# Inclusive Central Production and Evidence for Conformality

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# Outline

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- Background and Motivations
- Inclusive Cross Sections
- Pomeron and AdS/CFT
- Inclusive 1P Production
- Evidence of Conformality at LHC

Goals

- Motivate why understanding CFT is important for scattering
- Inclusive distributions are well described as Wightman discontinuities
- CFT "cross sections" can also be described as discontinuities
- Non-perturbative Pomeron can be used to show conformal behavior at the LHC

# Physical Motivations: The Issues that Keep Me Up at Night

QCD has been a resounding success for describing some areas of strong-force physics: Flavor flow, Color flow, Asymptotic Freedom ($\beta < 0$ CFT), etc.. But there are still physical regimes that are not well understood: n-particle scattering (amplitudes), strong coupling, confinement, etc.

Object of interest (observables) are usually related to "scattering amplitudes" (correlation functions) which tell us what particles, interactions, symmetries, etc...

QCD, QCD-extensions, holographic models, gravity, ... it's all *complicated!*. So let's look for (model-independent) way's to simplify the physics.

High energy scattering exhibits comparatively distinct and simple physical and analytic behavior: scaling, unitarity, pole structure, etc.

What scattering processes probe this physics: Deep Inelastic Scattering using a simple probe to better understand hadrons, Dijets with a rapidity gap or tagged proton(s), particle scattering near black hole horizon (SYK), etc..

# Wightman Functions

Scattering amplitudes are traditionally written as the correlation of *time-ordered* fields, connected to physical observables via the LSZ reduction formalism.

$$\langle | \, \mathcal{T}\{\phi(x_1)\phi(x_2)...\} \, | \rangle$$

*Inclusive* cross sections can be conveniently written as a forward discontinuity of time-ordered correlation functions, which in turn corresponds to an *un-ordered* correlation function

$$Disc_{forward} \left[ \langle | \, \mathcal{T}\{\phi(x_1)\phi(x_2)...\} \, | \rangle \right] \simeq \langle | \, \phi(x_1)\phi(x_2)... \, | \rangle$$

Most familiar example: *traditional* optical theorem $a + b \to X$

$$\sigma^{ab}_{total}(s) \simeq \frac{1}{s} Im\, T(s, t = 0) = \frac{1}{s} Disc_{t=0} T$$

"Simpler" example that can be extended to CFT: 2-point function $a \to b$

$$G_F^{FT}(p^2) = i \int d^4x e^{ip\cdot x} \langle 0 | T(\phi(x)\phi(0)) | 0 \rangle = -\frac{1}{p^2 - m^2 + i\epsilon},$$

$$G_W^{FT}(p^2) = \int d^4x e^{ip\cdot x} \langle 0 | \phi(x)\phi(0) | 0 \rangle = 2\pi\delta(p^2 - m^2)\theta(p^0)$$

$$G_F^{CFT}(p^2) = i \int d^4x \frac{e^{ipx}}{[\vec{x}^2 - t^2 + i\epsilon)^2]^\Delta} = -d(\Delta)(-p^2)^{\Delta-2},$$

$$G_W^{CFT}(p^2) = \int d^4x \frac{e^{ipx}}{[\vec{x}^2 - (t - i\epsilon)^2]^\Delta} = c(\Delta)\theta(p^2)\theta(p^0)\,(p^2)^{\Delta-2},$$

The Wightman function corresponds to the discontinuity of the time-ordered function across the appropriate cut.

