

Transverse-momentum dependence of gluon distributions at small- x

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Contents of the talk

- The need for TMDs at colliders

TMDs = transverse-momentum-dependent parton distributions

- The context for this talk: forward di-jets at the LHC

their structure of may be modified in p+Pb vs p+p collisions

- Gluon TMDs in the small-x limit

their (non-linear) QCD evolution can be obtained from the so-called JIMWLK equation

- Numerical results

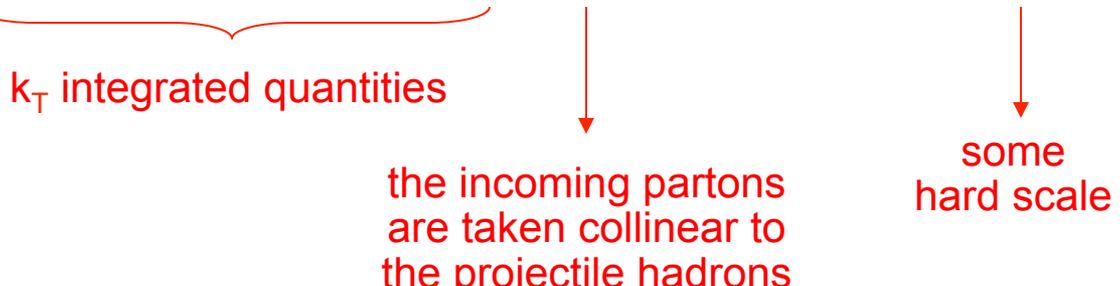
new insight regarding the low-momentum behavior (gluon saturation regime)

The need for TMDs at
hadron (and other) colliders

Collinear factorization

in standard pQCD calculations, the incoming parton transverse momenta are set to zero in the matrix element and are integrated over in the parton densities

$$d\sigma_{AB \rightarrow X} = \sum_{ij} \int dx_1 dx_2 \underbrace{f_{i/A}(x_1, \mu^2) f_{j/B}(x_2, \mu'^2)}_{\text{k}_\perp \text{ integrated quantities}} d\hat{\sigma}_{ij \rightarrow X} + \mathcal{O}(\Lambda_{QCD}^2/M^2)$$



the incoming partons
are taken collinear to
the projectile hadrons

some
hard scale

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↓

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↓

some
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in general for a hard process, this approximation is accurate
in some cases however, this is not good enough (examples follow)

TMD factorization is a more advanced QCD factorization framework
which can be useful and sometimes is even necessary

Drell-Yan process

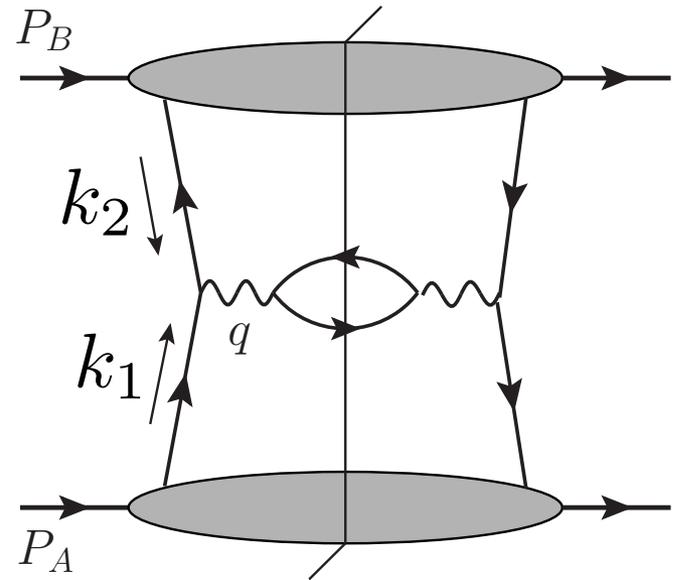
the transverse momentum of the lepton pair q_T is the sum of the transverse momenta of the incoming partons

