Aspects of Large D Matrix Models and SYK-like Physics

Tatsuo Azeyanagi
(ULB, Brussels)

Based on works
with F. Ferrari, P. Gregori, L. Leduc, G. Valette and F. Schaposnik

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SYK Model

$$H = \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \psi_i^\dagger \psi_j^\dagger \psi_k \psi_l$$

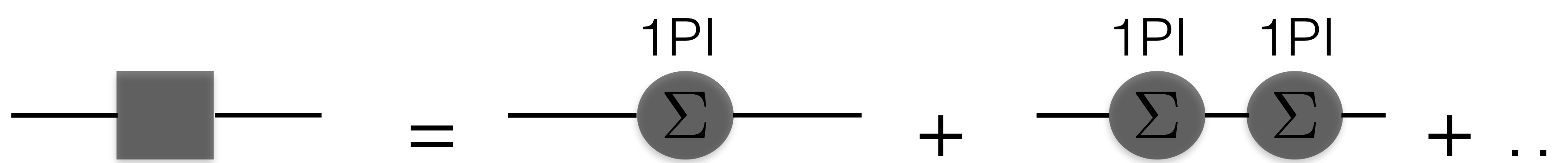
$\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$ $\quad$ J_{ijkl} : disorder coupling $\quad$ $\langle J_{ijkl}^2 \rangle \sim J^2/N^3$

Sachdev-Ye, Kitaev, Maldacena-Stanford
Polchinski-Rosenhaus, …

- 1d fermionic system with disorder
- Holographic description of near extremal black hole

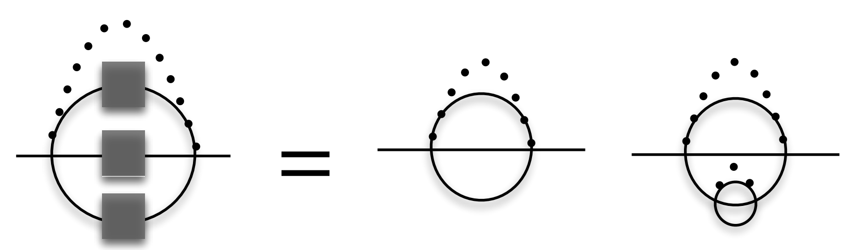
Dominance by Melons

- Schwinger-Dyson eq.



$$1/G_\omega = 1/G_\omega^{(0)} + \Sigma_\omega$$

$\Sigma(t) =$



$= J^2 G(t)^2 G(-t)$

Dominance by melon diagrams

- Approximate scaling behavior in IR

$$G(t) \sim \frac{sign(t)}{|t|^{2\Delta}} \qquad \Delta = 1/4$$

Motivation

- Random coupling in SYK model

→ unfamiliar object in AdS/CFT J_{ijkl}

- Tensors in SYK-like tensor models ϕ^{abc}

Gurau, Witten, Carozza-Tanasa, Klebanov-Tarnopolsky…

→ unfamiliar object in string theory

SYK-like models closer to string-based holographic models?

SYK-like models based on matrix?

Large D Matrix Model

Ferrari, TA-Ferrari-Gregori-Leduc-Valette, Ferrari-Rivasseau-Valette

Large D Matrix Model

$$H_{int} = NDg\,\mathrm{tr}\left(\phi_\mu \phi_\nu^\dagger \phi_\mu \phi_\nu^\dagger\right)$$

ϕ_μ : NxN fermionic/bosonic matrix $(\mu = 1, 2, \cdots D)$

- 1d QM with D matrices
- U(N)xU(N)xO(D) symmetry
- Large N limit + (enhanced) large D limit:

$N \to \infty, D \to \infty$ with $\lambda = gD^{-\frac{1}{2}}$ fixed

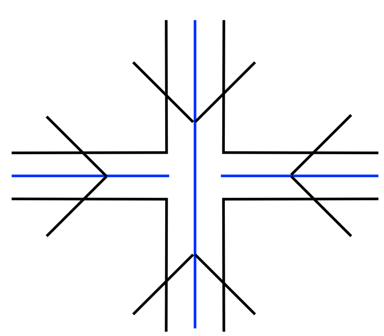
Large D Matrix Model

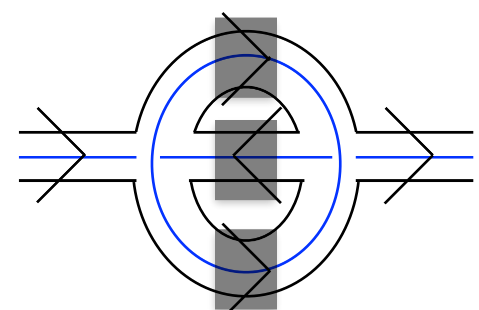
- Free energy

$$F = \sum_{g_{\text{genus}}=0}^{\infty} N^{2-2g_{\text{genus}}} \sum_{\ell=0}^{\infty} D^{\underline{1+g_{\text{genus}}-\frac{\ell}{2}}} F_{g_{\text{genus}},\ell}(\lambda)$$