# CFT $\sigma$

Using a CFT to describe scattering has been partially described before Strassler [0801.0629], Maldacena et. al. [0803.1467], & Balitsky et.al. [1309.0769, 1309.1424, 1311.6800]and we extend the analysis. The general idea is to consider infrared safe observables, general "event shapes", or to add mass deformations.
First type of interesting amplitude involves a single local source (e.g. a decay $\gamma^* \to c_1 + c_2 + ... + X$)

$$\langle \widetilde{O}_w \rangle = \frac{\sigma_w(p)}{\sigma_{\mathcal{O}}(p)} = \frac{\int d^4x e^{ipx} \langle 0|\mathcal{O}^\dagger(x)\widetilde{O}_w \mathcal{O}(0)|0\rangle}{\int d^4x e^{ipx} \langle 0|\mathcal{O}^\dagger(x)\mathcal{O}(0)|0\rangle} = \frac{\langle \mathcal{O}(p)|\widetilde{O}_w|\mathcal{O}(p)\rangle}{\langle \mathcal{O}(p)|\mathcal{O}(p)\rangle}$$

The normalization is chosen to ensure infrared safety, but we can generalize this approach to involve a set of local operators

$$\sigma_w(p) = \int d^4x e^{-ipx} \langle 0|\mathcal{O}^\dagger(x)\mathcal{D}[w]\mathcal{O}(0)|0\rangle$$

Generalizations: This approach can be used to describe more general observable flows/event shapes

$$\sigma_E(\hat{n}) = \sum_c \int d^4 p_c \, \frac{1}{2i} \, p_c^0 \, \delta^2(\hat{p}_c - \hat{n}) \, \mathrm{Disc}_{M^2} \; T_{\gamma^* c' \to \gamma'^* c}$$

as well as higher order correlation functions

$$\sigma_w(\hat{n}_1, \hat{n}_2, \cdots) =$$
$$= \sum_{c_1, c_2, \cdots} \int d^4 p_{c_1} \int d^4 p_{c_2} \cdots \frac{1}{2i} \, w(p_{c_1}, p_{c_2}, \cdots) \mathrm{Disc}_{M^2} \; T_{\gamma^* c_1' c_2' \cdots \to \gamma'^* c_1 c_2 \cdots}$$

Now that we have some new formalism, what can we do with it?

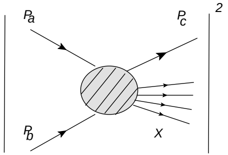
Combine **AdS/CFT** (strong coupling CFT), the **high energy limit** (Regge behavior simplifies amplitudes and has some model independent features), and **new inclusive methods** to model processes at the LHC.

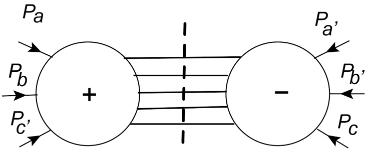
# 1PI Process

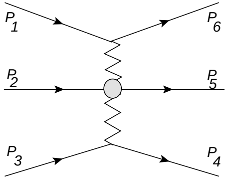
Process of interest is single particle inclusive scattering: $P + P \to \pi + X$ The differential cross section is related to the discontinuity in "missing mass", $M^2$, [Mueller ,et al.]of a related 6 point amplitude.

$$\frac{d\sigma_{ab\to cX}}{d^3P_c dE_c} \approx \frac{1}{2is} Disc_{M^2>0}\mathcal{A}_{abc'\to a'b'c}$$

In the appropriate Regge limit, this amplitude is described via the exchange of two Pomeron kernels and a Pomeron-Pomeron-particle-particle central vertex.

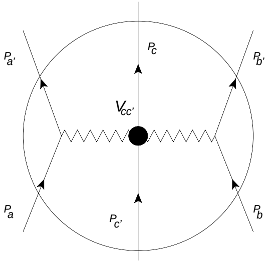


# Holographic Description

To describe this process
in the strong coupling limit we can use the AdS/CFT correspondence: we will describe the strongly coupled gauge amplitude with a dual gravity amplitude using "Witten diagrams"




The
amplitude can be written in a factorized form

$$T_{abc' \to a'b'c} = \Phi_{13} * \widetilde{\mathcal{K}}_P * V_{c\bar{c}} * \widetilde{\mathcal{K}}_P * \Phi_{24}$$

The appropriate discontinuity takes the form

$$(1/2i)\mathrm{Disc}_{M^2} T_{abc' \to a'b'c} = \Phi_{13} * [\mathrm{Im}\, \widetilde{\mathcal{K}}_P] * [\mathrm{Im}\, V_{c\bar{c}}] * [\mathrm{Im}\, \widetilde{\mathcal{K}}_P] * \Phi_{24}\,.$$