$$d\hat{\sigma} \propto \delta(k_{1t} + k_{2t} - q_T)$$

so in collinear factorization

$$d\sigma^{AB \rightarrow l^+ l^- X} \propto \delta(q_T) + \mathcal{O}(\alpha_s)$$

and TMDs could be useful here



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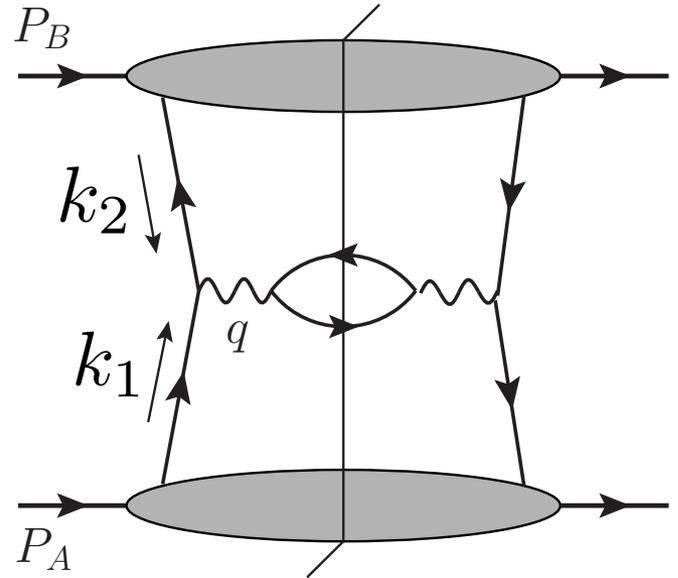
$$d\sigma^{AB \rightarrow l^+ l^- X} \propto \delta(q_T) + \mathcal{O}(\alpha_s)$$

and TMDs could be useful here

naively, TMD factorization is

$$d\sigma^{AB \rightarrow l^+ l^- X} = \sum_{i,j} \int dx_1 dx_2 d^2 k_{1T} d^2 k_{2T} f_{i/A}(x_1, \mathbf{k}_{1T}) f_{j/B}(x_2, \mathbf{k}_{2T}) d\hat{\sigma}^{ij \rightarrow l^+ l^- X}$$

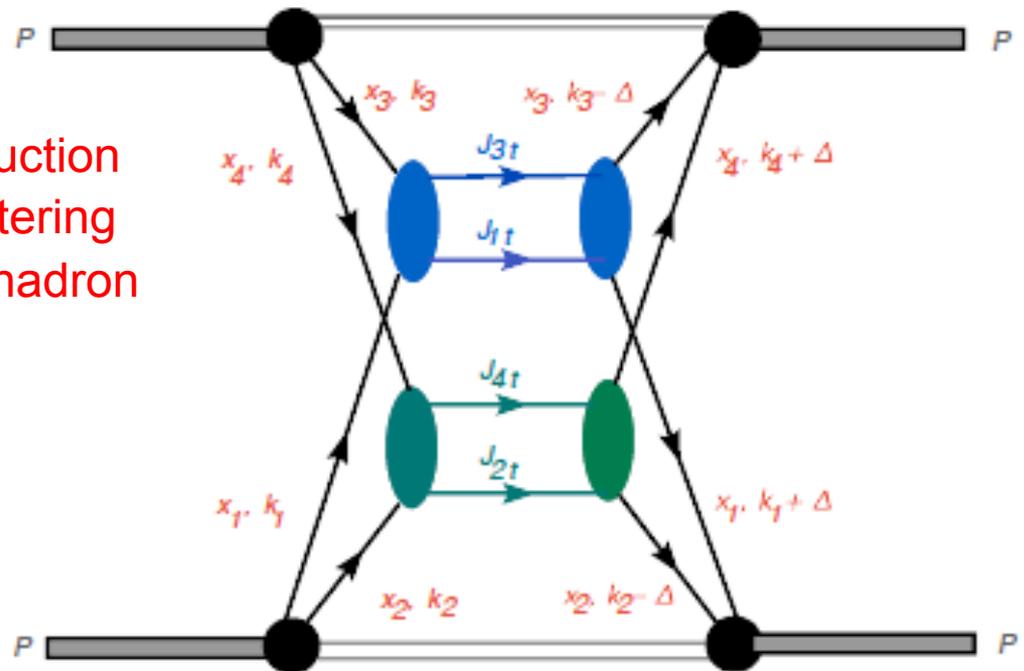
(but unfortunately, there are complications)



