

# Test of an Analytic, Finite, Gauge-Invariant, Non-Perturbative QCD formulation. with elastic proton-proton scattering Experiments at the ISR and LHC energies

Peter. H. Tsang<sup>†</sup>

Herbert. M. Fried<sup>†</sup>, Thierry Grandou<sup>‡</sup>, Yves Gabellini<sup>‡</sup>,  
Ralf Hofmann, Yeuan-Ming Sheu<sup>†‡</sup>

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<sup>†</sup> Brown University

# Analytic, Finite, Gauge-Invariant, Non-Perturbative QCD formulation obtained

- 1) First first-principled dynamical quark confining potential for quarks, [Fried et al. Eur. Phys. J. C 65, 395 2010]
- 2) A potential that allows nucleons to be bounded, thus provided the first-principled model deuteron, [Fried et al. Ann. Phys. 338, 2013]
- 3) New property of Effective Locality, provides the summation of all gluonic exchanges between two quarks, but moreover, the interaction becomes a Local Interaction. [Thierry Grandou et al. Ann. Phys. 327 (2012), Mod. Phys. Lett. A, Vol.32(2017); 1706.02264 ]
- 4) Extended Asymptotic Freedom as supported by Dyson Schwinger Equation ( $g \gg 1$  and  $g$  small) [Fried, Grandou, Sheu 1207.5017],
- 5) A qualitative description of Hadron Confinement mass scale(s), [Fried, PT 2015]
- 6) The full SU(3) algebraic content of QCD amplitudes.
- 7) First-principled calculation of elastic proton-proton scattering at ISR and LHC energies.

# Schwinger Generating Functional for QCD

a) Starting Point: Schwinger Generating Functional (GF) for QCD, with gluon operators in an Arbitrary (Relativistic) Gauge.

b) Re-arrange this GF in terms of a “Reciprocity Relation”, and a “Gaussian Linkage Operation”; and the GF now depends upon two functionals of A,

$$\mathcal{Z}_{\text{QCD}}[j, \bar{\eta}, \eta] = \mathcal{N} e^{-\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \mathbf{D}_c^{(0)} \cdot \frac{\delta}{\delta A}} \cdot e^{-\frac{i}{4} \int \mathbf{F}^2 + \frac{i}{2} \int A \cdot (-\partial^2) \cdot A} \cdot e^{i \int \bar{\eta} \cdot \mathbf{G}_c[A] \cdot \eta + \mathbf{L}[A]} \Big|_{A = \int \mathbf{D}_c^{(0)} \cdot j}$$

Quark line

$$G_c(x, y|A) = [m + \gamma \cdot (\delta - igA\tau)]^{-1}$$

Virtual quark loop

$$L[A] = \ln[1 - i\gamma A\tau_c[0]]$$

c) Insert Halpern's half-century-old representation for the  $\mathbf{F}^2$

Halpern's Representation

$$e^{-\frac{i}{4} \int F^2} = N \int d[\chi] e^{\frac{i}{4} \int \chi^2 + \frac{i}{2} \int F \cdot \chi}$$

Eur. Phys. J. C65, 395 (2010) arxiv 1103.4179

$$\chi_{\mu\nu}^a = -\chi_{\nu\mu}^a$$

The next two steps were overlooked for decades:

1

2

# 1

## GAUGE INVARIANCE BY GAUGE INDEPENDENCE

Trivial re-arrangement can now be made to formally insure gauge-invariance, Even though the GF still apparently contains gauge-dependent gluon propagators.

Functional derivatives on Generative Functional,  $Z_{QCD}$  ,

$$Z_{QCD}[j, \bar{\eta}, \eta] = N \int d[\chi] e^{\frac{i}{4} \int \chi^2} e^{\mathcal{D}_A^{(0)}} \cdot e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (-\partial^2) \cdot A} e^{i \int \bar{\eta} \cdot G_c[A] \cdot \eta + L[A]} \Big|_{A = \int D_c^{(0)} \cdot j}$$

2n-point functions

Gluonic ghosts leads to  $\exp \left[ \frac{i}{2} \int j \cdot D_c^{(0)} \cdot j \right]$  To be removed by

$$= N \int d[\chi] e^{\frac{i}{4} \int \chi^2} e^{\mathcal{D}_A^{(0)}} e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (D_c^{(0)})^{-1} \cdot A} G_c(1|gA) G_c(2|gA) e^{L[A]} \Big|_{A=0}$$

$$e^{\mathcal{D}_a F_1[A]} = \exp \left[ \frac{i}{2} \int \bar{Q} \cdot D_c^{(0)} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1} \cdot \bar{Q} - \frac{1}{2} \text{Tr} \ln(1 - D_c \cdot \tilde{K}) \right]$$

