Integrability of Conformal Fishnet Theory

Gregory Korchemsky

IPhT, Saclay

In collaboration with

David Grabner, Nikolay Gromov, Vladimir Kazakov

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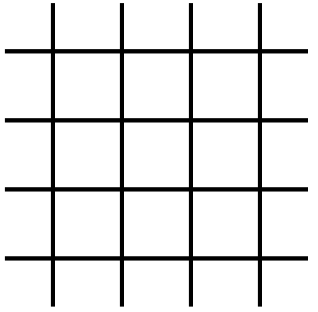
Integrability and fishnet graphs

- ✔ Fishnet graphs are four-dimensional scalar conformally invariant Feynman diagrams



- ✔ Define completely integrable lattice model [Zamolodchikov'80]
- ✔ Appear everywhere in planar $\mathcal{N}=4$ SYM:
  - ✗ Scattering amplitudes
  - ✗ Correlation functions
- ✔ What is the relation between integrability of planar $\mathcal{N}=4$ SYM and fishnet Feynman diagrams?
- ✔ Simplified model: $\gamma-$deformed $\mathcal{N}=4$ SYM
  - ✗ A non-unitary 'chiral' (almost) CFT dominated by fishnet graphs
  - ✗ Integrable in planar limit, related to conformal $SU(2,2)$ spin chain

This talk: *compute exactly correlation functions in $\gamma-$deformed $\mathcal{N}=4$ SYM*

Strongly twisted $\mathcal{N} = 4$ SYM

$$L = -\frac{1}{4}F_{\mu\nu}^2 + D^\mu \phi_i^\dagger D_\mu \phi^i + i\bar{\psi}_A D\psi^A + L_{\rm int}$$ [Leigh,Strassler][Frolov]

$$L_{\rm int} = g^2 \Bigg(\frac{1}{4}\{\phi_i^\dagger, \phi^i\}\{\phi_j^\dagger, \phi^j\} - {\rm e}^{-i\epsilon^{ijk}\gamma_k}\, \phi_i^\dagger \phi_j^\dagger \phi^i \phi^j$$
$$- {\rm e}^{-\frac{i}{2}\gamma_j^-}\, \bar{\psi}_j \phi^j \bar{\psi}_4 + {\rm e}^{\frac{i}{2}\gamma_j^-}\, \bar{\psi}_4 \phi^j \bar{\psi}_j + i\epsilon_{ijk}\, {\rm e}^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\, \bar{\psi}^k \phi^i \bar{\psi}^j + c.c. \Bigg)$$

Twist parameters $\gamma_1^\pm = -(\gamma_3 \pm \gamma_2)/2$, $\gamma_2^\pm = -(\gamma_1 \pm \gamma_3)/2$, $\gamma_3^\pm = -(\gamma_2 \pm \gamma_1)/2$

Is expected to be integrable in the planar limit

Double scaling limit: strong twist + weak coupling [Gurdogan,Kazakov]

$$g^2 \to 0\,, \qquad \gamma_{1,2} = \text{fixed}\,, \qquad \gamma_3 \to +i\,\infty\,, \qquad \xi^2 = g^2\,{\rm e}^{-i\gamma_3} = \text{fixed}$$

Gauge field, fermions and one scalar decouple

$$L = {\rm tr}\left[\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + \xi^2 \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right]$$

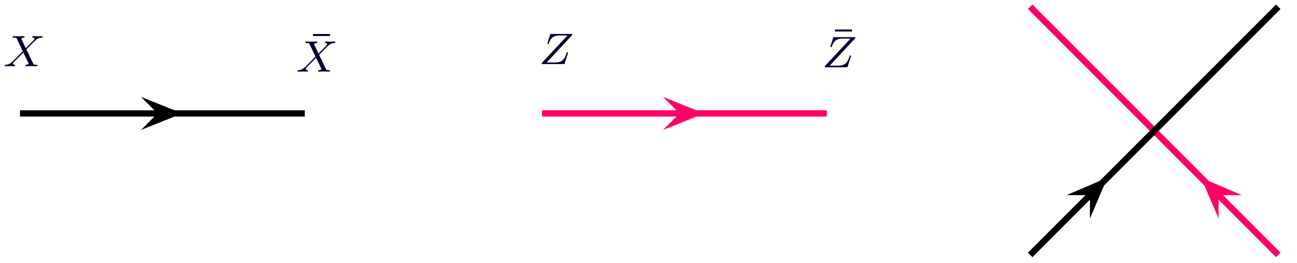
Supersymmetry and $R-$symmetry is broken $PSU(2,2|4) \to SU(2,2) \times U(1) \times U(1)$