Multiple parton interactions

keeping track of partonic transverse momenta is also crucial to describe multiple partonic interactions

consider for instance: 4-jet production coming from a double hard scattering of two partons in each incoming hadron



Spin physics

TMDs are crucial to describe hard processes in polarized collisions
(e.g. Drell-Yan and semi-inclusive DIS)

8 leading-twist TMDs

Sivers function

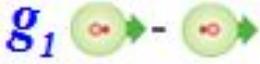
correlation between transverse spin of the nucleon and transverse momentum of the quark

Boer-Mulders function

correlation between transverse spin and transverse momentum of the quark in unpolarized nucleon

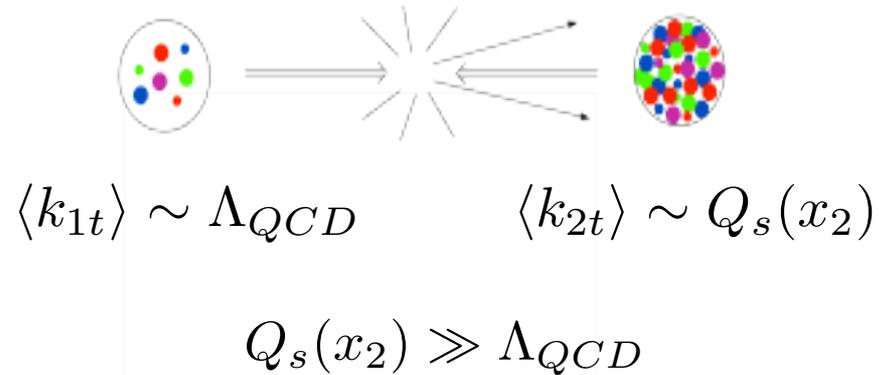
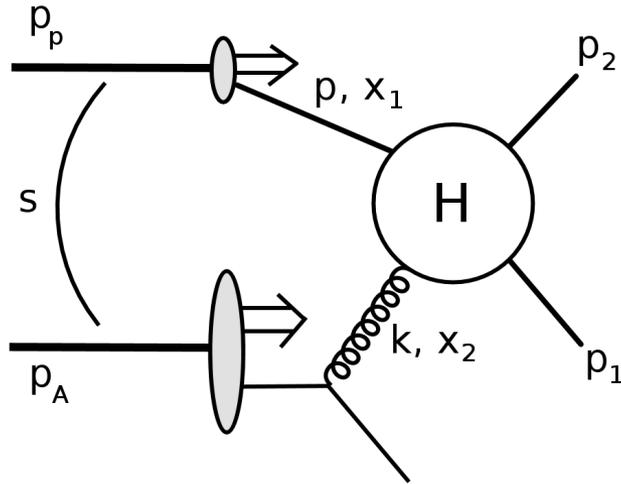
nucleon polarization

quark polarization

	U	L	T
U	f_1 number density q 		f_{1T}^\perp Sivers 
L		g_1 helicity Δq 	g_{1T} 
T	h_1^\perp Boer Mulders 	h_{1L}^\perp 	h_1 transversity  h_{1T}^\perp 

Our context: forward di-jets

- large-x projectile (proton) on small-x target (proton or nucleus)



so-called “dilute-dense” kinematics

Incoming partons' energy fractions:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}) \quad \xrightarrow{y_1, y_2 \gg 0} \quad x_1 \sim 1$$

$$x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}) \quad \xrightarrow{y_1, y_2 \gg 0} \quad x_2 \ll 1$$