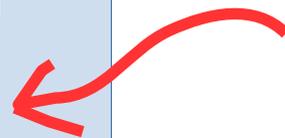
$$\cdot \exp \left[ \frac{1}{2} \int A \cdot \bar{K} \cdot (1 - D_c^{(0)} \cdot \bar{K})^{-1} \cdot A + i \int \bar{Q} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1} \cdot A \right]$$

$$D_c^{(0)} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1}$$

$$= D_c^{(0)} \cdot [1 - (\hat{K} + (D_c^{(0)})^{-1}) \cdot D_c^{(0)}]^{-1}$$

$$= -(\tilde{K}_{\mu\nu}^{ab} + g f^{abc} \chi_{\mu\nu}^c)^{-1} = -\hat{K}^{-1}$$

**Gluon Bundle**



$$\begin{aligned}
e^{\mathcal{D}^A} F_1[A] F_2[A] &= \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \bar{Q} + \frac{1}{2} \text{Tr} \ln \hat{\mathbf{K}} + \frac{1}{2} \text{Tr} \ln(-D_c^{(0)})\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot \mathbf{D}_c^{(0)} \cdot \frac{\delta}{\delta A'}\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A'} - \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A'}\right] \cdot (e^{\mathcal{D}^A} F_2[A'])
\end{aligned}$$

$$\begin{aligned}
e^{\mathcal{D}^A} F_1[A] F_2[A] &= N \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \bar{Q} + \frac{1}{2} \text{Tr} \ln \hat{\mathbf{K}}\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A} - \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A}\right] \cdot \exp(L[A])
\end{aligned}$$

**No gauge dependence.**

*(H.M.Fried, Modern Physics Letters A, 2013)*

$-\hat{\mathbf{K}}^{-1}$  or also written as  $(f \cdot \chi)^{-1}$



**All gluons exchanges summed!**

2

# Use Faddeev Representation for $G[A]$ and $L[A]$

$$\begin{aligned}
 G_c(x, y|A) &= i \int_0^\infty ds e^{-ism^2} e^{-\frac{1}{2}\ln(2h)} \int d[u] e^{\frac{i}{4} \int_0^s ds' [u'(s')]^2} \delta^{(4)}(x - y + u(s)) \\
 &\times \left[ m - \gamma_\mu \frac{\delta}{\delta u'_\mu(s)} \right] \mathcal{N}_\Omega \mathcal{N}_\Phi \int d[\alpha] \int d[\Xi] \int d[\Omega] \int d[\Phi] \left( e^{i \int_0^s ds' [\alpha^a(s') - i\sigma_{\mu\nu} \Xi_{\mu\nu}^a(s')] \tau^a} \right)_+ \\
 &\times e^{-i \int ds' \Omega^a(s') \alpha^a(s') - i \int ds' \Phi_{\mu\nu}^a(s') \Xi_{\mu\nu}^a(s')} \\
 &\times e^{-ig \int ds' u'_\mu(s') \Omega^a(s') A_\mu^a(y - u(s')) + ig \int ds' \Phi_{\mu\nu}^a(s') F_{\mu\nu}^a(y - u(s'))},
 \end{aligned}$$

**Gaussian**

$$1 = \int d[\alpha] \delta[\alpha^a(s') + g u'_\mu(s') A_\mu^a(y - u(s'))],$$

$$1 = \int d[\Xi] \delta[\Xi_{\mu\nu}^a(s') - g F_{\mu\nu}^a(y - u(s'))]$$

$$\begin{aligned}
 L[A] &= -\frac{1}{2} \int_0^\infty \frac{ds}{s} e^{-ism^2} e^{-\frac{1}{2}\ln(2h)} \times \mathcal{N}_\Omega \mathcal{N}_\Phi \int d^4x \int d[\alpha] \int d[\Omega] \int d[\Xi] \int d[\Phi] \\
 &\times \int d[v] \delta^{(4)}(v(s)) e^{\frac{i}{4} \int_0^s ds' [v'(s')]^2} \\
 &\times e^{-i \int ds' \Omega^a(s') \alpha^a(s') - i \int ds' \Phi_{\mu\nu}^a(s') \Xi_{\mu\nu}^a(s')} \cdot \left( e^{i \int_0^s ds' [\alpha^a(s') - i\sigma_{\mu\nu} \Xi_{\mu\nu}^a(s')] \tau^a} \right)_+ \\
 &\times e^{-ig \int_0^s ds' v'_\mu(s') \Omega^a(s') A_\mu^a(x - v(s')) - 2ig \int d^4z (\partial_\nu \Phi_{\nu\mu}^a(z)) A_\mu^a(z)} \\
 &\times e^{+ig^2 \int ds' f^{abc} \Phi_{\mu\nu}^a(s') A_\mu^b(x - v(s')) A_\nu^c(x - v(s'))}
 \end{aligned}$$