Bi-scalar chiral CFT

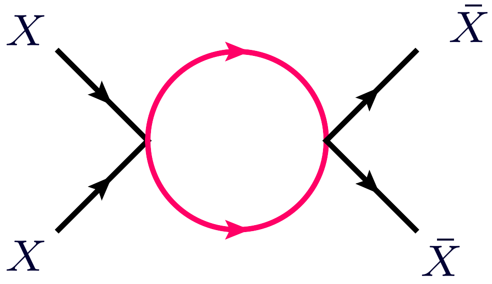
$$\mathcal{L} = N_c \,\mathrm{tr}\left[\partial^\mu \bar{X} \partial_\mu X + \partial^\mu \bar{Z} \partial_\mu Z + (4\pi)^2 \xi^2 \bar{X} \bar{Z} X Z\right]$$

Non-unitary theory, chiral vertex

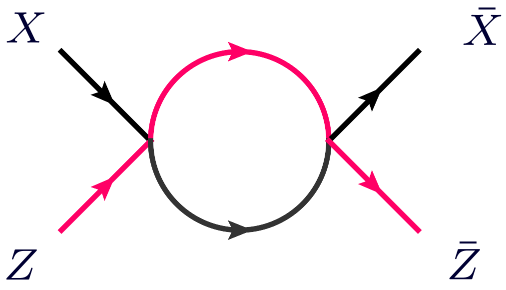
Feynman rules:



The theory is not complete at the quantum level [Fokken,Sieg,Wilhelm]



$$\sim \frac{1}{\epsilon}\,\mathrm{tr}(X^2)\,\mathrm{tr}(\bar{X}^2)$$



$$\sim \frac{1}{\epsilon}\,\mathrm{tr}(XZ)\,\mathrm{tr}(\bar{X}\bar{Z})$$

Double-trace counter terms have to added

Beta functions

- Quantum corrections induce double-trace interaction vertices

$$\mathcal{L}_{\rm dt}/(4\pi)^2 = \alpha_2^2 \left[\mathrm{tr}(X^2)\,\mathrm{tr}(\bar{X}^2) + \mathrm{tr}(Z^2)\,\mathrm{tr}(\bar{Z}^2)\right] - \alpha_1^2 \left[\mathrm{tr}(XZ)\,\mathrm{tr}(\bar{X}\bar{Z}) + \mathrm{tr}(X\bar{Z})\,\mathrm{tr}(\bar{X}Z)\right]$$

- ξ^2 does not run, but new couplings develop beta-functions $\beta_i = d\alpha_1^2/d\ln\mu \neq 0$– conformal anomaly!

$$\beta_1 = 2\left(\alpha_1^2 - \xi^2\right)^2,$$

$$\beta_2 = a(\xi) + \alpha_2^2\, b(\xi) + \alpha_2^4\, c(\xi)$$

Coefficient functions $a = -\xi^4 + \xi^8 + \dots, b = -4\xi^4 + 4\xi^8 + \dots, c = -4 - 4\xi^4 + \dots$

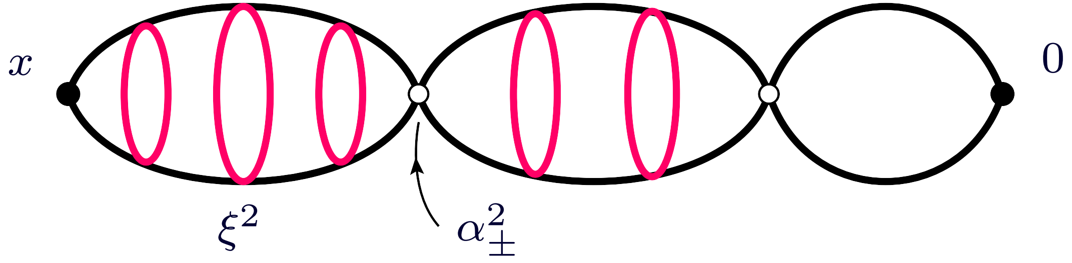
- The theory has two lines of fixed points

$$\alpha_1^2 = \xi^2\,, \qquad \alpha_2^2 = \alpha_\pm^2 = \pm\frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \xi^8 \pm \frac{65i\xi^{10}}{48} - \frac{19\xi^{12}}{10} + O\left(\xi^{14}\right)$$