Gluon's transverse momentum (p_{1t} , p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos \Delta\phi \quad |p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

prediction: modification of the k_t distribution in p+Pb vs p+p collisions

The gluon TMDs involved
in the di-jet process

TMD gluon distributions

- the naive operator definition is not gauge-invariant

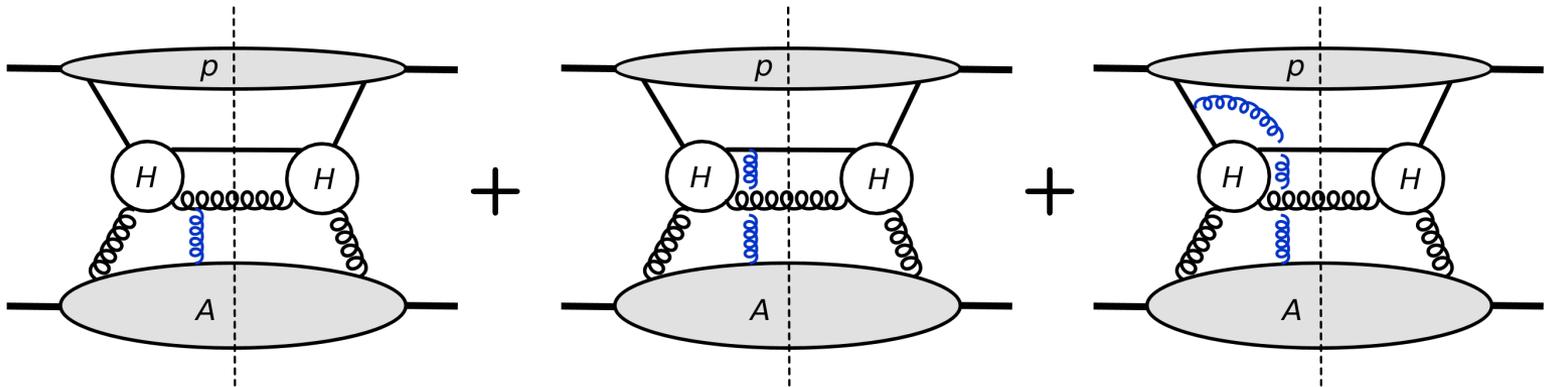
$$\mathcal{F}_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{i-}(0)] | A \rangle$$

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- a theoretically consistent definition requires to include more diagrams



+ similar diagrams with 2, 3, ... gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s) some gauge link $\mathcal{P} \exp \left[-ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

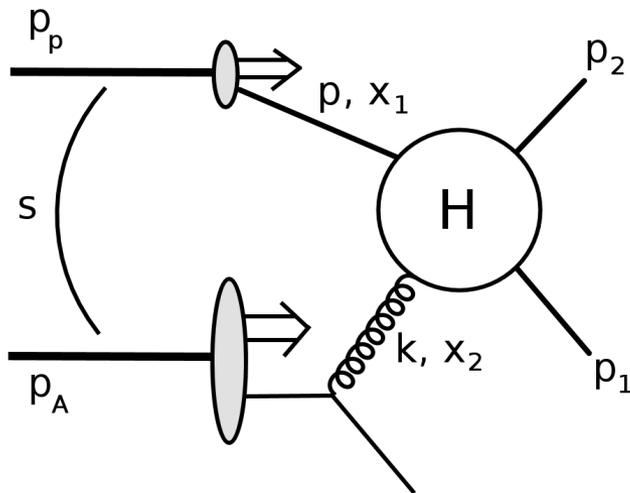
- ▶ $U_{[\alpha, \beta]}$ renders gluon distribution gauge invariant

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however, the precise structure of the gauge link is process-dependent:

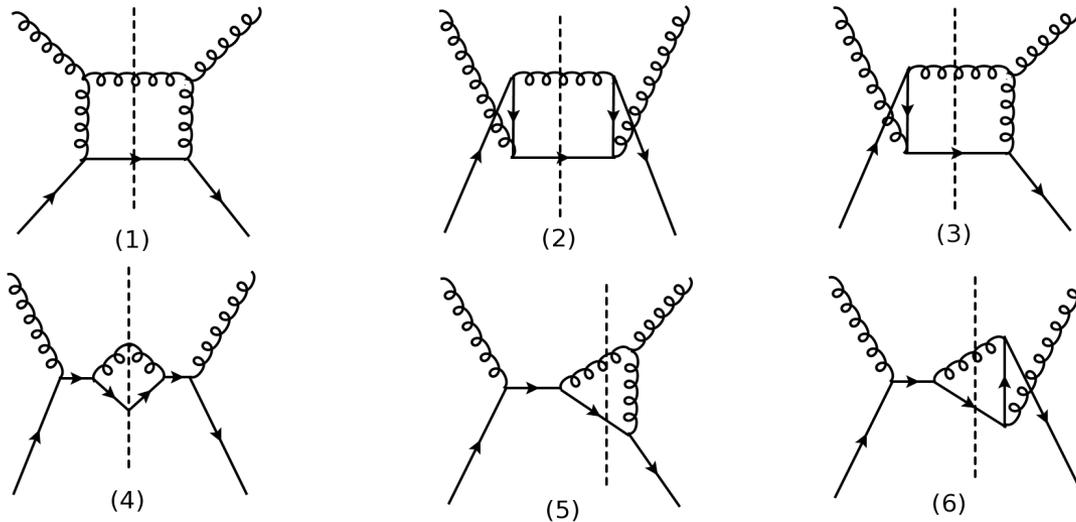
it is determined by the color structure of the hard process H

- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

TMDs for forward di-jets

- several gluon distributions are needed already for a single partonic sub-process

example for the $qg^* \rightarrow qg$ channel

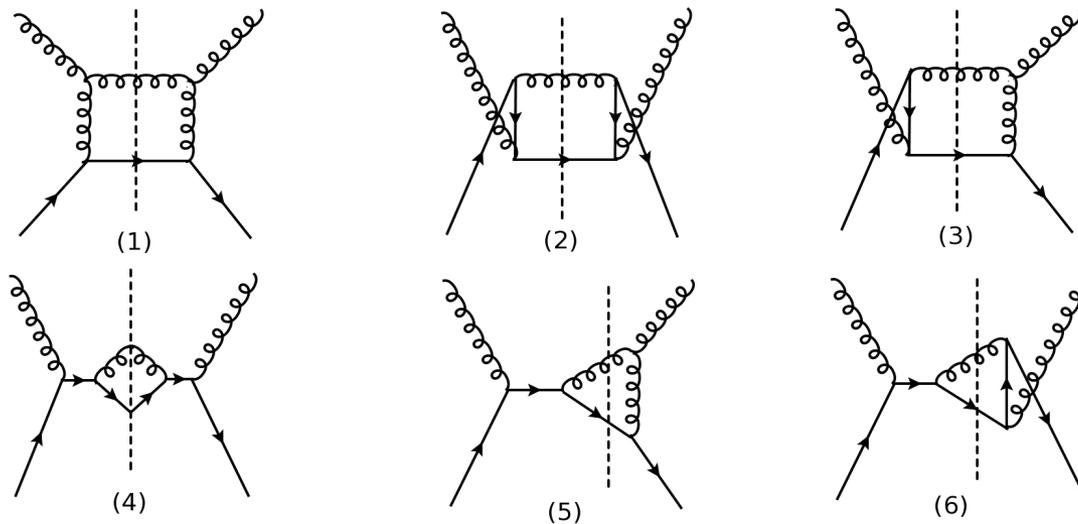


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2 unintegrated gluon distributions per channel, 6 in total: $\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$

$$qg^* \rightarrow qg \quad gg^* \rightarrow q\bar{q} \quad gg^* \rightarrow gg \quad i = 1, 2$$