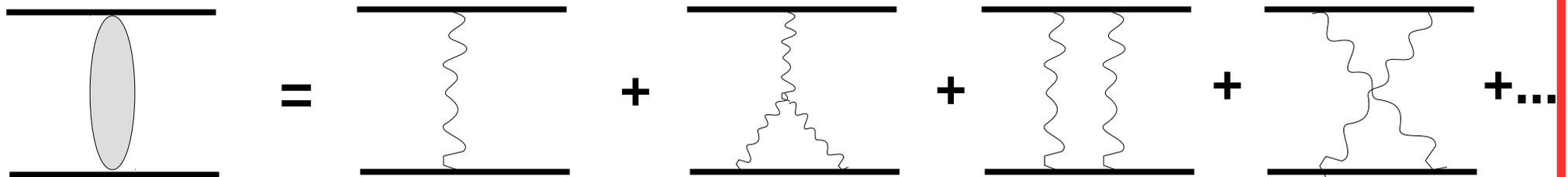
**Gaussian**

*H. M. Fried, Y. Gabellini, T. Grandou, Y.-M. Sheu, Eur. Phys. Journal C 65 (2010).*

*H. M. Fried, Y. Gabellini, T. Grandou, and Y.-M. Sheu, Ann. Phys. 338 (2013)*

All the Gaussian Linkage operations can then be carried through exactly, corresponding to the summation of all gluons exchanged between any pair of quark (and/or antiquark) lines, and including cubic and quartic gluon interactions.

Define a “Gluon Bundle” (GB) as the Sum over all gluon exchanges between any pair of quark lines,



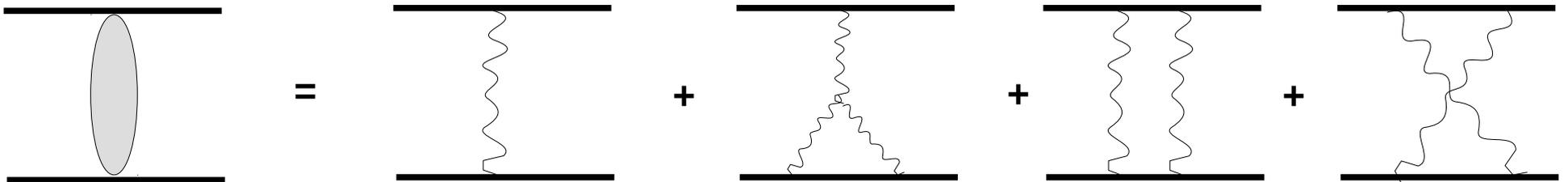
**THE RESULT:**

Explicit cancellation of all gauge-dependent gluon propagators, with resulting GF exhibiting Manifest Gauge Independence.. and one finds a new, exact property of Non-Perturbative, Gauge-Invariant QCD,

$\mathcal{D}_A^{(0)}$  Is gaussian functional operation,  $G[A]$  and  $L[A]$  are gaussian in Fradkin representation  
 Functional derivatives operations with respect to sources  $\bar{\eta}, \eta$  can be performed exactly  
 This produces sum of all Feynman graphs corresponding to the exchange of infinite number of gluons between quarks.

# New QCD Property: Effective Locality (EL)

Define a “Gluon Bundle” (GB) as the Sum over all gluon exchanges between any pair of quark lines,



See Thierry Grandou's talk

The space-time coordinates of both ends of a GB are equal, modulo small uncertainties in their transverse coordinates.

What this means is that, at high energies, the Halpern FI can be reduced to sets of Ordinary integrals, yielding a vast simplification in the calculation of all QCD Correlation functions.

(Pencil and paper + desktop computer can now replace huge, Multi-processing lattice calculations.)