- In the planar limit, the bi-scalar theory with appropriately tuned double-trace couplings is a genuine non-unitary CFT

Bi-scalar theory at the fixed point

$$\langle \mathrm{tr}[X^2(x)]\,\mathrm{tr}[\bar{X}^2(0)]\rangle \;=$$



✔ Expected behaviour at the fixed points

$$\langle \mathrm{tr}[X^2(x)]\,\mathrm{tr}[\bar{X}^2(0)]\rangle \;\sim\; \frac{1}{(x^2)^{\Delta_\pm}}$$

✔ Scaling dimensions of operators at weak coupling

$$\Delta_\pm = 2 \mp 2i\xi^2 \pm i\xi^6 \mp \frac{7i}{4}\xi^{10} + O\left(\xi^{14}\right)$$

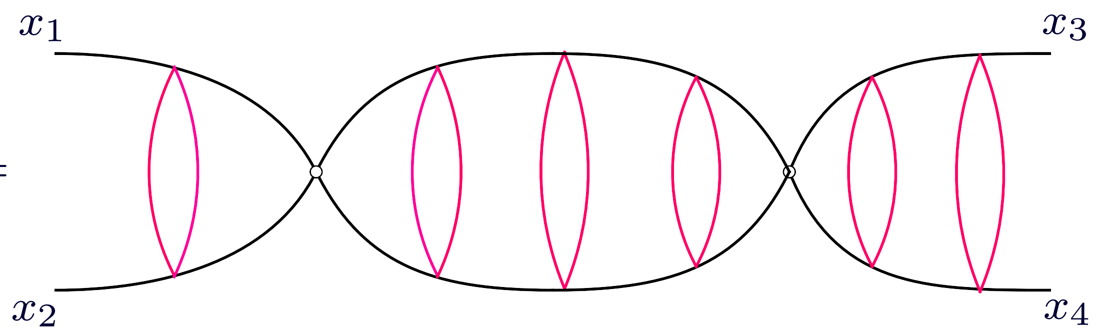
✔ Satisfies remarkably simple *exact* relation

$$(\Delta-4)(\Delta-2)^2\Delta = 16\xi^4\,.$$

Hint for integrability of the theory

Four-point correlation function

Exploit the conformal symmetry to compute the four-point correlation function

$\langle \mathrm{tr}[X(x_1)X(x_2)]\,\mathrm{tr}[\bar{X}(x_3)\bar{X}(x_4)]\rangle =$ 

Is obtained from $\langle \mathrm{tr}[X^2(x)]\,\mathrm{tr}[\bar{X}^2(0)]\rangle$ by point splitting the scalar fields

$$G(x_1, x_2|x_3, x_4) = \frac{\mathcal{G}(u,v)}{x_{12}^2 x_{34}^2}\,, \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Admits the conformal partial wave expansion

$$\mathcal{G}(u,v) = \sum_{\Delta, S/2 \in \mathbb{Z}_+} C^2_{\Delta,S}\, u^{(\Delta-S)/2} g_{\Delta,S}(u,v),$$

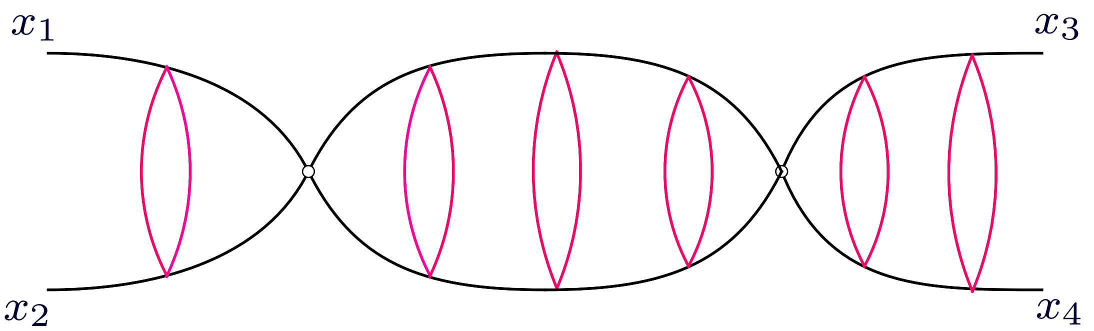
The sum runs over operators with scaling dimensions Δ and even Lorentz spin S.