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

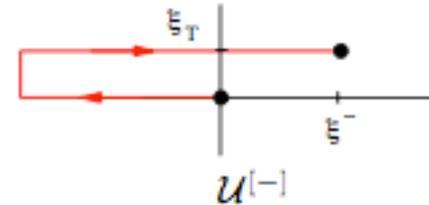
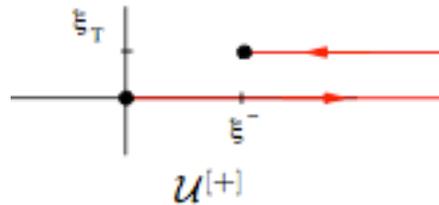
The six TMD gluon distributions

- correspond to a different gauge-link structure

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

several paths are possible for the gauge links

examples :



- when integrated, they all coincide

$$\int^{\mu^2} d^2 k_t \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2) = x_2 f(x_2, \mu^2)$$

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- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretation of the number density of gluons at small x_2

Evaluating the gluon TMDs at small- x

Gluon TMDs at small-x

- the gluon TMDs involved in the di-jet process are:

(showing here the $qg^* \rightarrow qg$ channel TMDs only)

$$\mathcal{F}_{qg}^{(1)} = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \left\langle \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(2)} = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \left\langle \text{Tr} \left[F^{i-}(\xi) \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

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- at small x they can be written as:

$$U_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \left\langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \right\rangle_{x_2}$$

$$\mathcal{F}_{qg}^{(2)}(x_2, |k_t|) = -\frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \left\langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \right\rangle_{x_2}$$

these Wilson line correlators also emerge directly in CGC calculations when $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ (the regime of validity of TMD factorization)

Outline of the derivation

- using $\langle p|p'\rangle = (2\pi)^3 2p^- \delta(p^- - p'^-) \delta^{(2)}(p_t - p'_t)$ and translational invariance

$$\int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A|O(0, \xi)|A\rangle = \frac{2}{\langle A|A\rangle} \int \frac{d^3\xi d^3\xi'}{(2\pi)^3} e^{ix_2 p_A^- (\xi^+ - \xi'^+) - ik_t \cdot (\xi - \xi')} \langle A|O(\xi', \xi)|A\rangle .$$

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- setting $\exp[ix_2 p_A^- (\xi^+ - \xi'^+)] = 1$ and denoting $\frac{\langle A|O(\xi', \xi)|A \rangle}{\langle A|A \rangle} = \langle O(\xi', \xi) \rangle_{x_2}$

we obtain e.g.

$$\mathcal{F}_{qg}^{(1)}(x_2, k_t) = 4 \int \frac{d^3x d^3y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} \left[F^{i-}(x) \mathcal{U}^{[-]\dagger} F^{i-}(y) \mathcal{U}^{[+]} \right] \right\rangle_{x_2}$$

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- then performing the x^+ and y^+ integrations using

$$\partial_i U_{\mathbf{y}} = ig \int_{-\infty}^{\infty} dy^+ U[-\infty, y^+; \mathbf{y}] F^{i-}(y) U[y^+, +\infty; \mathbf{y}]$$

we finally get $\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} \left[(\partial_i U_{\mathbf{y}}) (\partial_i U_{\mathbf{x}}^\dagger) \right] \right\rangle_{x_2}$

The other TMDs at small-x

- involved in the $gg^* \rightarrow q\bar{q}$ and $gg^* \rightarrow gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = \frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \rangle_{x_2} ,$$

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$$\mathcal{F}_{gg}^{(4)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{x}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{y}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger U_{\mathbf{x}} U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2} ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c^2} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2} .$$

with a special one singled out: the Weizsäcker-Williams TMD

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2\mathbf{x}d^2\mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \rangle_{x_2}$$

x evolution of the gluon TMDs

the evolution of Wilson line correlators with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d \ln(1/x_2)} \langle O \rangle_{x_2} = \langle H_{JIMWLK} O \rangle_{x_2}$$

