Tensor Models for Black Hole Probe

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Summary

- Recently there have been models of near extremal black holes (and possibly examples of NAdS_2/NCFT_1).
- Such black holes Hawking radiate.
  Model for a probe interacting with such black holes?
- We propose a class of tensor models as models of black hole interacting with a probe.
- Non- trivial features:
  symmetries of near horizon geometry,
  black hole (partially) inflicts maximal chaos into the probe.

Outline

- Introduction
- Our Model
- Future directions

Introduction

Near-extremal black holes

A. Pattern of symmetry breaking:

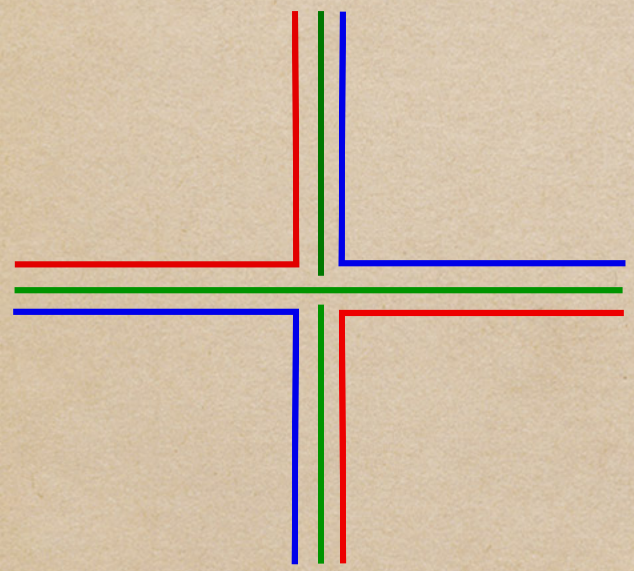
- Near horizon geometry of extremal black holes contains a AdS_2 factor.
  Near horizon geometry of near-extremal black holes contains a near-AdS_2
- AdS_2 => Asymptotic symmetries are all reparameterizations of asymptotic boundary circle, but AdS_2 metric preserves only a SL(2, R) subgroup of this.
  => Spontaneous breaking of reparameterization to SL(2,R).
- Near AdS_2 => spontaneous and explicit breaking of reparameterization symmetry.

B. Maximal Chaos:

(Non-extremal) black holes are maximally chaotic systems.
explicit breaking of reparameterization symmetry should be related to maximal chaos.

Tensor models for near-extremal black holes

- Certain tensor models exhibit the same pattern of symmetry breaking and maximal chaos, in "large N" limit and deep IR.
- Simplest such tensor model is Carozza-Tanasa-Klebanov-Tarnopolsky (CTKT) Model



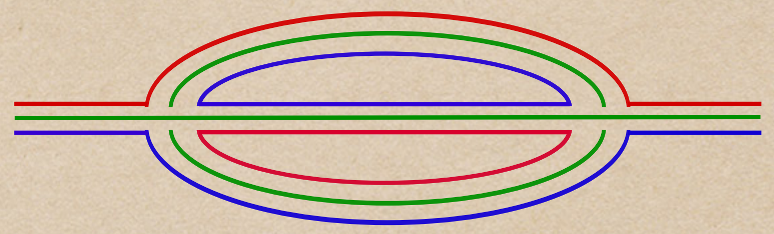
$$H_{CTKT} = \frac{J}{4N^{3/2}}\psi_{abc}\psi_{ab'c'}\psi_{a'bc'}\psi_{a'b'c}$$

ψ_{abc} is a real fermion.

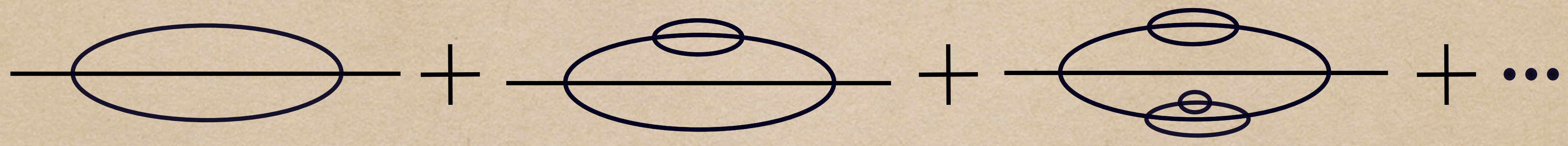
a,b,c = 1, …,N

Two point function in CTKT Model

- Simplest leading correction to free propagator



- Replace any propagator by the above diagram



Two point functions in CTKT model (contd.)