$C_{\Delta,S}$ the OPE coefficient, $g_{\Delta,S}(u,v)$ the conformal block

For $u \to 0$ and $v \to 1$, the leading contribution comes from operators $O = \mathrm{tr}[\bar{X}^2(0)]$

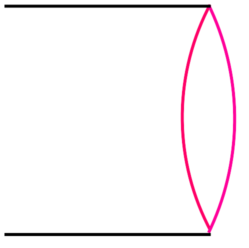
Four-point correlation function II

Feynman diagrams contributing in the planar limit

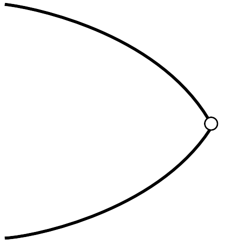


Sum of ladder diagrams glued together through double-trace vertices

$\mathcal{H} =$



$\mathcal{V} =$



Integral operators $\mathcal{V}$ and $\mathcal{H}$ insert quartic vertex and scalar loop, respectively

$$G \sim \langle x_1, x_2 | \frac{1}{1 - \alpha^2 \mathcal{V} - \xi^4 \mathcal{H}} | x_3, x_4 \rangle + (x_1 \leftrightarrow x_2)$$

α^2 the double-trace coupling at the fixed point

The operators $\mathcal{V}$ and $\mathcal{H}$ commute with the generators of the conformal group

Eigenvalues of graph generating kernels

$$\mathcal{V}\,\Phi_{\Delta,S}(x_1,x_2) = \delta(\Delta-2)\delta_{S,0}\Phi_{\Delta,S}(x_1,x_2)$$

$$\mathcal{H}\,\Phi_{\Delta,S}(x_1,x_2) = \frac{1}{h_{\Delta,S}}\Phi_{\Delta,S}(x_1,x_2)$$

Conformal symmetry fixes the form of eigenstates of $\mathcal{V}$ and $\mathcal{H}$

$$\Phi_{\Delta,S,n}(x_{10},x_{20}) = \langle \mathrm{tr}[X(x_1)X(x_2)]O_{\Delta,S,n}(x_0)\rangle$$

$$= \frac{1}{x_{12}^2}\left(\frac{x_{12}^2}{x_{10}^2 x_{20}^2}\right)^{(\Delta-S)/2}\left((n\partial_{x_0})\ln\frac{x_{20}^2}{x_{10}^2}\right)^S$$

All Lorentz indices are projected onto auxiliary light-cone vector n^μ

The operator $O_{\Delta,S,n}(x_0)$ carries the scaling dimension $\Delta = 2+2i\nu$ and Lorentz spin S

The states $\Phi_{\Delta,S,n}$ belong to the principal series of the conformal group

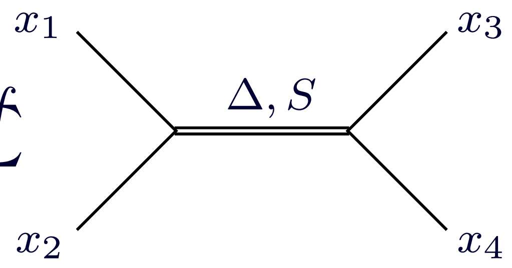
Eigenvalue of $\mathcal{H}$

$$h_{\Delta,S} = \frac{1}{16}(\Delta+S-2)(\Delta+S)(\Delta-S-2)(\Delta-S-4)$$

Correlation function

Decompose the four-point correlation function over the eigenstates

$$G(x_1, x_2|x_3, x_4) = \sum\!\!\!\!\!\!\!\int$$



$$= \sum_{S\geq 0} \int_0^\infty \frac{d\nu}{h(\nu, S) - \xi^4} \int d^4 x_0 \, \Phi_{\nu,S}^{\mu_1\cdots\mu_S}(x_{10}, x_{20}) \Phi_{-\nu,S}^{\mu_1\cdots\mu_S}(x_{30}, x_{40})$$

$$= \frac{1}{x_{12}^2 x_{34}^2} \sum_{S\geq 0} \int_{-\infty}^{\infty} d\nu \, \frac{1}{h(\nu, S) - \xi^4} \quad \underbrace{\mu(\nu, S)}_{\text{kinem.factor}} \quad \underbrace{g_{2+2i\nu,S}(u, v)}_{\text{4dconf.block}}$$