Large N first, large D second

- 2pt function



$\Sigma(t) =$  $\sim \lambda^2 G^3$

$$(N^2 D) \times \left(\frac{1}{ND}\right)^3 \times (ND^{\frac{3}{2}})^2 = ND$$

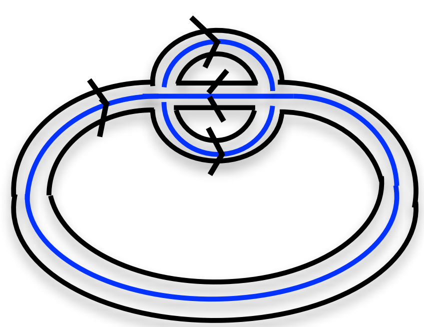
loop propagator vertex

Dominated by melon diagrams

Comparison

Usual vector-like large D limit

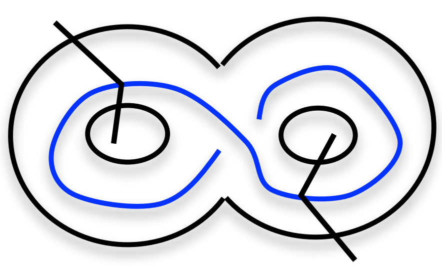
$$D \to \infty \quad \text{with} \quad \lambda = g \quad \text{fixed}$$



$$(N^4 D^2) \times \left(\frac{1}{ND}\right)^4 \times (ND)^2 = N^2 D^0$$

loop propagator vertex

enhancement factor $D^{\frac{1}{2}\times 2}$



$$(N^3 D^1) \times \left(\frac{1}{ND}\right)^2 \times (ND) = N^2 D^0$$

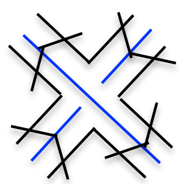
loop propagator vertex

$D^{\frac{1}{2}}$

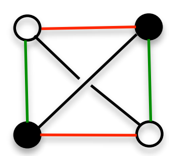
Melons dominate in the enhanced large D limit

Comment

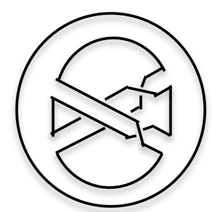
- Large D limit exists for general single trace interactions



Stranded Graph



Colored Graph



Ribbon Graph

$g_{vertex} = 1/2$ → Large D scaling $\lambda = g D^{-g_{vertex}}$

Genus of Ribbon Graph

- Generalization to multi-trace interaction exists

$$Z = \int D\phi \exp\left(-S_{\text{single}} - \tilde{\lambda} \int \text{tr}(...)\text{tr}(...)...\text{tr}(...)\right)$$

→ Large D expansion well-defined for correlation functions

- Hermitian models well-defined in planar limit

- Closely related to Klebanov-Tarnopolsky tensor models

Phase Structure

TA-Ferrari-Schaposnik

Complex Model with Mass

- Large D model with mass

$$H = NDtr\left(m\psi^\dagger_\mu\psi_\mu + \frac{1}{2}\lambda D^{1/2}\psi_\mu\psi^\dagger_\nu\psi_\mu\psi^\dagger_\nu\right)$$

- Schwinger-Dyson eq.