# Ingredients

$\widetilde{\mathcal{K}}_P$ Pomeron kernel: the AdS/CFT Pomeron [BPST]has been identified as the Regge trajectory associated with the AdS graviton.

$$\widetilde{\mathcal{K}}_P(s, 0, z, z') = -(\frac{1 + e^{-i\pi j_0}}{\sin \pi j_0})(\alpha' \widetilde{s})^{j_0}$$

$\Phi_{ab}$ Wave functions: The vertex couplings $\Phi_{ab}(z) \sim \phi_a(z)\,\phi_b(z)$ can be described by confined (hard wall) glueball wave functions $\phi_a(z) \sim z^2 J_{(\Delta-2)}(m_a z)$

$V_{c\bar{c}}$ Central vertex: The 6 point amplitude in the double Regge limit [DeTar, et.al.]can be constructed by generalizing flat space amplitudes. Following the prescription [Herzog, et.al.]we find

$$\mathcal{V}_{c\bar{c}}\,(\widetilde{\kappa}, 0, 0) \sim e^{-2\alpha' \kappa z^2/R^2} \sim e^{-2(z^2/\sqrt{\lambda})\kappa},$$

# AdS Calculation Cont'd

The explicit bulk six-point amplitude can be expressed as

$$T_{abc'\to a'b'c}(\kappa, s_1, s_2, t_1, t_2)$$
$$= \frac{g_0^2}{R^4}\int_0^{z_{max}} dz_1 \sqrt{|g(z_1)|}[z_1^2\phi_a(z_1)\phi_{a'}(z_1)]\int_0^{z_{max}} dz_2\sqrt{|g(z_2)|}[z_2^2\phi_{b'}(z_2)\phi_b(z_2)]$$
$$\times \int_0^{z_{max}} dz_3\sqrt{|g(z_3)|}\,\widetilde{\mathcal{K}}_P(-\tilde{s}_1, \tilde{t}_1, z_1, z_3)\; I(\tilde{\kappa}, \tilde{t}_1, \tilde{t}_2, z_3)\,\widetilde{\mathcal{K}}_P(-\tilde{s}_2, \tilde{t}_2, z_2, z_3)$$

where the dependence on the central vertex is collected as

$$I(\tilde{\kappa}, \tilde{t}_1, \tilde{t}_2, z_3) = (z_3^2\phi_c(z_3))\,V_{c\bar{c}}(\tilde{\kappa}, \tilde{t}_1, \tilde{t}_2)\,(z_3^2\phi_{c'}(z_3))\,.$$

Putting this all together we find

$$
\begin{aligned}
\rho(\vec{p}_T, y, s) &\equiv \frac{1}{\sigma_{total}} \frac{d^3\sigma_{ab\to c+X}}{d\mathbf{p}_c^3/E} = \frac{1}{2is\,\sigma_{total}(s)} \mathrm{Disc}_{M^2}\, T_6\,(\kappa, s_1, s_2, 0, 0) \\
&= \beta \int_0^{z_{max}} \frac{dz_3}{z_3}\, \tilde{\kappa}^{j_0}\, [\phi_c(z_3)]^2\, [\,\mathrm{Im}\, \mathcal{V}_{c\bar{c}}\,(\tilde{\kappa}, 0, 0)] \\
&= \beta \int_0^{z_s} \frac{dz}{z} z^{2\tau_c} (\kappa z^2/R^2)^{j_0} e^{-(2\kappa/\lambda^{1/2})z^2} \\
&\simeq \beta'\, \kappa^{-\tau_c},
\end{aligned}
$$

Where we have absorbed coefficients into overall constants.