- Schwinger Dyson eqn:

$$\text{large N} \Rightarrow \Sigma(t_1, t_2) = J^2 G(t_1, t_2)^3$$

$$\text{deep IR} \Rightarrow G(\omega) = \frac{1}{-i\omega - \Sigma(\omega)}$$

$$\text{large N and deep IR} \Rightarrow J^2 \int dt\, G(t_1, t)G(t, t_2)^3 = -\delta(t_1 - t_2)$$

- Reparameterization symmetry:

$$G(t_1, t_2) \to \left| \frac{df(t_1)}{dt_1} \frac{df(t_2)}{dt_2} \right|^{1/4} G(f(t_1), f(t_2))$$

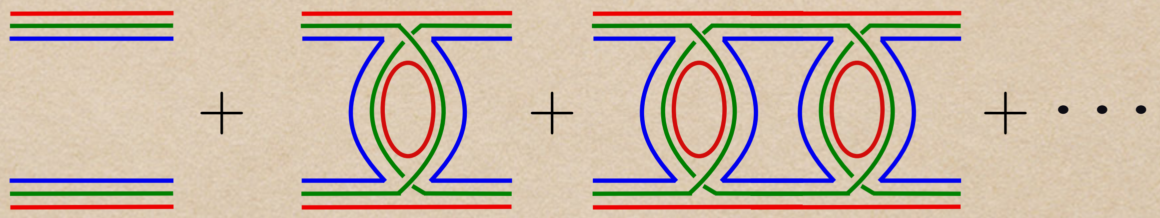
spontaneously broken down to SL(2,R) by solution

$$G(t) = \frac{1}{(4\pi J^2)^{1/4}} \frac{\operatorname{sgn}(t)}{|t|^{1/2}}$$

Four point functions in CTKT model

- connected four point function

$$\frac{1}{N^6}\langle\psi_{abc}(t_1)\psi_{abc}(t_2)\psi_{a'b'c'}(t_3)\psi_{a'b'c'}(t_4)\rangle = G(t_1,t_2)G(t_3,t_4) + \frac{1}{N^3}\mathcal{F}(t_1,t_2;t_3,t_4)$$



$\mathcal{F}_{n+1} \sim \int K * \mathcal{F}_n$ where K =  commutes with SL(2,R) gen.

$$\mathcal{F} = (1 + K + K^2 + \dots)\mathcal{F}_0 = \frac{1}{1-K}\mathcal{F}_0$$

- K has an eigenvalue 1, when evaluated in conformal limit.
  For this eigenspace, K must be evaluated away from conformal limit, thus reparameterization symmetry is explicitly broken.

Spectrum of primaries

- K commutes with SL(2,R) generators.

  Utilise conformal symmetry:

$$\mathcal{F}(t_1, t_2; t_3, t_4) = G_\psi(t_{12}) G_\psi(t_{34}) \mathcal{F}(\chi), \quad \text{where} \quad \chi = \frac{t_{12}t_{34}}{t_{13}t_{24}}$$

- In the limit $\chi \to 0$, $\mathcal{F}(\chi) \sim \sum_{m=1}^{\infty} c_m^2 \chi^{h_m}$

primary of dimension $h_n \Rightarrow$ bulk scalar of mass $m_n^2 = h_n(h_n - 1)$

- NOT a sparse spectrum.

  No truncation to SUGRA expected.

  (see 1712.02725, 1711.09839 for discussion on holographic dual)

A Tensor Model for Black Hole Probe

Guessing a probe model

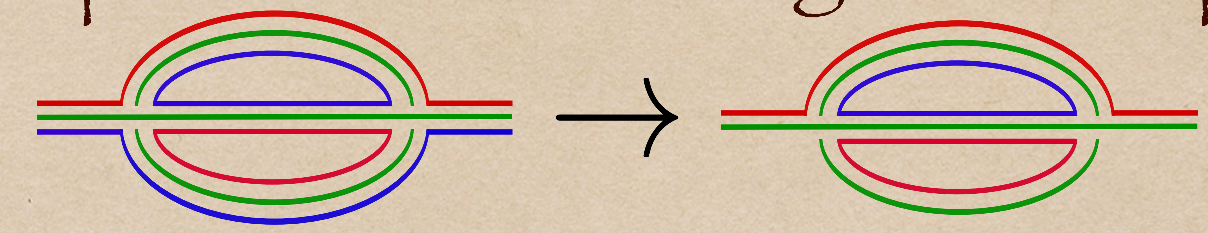
- Had these tensors been matrices, (which they are not), one would think of the black hole as a bound state of D0 branes and them matrices describing strings stretched between D0 branes.
- An external D0 brane would be a probe.
  probes would be described by vectors.
- Naive tensor analog: probes could be tensors of lower rank.

Desirable properties for the Black Hole Probe

- Probe does not affect the black hole dynamics to leading order.
- Emergent reparameterization symmetry and the pattern of its breaking should be intact.
  non trivial because this symmetry is not visible in the Lagrangian.
- Infliction of maximal chaos.