Jalilian-Marian, Iancu,
McLerran, Weigert,
Leonidov, Kovner

a functional RG equation that resums the leading logarithms in $y = \ln(1/x_2)$

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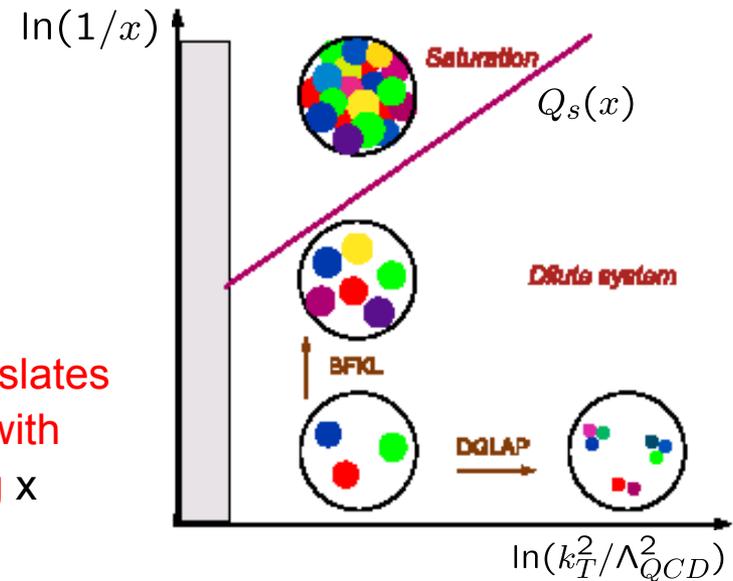
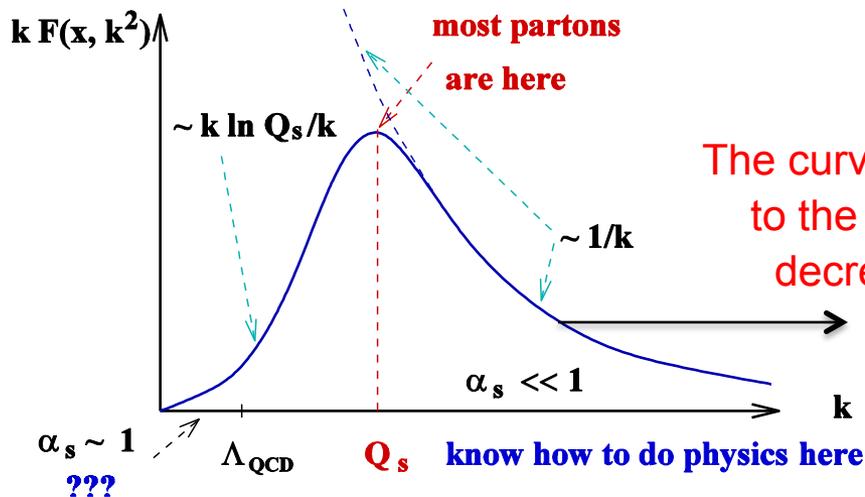
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Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in $y = \ln(1/x_2)$

- qualitative solutions for the gluon TMDs:



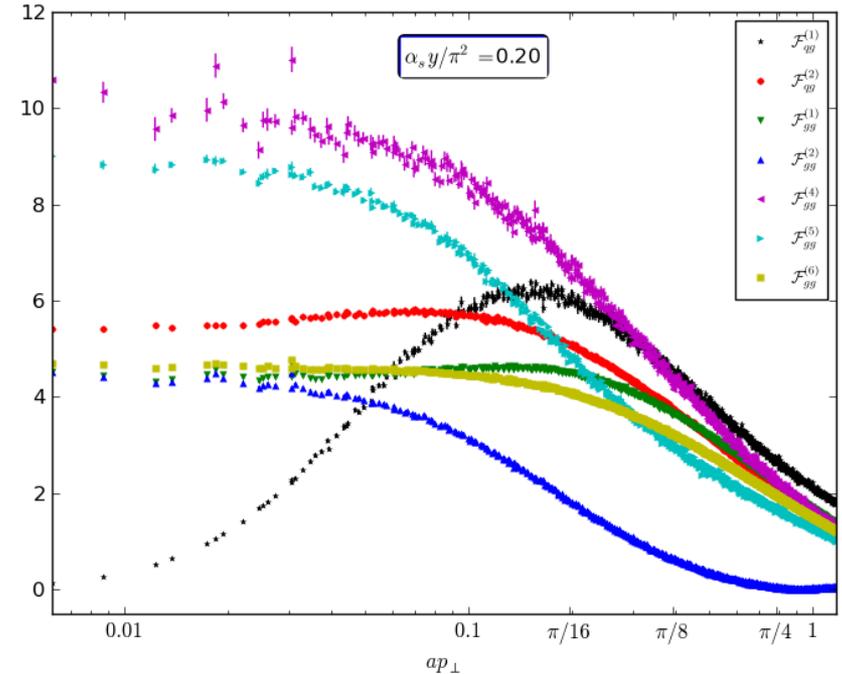
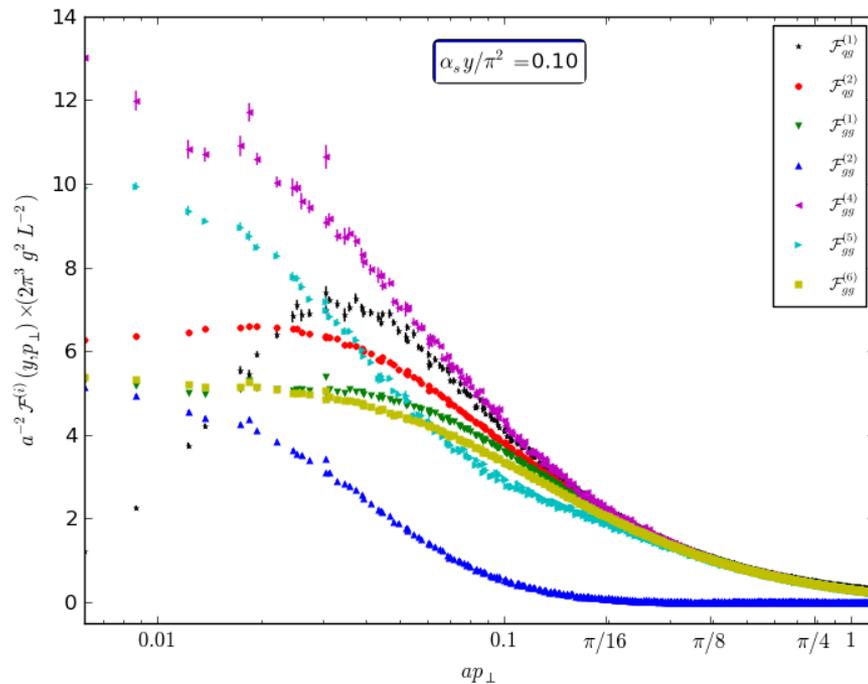
the distribution of partons as a function of x and k_T

JIMWLK numerical results

using a code written by Claude Roiesnel

initial condition at $y=0$: MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)



saturation effects impact the various gluon TMDs in very different ways

Conclusions

- different processes involve different gluon TMDs, with different operator definitions
- given an initial condition, they can all be obtained at smaller values of x , from the JIMWLK equation
- as expected, the various gluon TMDs coincide at large transverse momentum, in the linear regime
- however, they differ significantly from one another at low transverse momentum, in the non-linear saturation regime
- we have quantified these differences and they are not negligible