The sum runs over the states with $\Delta = 2 + 2i\nu$ and Lorentz spin S

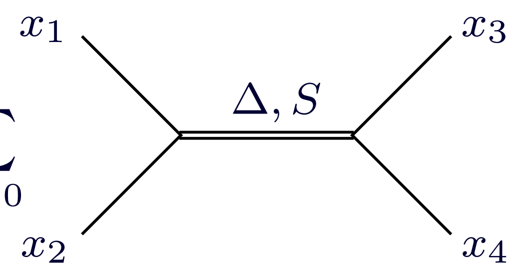
Close the integration contour to the lowest half-plane and pick up residues at

$$h(\nu, S) = (\nu^2 + S^2/4)(\nu^2 + (S + 2)^2/4) = \xi^4 \,, \qquad \text{Im}\, \nu < 0$$

Two solutions $i\nu_2 = S/2 + O(\xi^4)$ and $i\nu_4 = (S + 2)/2 + O(\xi^4)$

Exact scaling dimensions

$$G(x_1, x_2|x_3, x_4) = \sum_{\Delta=\Delta_2,\Delta_4} \sum_{S\geq 0}$$



Exact scaling dimensions

$$\Delta_2(S) = 2 + \sqrt{(S+1)^2 + 1 - 2\sqrt{(S+1)^2 + 4\xi^4}}\,,$$

$$\Delta_4(S) = 2 + \sqrt{(S+1)^2 + 1 + 2\sqrt{(S+1)^2 + 4\xi^4}}\,,$$

Describe conformal operators of twist 2 and 4

Special case: operators with $S = 0$

$$\Delta_2(0) = 2 + \frac{2i\sqrt{2}\xi^2}{\sqrt{1+\sqrt{4\xi^4+1}}} = 2 - 2i\xi^2 + i\xi^6 - \frac{7i}{4}\xi^{10} + O\left(\xi^{14}\right)$$

Agrees with the result of explicit calculation at 7 loops!

Exact OPE coefficients

$$G(x_1, x_2|x_3, x_4) = \frac{1}{x_{12}^2 x_{34}^2} \sum_{S\geq 0} C_{\Delta_2,S}\, g_{\Delta_2,S}(u,v) + C_{\Delta_4,S}\, g_{\Delta_4,S}(u,v)$$

The OPE coefficients

$$C_{\Delta,S} = -2\pi i \times \mathrm{res}_\nu \, \frac{\mu(\nu,S)}{h(\nu,S)-\xi^4}$$

The residue at the physical pole $h(\nu,S) = \xi^4$

$$C_{\Delta,S} = \frac{4^{3-\Delta}(-1)^S(S+1)\Gamma\left(\frac{1}{2}(S-\Delta+5)\right)\Gamma\left(\frac{1}{2}(S+\Delta)\right)}{[(4-\Delta)\Delta+S(S+2)-2]\Gamma\left(\frac{1}{2}(S-\Delta+4)\right)\Gamma\left(\frac{1}{2}(S+\Delta-1)\right)}$$

The dependence on the coupling constant enters through the scaling dimensions

The operators with zero Lorentz spin

$$C_{\Delta,0} = -\frac{4^{3-\Delta}\Gamma\left(\frac{5-\Delta}{2}\right)\Gamma\left(\frac{\Delta}{2}\right)}{((\Delta-4)\Delta+2)\Gamma\left(\frac{4-\Delta}{2}\right)\Gamma\left(\frac{\Delta-1}{2}\right)}$$

The exact conformal data for any coupling ξ^2 *!*

Conclusions and open questions

- ✔ Strongly $\gamma-$deformed planar $\mathcal{N}=4$ SYM has two lines of fixed points
- ✔ The corresponding non-unitary four-dimensional conformal field theory is integrable
- ✔ Closed expression for the four-point correlation function of the simplest protected operators, the exact conformal data
- ✔ Do conformal symmetry and integrability survive in $\gamma-$deformed planar $\mathcal{N}=4$ SYM for arbitrary values of the deformation parameters?
- ✔ Does the bi-scalar theory admit a dual AdS description?