Some things to note: (1) We assumed a confinement model to get finite results, but the answer is independent of the scale. (2) there is a simple scaling behavior that scales as power of the twist (3) The scaling is independent of initial sources

# Approach

The dominant contribution comes from tensor glueballs leading to the expected behavior

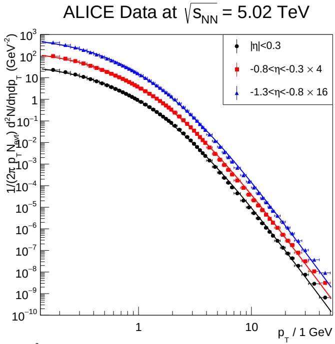
$$\rho(p_\perp, y, s) \sim p_\perp^{-8} \xrightarrow{\text{fit ansatz}} \frac{A}{(p_\perp + C)^B}$$

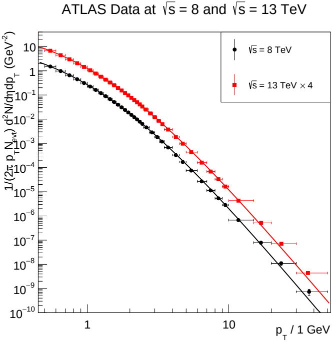
Can compare to:

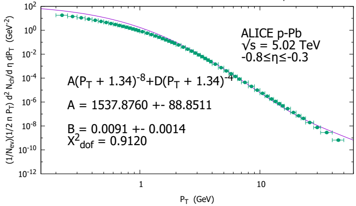
ATLAS p-p $\sqrt{s} = 8\ TeV$ and $\sqrt{s} = 13\ TeV$

ALICE p-Pb $\sqrt{s} = 5.02\ TeV$

# Plots!




For $d\sigma \sim A/(P_T + C)^B$

$B \sim 7$, $C \sim 1$GeV

This is above $\Lambda_{QCD}$, small $p_T$ behavior might be different. Not exactly the expected $p_T^{-8}$ behavior expected! Still, $\chi^2_{dof} \sim 1$

# Conclusions and Future Work

Conclusions:

- Conformal symmetry shows it's use in a wide range of collider physics, not limited to just AdS/CFT Regge physics [Randall, Sundrum][Georgi][Strassler, et. al.]
- 1P inclusive production in the central region can both be well modeled using the AdS/CFT. (Just like DIS in the past)
- Single particle inclusive production behaves like the exchange of a pair of operators in region $P_T > \Lambda_{QCD}$

Future Directions:

- Compute with softwall to see *true* model independent features
- AdS EOM to higher order in $\lambda$ (Hard string calculation!)[Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka]
- Extend to meson exchange.[Karch, Katz, Son, Stephanov] [Brodsky, de Teramond]
- Incorporate higher order anomolous dimension, $\Delta(j)$, results. [Brower, Costa, Djuric, **TR**, Tan] [Gromov, et. al][Lipatov, et. al.][Gromov, et. al.]
- More robust Ads wavefunctions and PDFs
- New processes and data sets

# Can you do anything else?

Similar approach can be used to describe DIS at small-x ($\gamma^* p$).

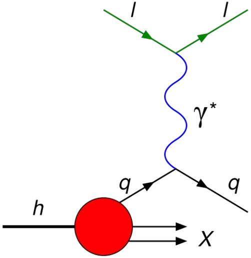
$$\sigma_{total} = \frac{1}{s} \text{Im}\left[\mathcal{A}(s, t=0)\right] \sim \frac{1}{s} \text{Im}\left[\chi(s, t=0)\right]$$

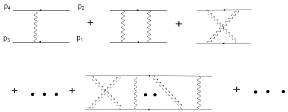
We can use this to calculate total cross sections and to determine the proton structure function

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left(\sigma_{trans} + \sigma_{long}\right)$$

Finally we must be wary of saturation where we must consider multipomeron exchange via eikonalization

$$\chi \to 1 - e^{i\chi}$$




[Cornalba, et. al.][Brower et.al.]

