\mathbb{Z}_{2N_F})_A$ can be written as

$$\mathrm{e}^{\frac{2\pi}{2N_F}\mathrm{i}\gamma_5}\mathbf{1}_{N_F} = \exp\left(\frac{2\pi}{N_F}\mathrm{i}\frac{1+\gamma_5}{2}\mathrm{diag}[1,\dots,1,1-N_F]\right)\mathrm{e}^{-2\pi\mathrm{i}/(2N)}$$

Quick message: SSB of $(\mathbb{Z}_{2N_F})_A$ implies continuous ChSB.

Gauging vector-like symmetry: Part 1

We pay attention to the vector-like symmetry,

$$\frac{SU(N_F)_V \times U(1)_V}{(\mathbb{Z}_{N_C}) \times (\mathbb{Z}_{N_F})},$$

and $(\mathbb{Z}_{2N})_{\mathrm{axial}}$.

To detect the 't Hooft anomaly, we introduce the gauge fields for vector-like symmetry (1710.08923, 1711.10487) (ref. Kapustin, Seiberg (2014));

- $SU(N_F)_V$ one-form gauge field: A_f
- $U(1)_V$ one-form gauge field: A_q
- $(\mathbb{Z}_{N_C})$ two-form gauge field: B_c
- $(\mathbb{Z}_{N_F})$ two-form gauge field: B_f

Gauging vector-like symmetry: Part 2

Question What is the meaning of two-form gauge fields?

Connection formula on double overlaps of patches $U_i \cap U_j$ $(g_{ij}^{(c)} \in SU(N_c), g_{ij}^{(f)} \in SU(N_f), g_{ij}^{(q)} \in U(1)_V)$:
For gauge field,

$$a^{(j)} = {g_{ij}^{(c)}}^{-1} a^{(i)} g_{ij}^{(c)} + {g_{ij}^{(c)}}^{-1} \mathrm{d} g_{ij}^{(c)},$$
$$A_f^{(j)} = {g_{ij}^{(f)}}^{-1} A_f^{(i)} g_{ij}^{(f)} + {g_{ij}^{(f)}}^{-1} \mathrm{d} g_{ij}^{(f)},$$
$$A_q^{(j)} = {g_{ij}^{(q)}}^{-1} A_q^{(i)} g_{ij}^{(q)} + {g_{ij}^{(q)}}^{-1} \mathrm{d} g_{ij}^{(q)}.$$

For quark field,

$$q^{(j)} = \left(g_{ij}^{(c)} \otimes g_{ij}^{(f)} \otimes g_{ij}^{(q)} \right) q^{(i)}.$$

Gauging vector-like symmetry: Part 3

Consistency requires the cocycle condition for $\left(g^{(c)}_{ij}\otimes g^{(f)}_{ij}\otimes g^{(q)}_{ij}\right)$, but not for each of them:

$$g^{(c)}_{ij}g^{(c)}_{jk}g^{(c)}_{ki}=\exp\left(\frac{2\pi \mathrm{i}}{N_C}n^{(c)}_{ij}\right)\in\mathbb{Z}_{N_C},$$

$$g^{(f)}_{ij}g^{(f)}_{jk}g^{(f)}_{ki}=\exp\left(\frac{2\pi \mathrm{i}}{N_F}n^{(f)}_{ij}\right)\in\mathbb{Z}_{N_F},$$

$$g^{(q)}_{ij}g^{(q)}_{jk}g^{(q)}_{ki}=\exp\left(-\frac{2\pi \mathrm{i}}{N_C}n^{(c)}_{ij}-\frac{2\pi \mathrm{i}}{N_F}n^{(f)}_{ij}\right)\in U(1)_V.$$

$[\{(n^{(c)}_{ij},U_i\cap U_j)\}_{ij}]$ and $[\{(n^{(f)}_{ij},U_i\cap U_j)\}_{ij}]$ are characterized by

$$B_c\in H^2(M,\mathbb{Z}_{N_C}),\ B_f\in H^2(M,\mathbb{Z}_{N_F}).$$

Mixed 't Hooft anomaly for massless QCD

$(\mathbb{Z}_{2N})_{\mathrm{axial}}$ can change $\mathcal{Z}$ as

$$\mathcal{Z}[(A_\mathrm{f}, A_\mathrm{q}, B_\mathrm{c}, B_\mathrm{f})] \mapsto \mathcal{Z}[(A_\mathrm{f}, A_\mathrm{q}, B_\mathrm{c}, B_\mathrm{f})] \exp(\mathrm{i}\mathcal{A}).$$

The 't Hooft anomaly $\mathcal{A}$ is given by Fujikawa method:

$$\mathcal{A} = 2\frac{2\pi}{2N}\frac{1}{8\pi^2}\int \mathrm{tr}_{\mathrm{c,f}}\left[F \wedge F\right],$$

F: field strength of $[SU(N_C) \times SU(N_F) \times U(1)_V]/[\mathbb{Z}_{N_C} \times \mathbb{Z}_{N_F}]$.

For simplicity of expression, let us set $N_C = N_F = N$, then

$$\mathcal{A} = -\frac{N}{2\pi}\int B_\mathrm{c} \wedge B_\mathrm{f} \bmod 2\pi\mathrm{i}.$$

After gauging the vector symmetry correctly, $(\mathbb{Z}_{2N})_A$ is broken by $\mathcal{A}$.