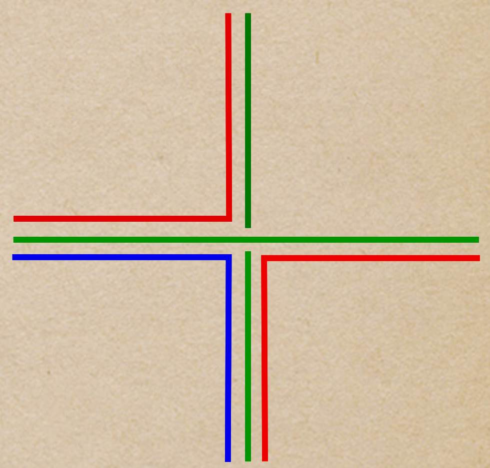
Our Model

- removing an open line do not change the N dependence.



One can go on inserting this diagram in internal propagators to get new leading diagrams.

- These diagrams can come from a vertex like



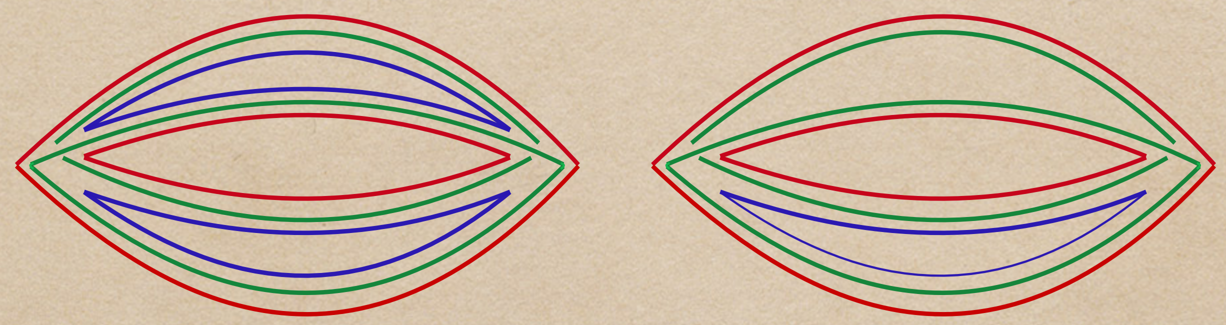
Above method generates all leading diagrams

- Suggests the Hamiltonian

$$H = H_{CTKT} + \frac{g_1}{N^{3/2}} \psi_{abc}\psi_{a'b'c}\kappa_{ab'}\kappa_{a'b}$$

ψ physics intact in large N limit

- Leading contributions to two and four point functions can be obtained by cutting "melons".
- κ melons are subleading compared to ψ melons.



Propagators

- New diagrams give subleading contribution to ψ propagators.

$$G_\psi(t) = \frac{1}{(4\pi g_0^2)^{1/4}} \frac{\text{sgn}(t)}{|t|^{1/2}}$$

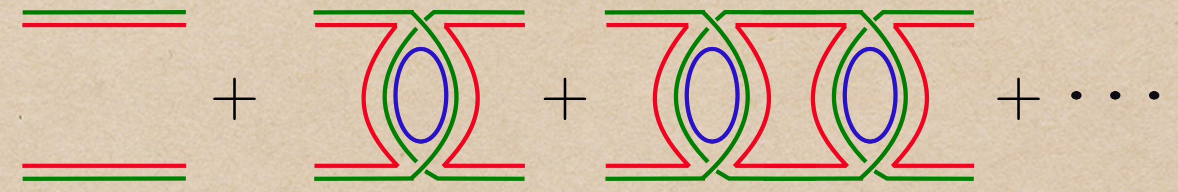
- G_κ satisfy same (up to constants) Schwinger Dyson equation as G_ψ

$$G_\kappa(t) = \frac{g_0}{g_1} G_\psi(t)$$

$\langle \kappa\kappa\kappa\kappa \rangle$

$$\frac{1}{N^6}\langle \kappa_{ab}(t_1)\kappa_{ab}(t_2)\kappa_{ab}(t_3)\kappa_{ab}(t_4)\rangle = G_\kappa(t_{12})G_\kappa(t_{34}) + \frac{1}{N^3}\mathcal{F}^\kappa(t_1,t_2;t_3,t_4)$$

- Diagrams contributing to $\mathcal{F}^\kappa$ are



$$\mathcal{F}^\kappa = \frac{g_0^2}{g_1^2}\frac{1}{1-\frac{1}{3}K}\mathcal{F}_0^\psi$$

- No breaking of conformal symmetry.