# Comparison With Previous Work

Can be used to identify the onset of strong-coupled/holographic saturation and confinement

| Model | $\rho$ | $g_0^2$ | $z_0$ | Q' | $\chi^2_{dof}$ |
|---|---|---|---|---|---|
| conformal | 0.774* | 110.13* | – | 0.5575*GeV | 11.7 (0.75*) |
| hard wall | 0.7792 | 103.14 | 4.96 $GeV^{-1}$ | 0.4333 GeV | 1.07 (0.69*) |
| softwall | 0.7774 | 108.3616 | 8.1798 $GeV^{-1}$ | 0.4014 GeV | 1.1035 |
| softwall* | 0.6741 | 154.6671 | 8.3271 $GeV^{-1}$ | 0.4467 GeV | 1.1245 |

Comparison of the best fit (including a $\chi$ sieve) values for the conformal, hard wall, and soft wall AdS models. The final row includes the soft wall with improved intercept. [Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka]The statistical errors (omitted) are all $\sim 1\%$ of fit parameters.

As expected, best fit values imply

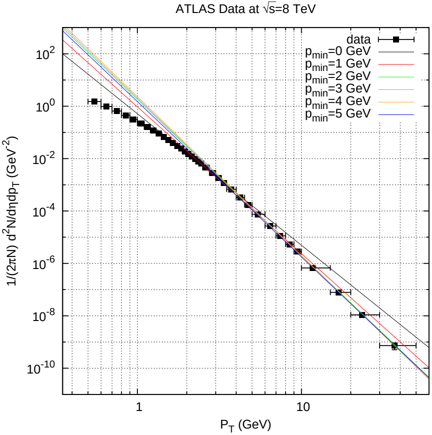
$$\rho \to \lambda > 1 \qquad 1/z_0 \sim \Lambda_{QCD} \quad \text{and} \quad Q' \sim m_{proton}$$

# Parameter Stability

We expect deviations from Regge behavior at low $p_\perp$. (Note our exact conformal solution diverges as $p_\perp \to 0$).

Q: Why choose our parameterization?

Two ideas: (1) Don't fit small $p_\perp$ behavior and/or (2) introduce a momentum offset



| $p_{min}/(1\text{ GeV})$ | A/10 ($GeV^{-2}$) | B | $\chi^2$/NDF |
|---|---|---|---|
| 0 | 0.0516 ± 0.00687 | 5.02 ± 0.164 | 51.2 |
| 0.5 | 0.0575 ± 0.00718 | 5.15 ± 0.148 | 29.8 |
| 1.0 | 0.0943 ± 0.0140 | 5.60 ± 0.139 | 3.21 |
| 1.5 | 0.153 ± 0.0585 | 5.88 ± 0.231 | 0.135 |
| 2.0 | 0.183 ± 0.131 | 5.97 ± 0.368 | 0.0412 |
| 2.5 | 0.199 ± 0.247 | 6.01 ± 0.578 | 0.0337 |
| 3.0 | 0.205 ± 0.291 | 6.027 ± 0.646 | 0.0316 |
| 3.5 | 0.218 ± 0.348 | 6.05 ± 0.712 | 0.0258 |
| 4.0 | 0.233 ± 0.416 | 6.07 ± 0.770 | 0.0189 |
| 4.5 | 0.253 ± 0.518 | 6.10 ± 0.846 | 0.0127 |
| 5.0 | 0.150 ± 0.736 | 5.93 ± 1.70 | 0.000621 |

# 1PI Kinematics Cont'd

For $a + b \to c + X$, treat $X$ effectively as a particle with mass

$$M^2 = (p_a + p_b - p_x)^2 = s + t + u - m_a^2 - m_b^2 - m_c^2$$

The final line is a constraint relating to the usual three Mandelstam invariants.