Summary of computation

We show that massless N-flavor QCD has a mixed 't Hooft anomaly between

$$\frac{SU(N_F)_V \times U(1)_V}{(\mathbb{Z}_{N_C}) \times (\mathbb{Z}_{N_F})} \quad \text{and} \quad (\mathbb{Z}_{2N})_{\mathrm{axial}}.$$

The anomaly is given as $(N_C = N_F = N)$

$$\mathcal{Z}[(A_\mathrm{f}, A_\mathrm{q}, B_\mathrm{c}, B_\mathrm{f})] \mapsto \mathcal{Z}[(A_\mathrm{f}, A_\mathrm{q}, B_\mathrm{c}, B_\mathrm{f})] \exp\left(-\frac{\mathrm{i}N}{2\pi}\int B_\mathrm{c}\wedge B_\mathrm{f}\right).$$

We will discuss consequences of this anomaly.

Application: Constraint on the phase diagram of massless $\mathbb{Z}_3$-QCD

Symmetry and Anomaly of massless $\mathbb{Z}_N$-QCD

There are three symmetries for massless $\mathbb{Z}_N$-QCD:

$$(\mathbb{Z}_N)_{\mathrm{shift,center}}, \quad \frac{U(1)_F^{N-1}\times U(1)_V}{(\mathbb{Z}_N)_C\times(\mathbb{Z}_N)_F}, \quad \text{and} \quad (\mathbb{Z}_{2N})_{\mathrm{axial}}$$

We can gauge $(\mathbb{Z}_N)_{\mathrm{shift}}$ and $[U(1)_F^{N-1}\times U(1)_V]/[(\mathbb{Z}_N)_C\times(\mathbb{Z}_N)_F]$ by introducing the following two-form gauge fields (+ ordinary ones):

$$B_{\mathrm{c}}=B_{\mathrm{c}}^{(1)}\wedge L^{-1}\mathrm{d}x^4+B_{\mathrm{c}}^{(2)}, \quad B_{\mathrm{f}}=B_{\mathrm{f}}^{(2)}.$$

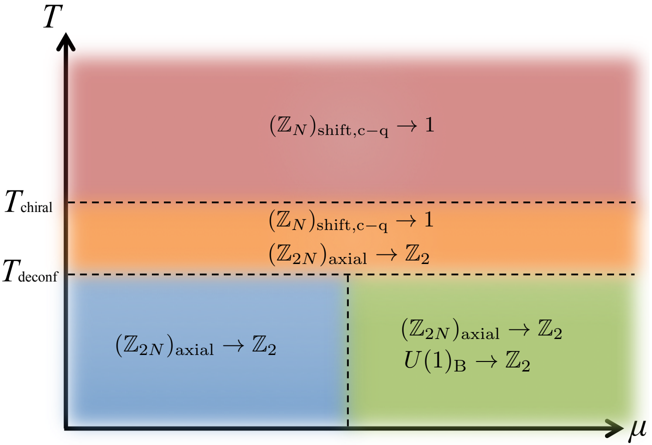
Substituting it into the four-dimensional anomaly, we obtain the anomaly for massless $\mathbb{Z}_N$-QCD:

$$\mathcal{A}=-\frac{N}{2\pi}\int B_{\mathrm{c}}^{(1)}\wedge B_{\mathrm{f}}^{(2)}\in\frac{2\pi}{N}\mathbb{Z}.$$

(1710.08923, 1711.10487)

One of possible phase diagrams

Anomaly matching says that the phase is nontrivial at any T and μ:



(1711.10487)

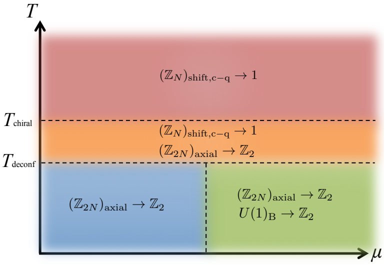
Comment on Possible phase diagrams

We have a mixed 't Hooft anomaly between $(\mathbb{Z}_N)_{\rm shift}$, $\frac{U(1)_F^{N-1}\times U(1)_V}{(\mathbb{Z}_N)_C\times(\mathbb{Z}_N)_F}$, and $(\mathbb{Z}_{2N})_{\rm axial}$.

1. System is conformal or breaks some of symmetries
2. When anomaly is matched by SSB,

$$T_{\rm deconf}(\mu)\le T_{\rm chiral}(\mu)$$

if flavor is unbroken.



Note: These constraints are not necessarily satisfied by PNJL model. Ginzburg-Landau description does not respect anomaly matching, so we must go beyond!

Summary

- We find a new 't Hooft anomaly of massless QCD.
  Anomaly matching gives nonperturbative constraints on low-energy physics.
- Application to massless $\mathbb{Z}_N$-QCD:
  The phase is always non-trivial at any T and μ.
- If the vector-like flavor is unbroken, $T_{\mathrm{deconf}} \leq T_{\mathrm{chiral}}$;

$$\text{Confinement} \Rightarrow \text{Chiral symmetry breaking}$$