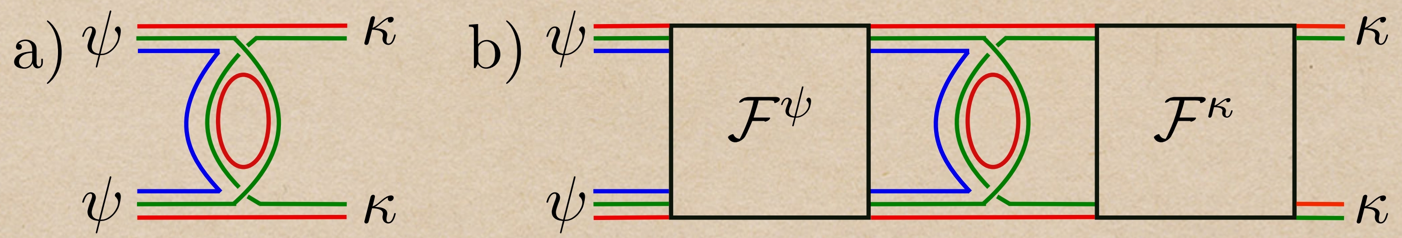
$$\mathcal{F}^\kappa(\tau_1,\ldots,\tau_4) = G_\kappa(\tau_{12})G_\kappa(\tau_{34})\mathcal{F}^\kappa(\chi)$$

$$\lim_{\chi\to 0}\mathcal{F}^\kappa(\chi) = \sum_{m=1}^{\infty}\tilde{c}_m^2\chi^{\tilde{h}_m}$$

- Primaries of different dimensions, but in natural correspondence with those of the black hole.

$\langle \psi\psi\kappa\kappa \rangle$

$$\frac{1}{N^5}\sum \langle \psi_{abc}(t_1)\psi_{abc}(t_2)\kappa_{a'b'}(t_3)\kappa_{a'b'}(t_4)\rangle = G^{\psi}(t_1,t_2)G^{\kappa}(t_3,t_4) + \frac{1}{N^3}\mathcal{F}^{\psi\kappa}$$



$$\mathcal{F}^{\psi\kappa} = \frac{2g_0}{3g_1}\frac{K}{(1-K)(1-\frac{1}{3}K)}\mathcal{F}_0^{\psi}$$

$$\frac{\mathcal{F}^{\psi\kappa}(\tau_2,\tau_2,\tau_3,\tau_4)}{G^{\kappa}(\tau_{12})G^{\psi}(\tau_{34})} = \mathcal{F}^{\psi\kappa}_{conf}(\chi) + \mathcal{F}^{\psi\kappa}_{non-conf}(\tau_1,\tau_2,\tau_3,\tau_4)$$

$$\lim_{\chi\to 0}\mathcal{F}^{\psi\kappa}_{conf}(\chi) = \sum_{m=1}^{\infty}\left[c_m^2\chi^{h_m} - \tilde{c}_m^2\chi^{\tilde{h}_m}\right]$$

- primaries of same dimensions appear.

Chaos

- Semi-classical intuition:

$$\frac{\partial q(t)}{\partial q(0)} \sim e^{\lambda t} \;\; \Rightarrow \langle [p(0), q(t)]^2 \rangle \sim e^{\lambda t}$$

- Replace position and momenta by general operators and separate two commutators along thermal circle.

$$tr\left[V(t+3i\beta/4)\ W(i\beta/2)\ V(t+i\beta/4)\ W(0)\right]_\beta$$

This is an Out of Time Order Correlator (OTOC).

- For us

$$\text{case 1: } V=\psi, W=\psi$$

$$\text{case 2: } V=\kappa, W=\kappa$$

$$\text{case 3: } V=\psi, W=\kappa$$

Chaos continues ...

- For four point functions are of the form
$$\mathcal{F} = \frac{f(K)}{(1-K)^l}\mathcal{F}_0$$
corresponding OTOC is maximally chaotic.
- $\mathcal{F}^{\psi}$, $\mathcal{F}^{\psi\kappa}$ are maximally chaotic.
$\mathcal{F}^{\kappa}$ turns out to be non-chaotic.

Some Cousin Models

- Restoring permutation symmetry
- Adding vector fields
- Coloured probe model

Disordered probe model

(arXiv: 1806.????)

- Probe fields are n in number.

  1<<n<<N.

$$H \sim H_{SYK} + j'\psi^{q-p}\kappa^p$$

- F^{ψ} and $F^{\psi\kappa}$ are maximally chaotic.

  F^{κ} has Lyapunov coefficient. smaller than, but order of the maximal value, $2\pi T$.

- In the limit $q \to 1, \frac{p}{q} \to 1$

  ALL four point functions are maximally chaotic.

  Probe primaries make a copy of the BH primaries.

Scorecard

- Our models satisfy the non trivial requirement of symmetry breaking and partially fulfil that of maximal chaos.
- Probe spectrum is in natural correspondence with black hole spectrum.
- Significant improvement for disordered probe model.

Future Directions

- Study thermalisation, i.e. test ETH.
- Higher point functions.


Merci Beaucoup !