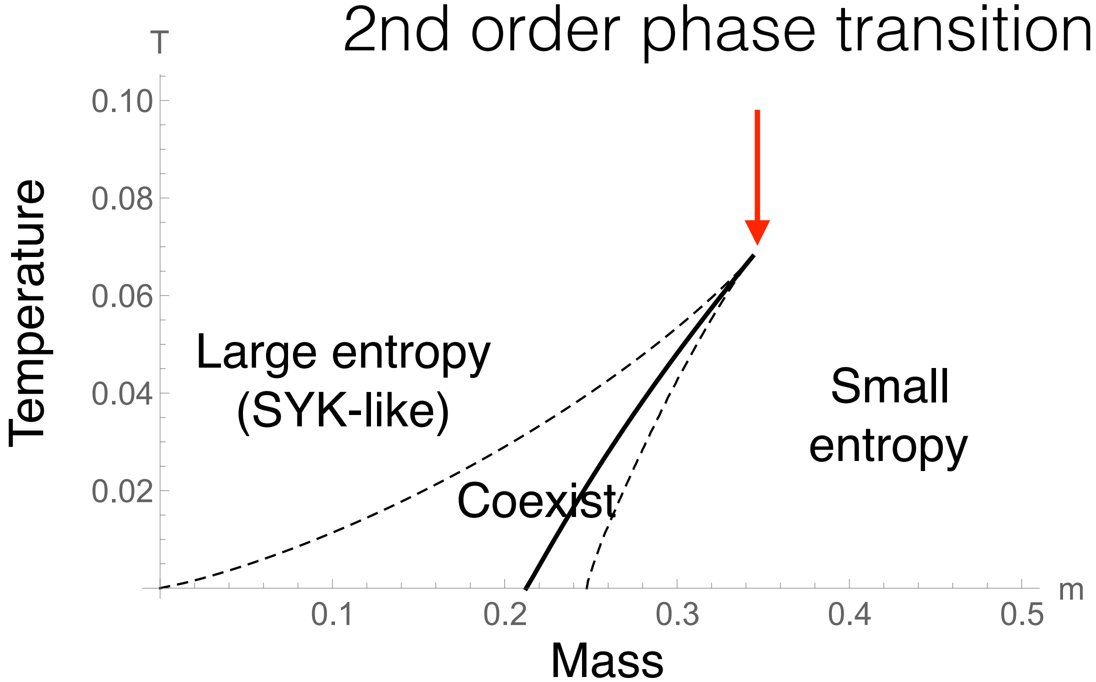
$$1/G_k = 1/G_k^{(0)} + \Sigma_k \quad \text{with} \quad \Sigma(t) = \lambda^2 G(t)^2 G(-t)$$

(Same for complex SYK and complex KT models)

Phase Diagram

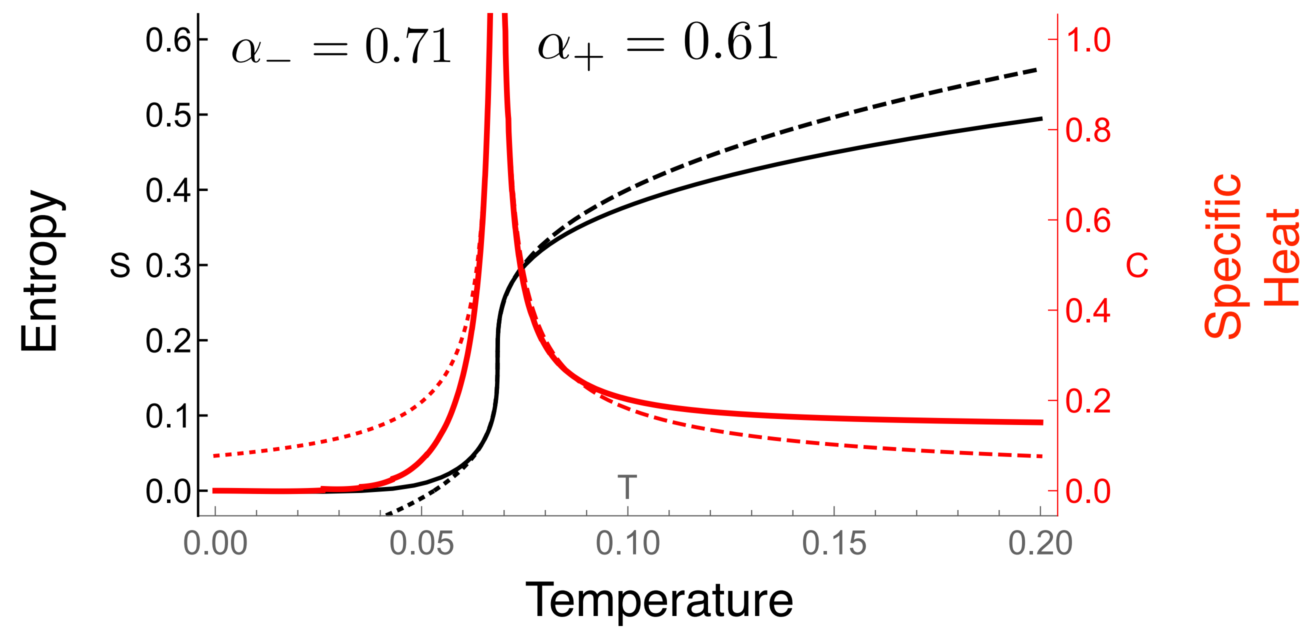
Dimensionless coupling $\frac{\lambda}{m}$ $\frac{\lambda}{T}$