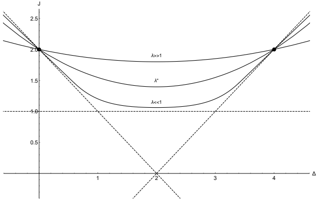
More convenient to pick a LC frame where $p_a = (p_a^+, p_a^-, \vec{p}_{\perp,a}) = (m_a e^{Y/2}, m_a e^{-Y/2}, 0)$ , $p_b = (m_b e^{-Y/2}, m_b e^{Y/2}, 0)$, where $Y$ is the rapidity. The Mandelstam $s$ becomes approx $s \sim m^2 e^Y$, and the produced particle has LC momentum given by

$$p_c = (m_\perp e^y, m_\perp e^{-y}, \vec{p}_\perp), \quad m_\perp^2 \equiv m_c^2 + \vec{p}_\perp^{\,2} \simeq \frac{(-t)(-u)}{M^2} \equiv \kappa$$

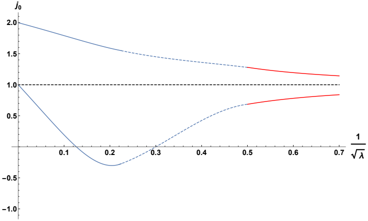
In the appropriate Regge limit

$$s \simeq \; M^2 \; \simeq m^2 e^Y \to +\infty \quad t \simeq -m m_\perp e^{Y/2-y} \to -\infty \quad u \simeq -m m_\perp e^{Y/2+y} \to -\infty$$

# More on AdS Reggeons



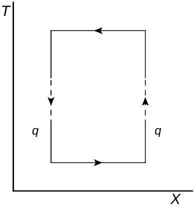

Reggeon propagators can be written in a form reminiscent of the weak coupling partonic description
$\chi_R \sim \int dj(\alpha' \hat{s})^j (1 + Cos(-i\pi j)) G_j(t, z, z')$
and the $G_j$
are AdS wavefunctions behaving like Bessel functions.
For a physical process like DIS we have hadronic structure functions that are singular as $x \to 0$.
These can be expressed in terms of a standard moment expansion: $M^\alpha(Q^2) = \int dx x^{1-\alpha} F_\alpha(x, Q^2)$. For large $Q^2$ this scales as a power of the anomalous dimension $\gamma$.




For AdS-Reggeons the operator dimensions admit a convergent expansion in terms of the objects twist, coupling, and anomalous dimension. These $\Delta - J$ curves admit non-trivial convergent expansions and can be calculated to high order using a mix of conformal, string, and integrability techniques. Minimizing these curves allows one to calculate Reggeon intercepts at strong coupling. [Gromov, et. al.][Lipatov, et. al.][Basso][Balitsky, et. al.] [Brower, et. al.]

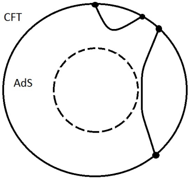
# More on Confinement

- Where are the single quarks? Naively, this could be explained by a quark-quark energy that grows with seperation. At large distance it becomes energetically favorable to create new quarks.




- Wilson originally used Wilson loops $W = \frac{1}{N} trPexp\left(ig \oint_C A\right)$ to try and describe confinement. In the limit of large times, a square path for a quark corresponds to the energy of two static quarks. In a confining theory, the expectation of the wilson loop to have an area dependence: $< W >\sim exp(-\sigma \text{Area})$

- In AdS Wilson loops in $\mathcal{N} = 4$ SYM are dual to minimal surfaces that extend into the bulk AdS.[Maldacena],[Polyakov]Note, in pure AdS, distances diverge at the boundary (small z) and become small in the interior of the bulk (large z).



- The original AdS/CFT conjecture predicts $< W > \sim exp(-\sigma T/x)$.[Maldacena] But it was quickly shown that deformations of the AdS space lead to confining behavior $exp(-\sigma Tx)$ [Polchinski, Strassler],[Andreev,et.al.]
- For us, it is sufficient to consider a purely geometric confinement deformation. However, to describe mesons it will be required to consider other dynamical fields in the bulk. [Karch, et.al.], [de Teramond, Brodsky], [Batell, Gherghetta][1]
- Thus the conformal description can be deformed to describe a confining theory

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx \cdot dx\right] + R^2 d\Omega_5$$

[1] For a definitive discussion on confinement via AdS dilaton see a series of papers by [Gürsoy, Kiritsis, et al.]