2nd order phase transition



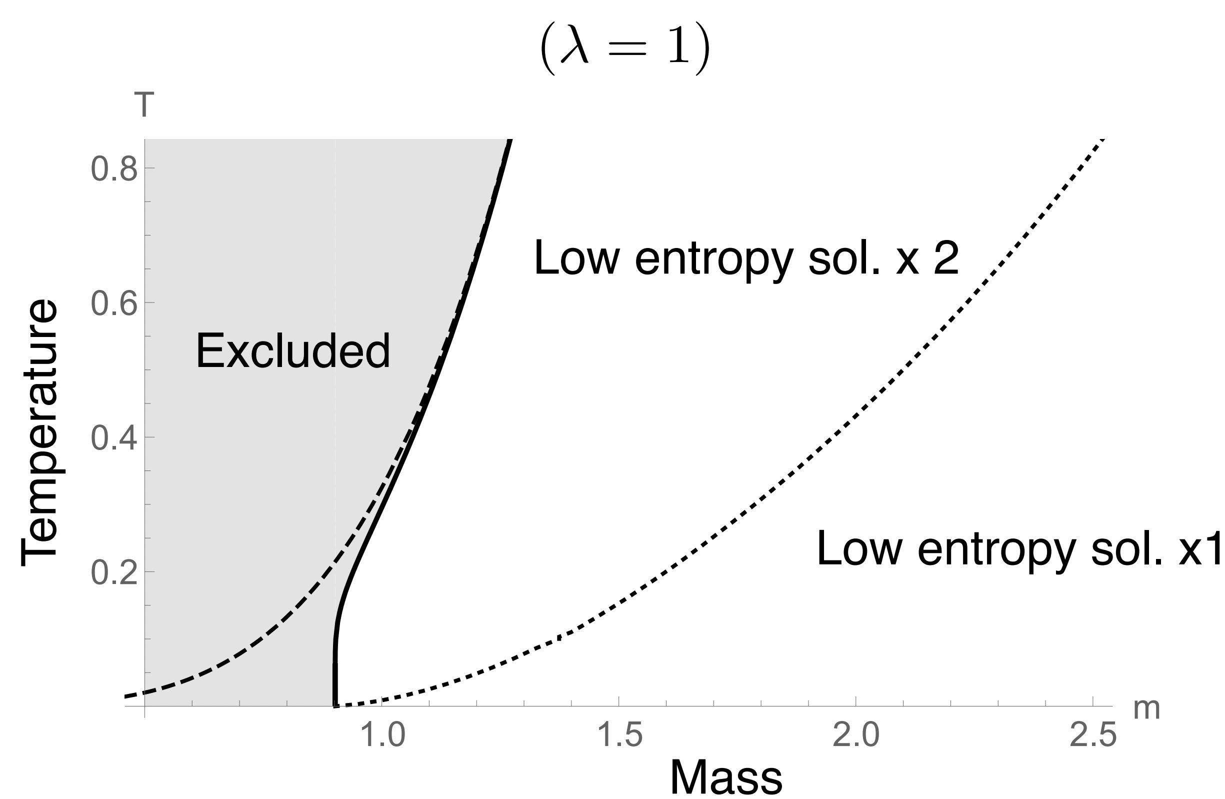
$(\lambda = 1)$

$C \sim |T - T_c|^{-\alpha}$



Critical exponents depend on how to approach the critical point

Bosonic Model



No SYK-like scaling region in IR

c.f. Klebanov-Tarnopolsky

Summary and Outlook

- SYK physics from large D matrix model

- Phase structure of large D matrix model
  - 2nd-order phase transition in fermonic model
  - No SYK-like IR scaling for bosonic model

- IR scaling with bosonic degrees of freedom?
  - Boson+fermion, SUSY ?

- Relation to large D gravity ?

Thank You!