# How to introduce transverse quark fluctuations from First Principles?

We believe we know how to do this, but work still underway

What we have done is to introduce phenomenological transverse fluctuation amplitude  $a$ .  
Every quark-gluon vertex, replacing the usual gluon-quark current interaction at the same space-time point,

$$\int d^4x \bar{\psi}(x) \gamma_\mu A_\mu^a(x) \tau_a \psi(x) \quad \text{by}$$
$$\int d^2x'_\perp \int d^4x a(x_\perp - x'_\perp) \bar{\psi}(x') \gamma_\mu \tau_a A_\mu^a(x) \psi(x'),$$

With  $a(x_\perp - x'_\perp)$  real and symmetric, and  $x'_\mu = (x'_\perp, x_L, x_0)$ .

The probability of finding two quarks separated by a transverse (or impact parameter) distance is then:

$$\varphi(b) = \int \frac{d^2q}{(2\pi)^2} e^{iq \cdot b} |\tilde{a}(q)|^2.$$

How to choose  $\varphi(b)$  ? It is directly related to quark binding; how does  $\varphi(b)$  produce  $V(r)$ , the  $q - \bar{q}$  binding potential whose lowest bound state represents the pion?

First try: A Gaussian,  $\varphi(b) \approx e^{-(\mu b)^2}$  where  $\mu^{-1}$  sets the scale of transverse fluctuations. Then, all absurdities of correlation functions disappear. But this distribution is “too symmetric”, and gives a zero  $V(r)$ .

Second try: A “deformed” Gaussian,  $\varphi(b) = \varphi(0)e^{-(\mu b)^{2+\xi}}$  with  $\xi$  a “deformation parameter”, real and small.

A straight-forward calculation yields, for small  $\xi$

$$V(r) \approx \xi \mu (\mu r)^{1+\xi}$$

## CONCERNING TRANSVERSE QUARK FLUCTUATIONS

Starting from conventional, quark-field-operator equations of motion, one can define “IN” and “OUT” operator fields,  $\psi_{out}^{in}(x)$  as in any Abelian Theory; QCD just has more complicated interaction, no?

But this assumption is wrong. For decades we have known that all asymptotic quark states are hadronic Bound states of quarks; and for such a bound state we can specify longitudinal and time Coordinates, but not transverse coordinates, since they are always fluctuating.

NB: The conventional “static quark” approximation used in lattice and other model Binding-potential calculations in all non-perturbative Amplitudes are plagued with divergences because the “static quarks” are not physical.

Without taking such “transverse imprecision” into account, all non-perturbative Amplitudes are plagued with absurdities.

# Mass of pion from QCD

stituting this potential into a Schrodinger binding equation, using the  
"WKB" approximation, then yields  $\mu \sim m_\pi, \xi \approx 0.1$ .

[1104.4663, and  
Levy-Leblond, Balibar, Quantics:  
Rudiments of Quantum Physics]

is sensible, since the max. fluctuations should be  $\leq$  than  $m_\pi^{-1}$ .

result encompasses two different lattice calculations,  $V \sim r$  and  
 $V \sim r \ln(r)$ . But all lattice and other model calculations of  $q - \bar{q}$   
amplitudes correspond to an amplitude containing only one of the two  
independent SU(3) invariants,  $C_2, C_3$ ; our amplitude contains both.

(H.M.Fried, T.Grandou, R.Hoffman, Euro. Journal Phys. 2015  
. 1504.05502)

What method do we use to pass from  $\varphi(b)$  to  $V(r)$ ?

Define that a  $q$  and a  $\bar{q}$  are scattering at high energy. One can  
use an Eikonal approximation, valid in the limit of  $s \gg |t|$ , for  
conventional scattering amplitude. (Details for QCD eikonals were  
worked out by HMF, YG, JA, and BMcK in two papers circa 1983. )

## Pion's mass from QCD

No static approximation required! Our analysis gives  $E(b)$  explicitly, in terms of  $\varphi(b)$ , so that we can calculate  $V(r)$  for any choice of  $\varphi(b)$ . The minimal bound state energy representing the pion shows that most of the pion mass comes from the gluons forming the GB, and relatively little from the quark masses.

# Eikonal to Potential

It has been well-known for a half-century that, assuming a specific  $V(r)$ , in ordinary QM, or in Abelian QFTs, the corresponding eikonal function  $E(\mathbf{b})$  is given by

$$E(b) = \gamma(s) \int_{-\infty}^{+\infty} dz_L V(\vec{b} + \hat{p}_L z_L),$$

where  $\gamma(s)$  is a constant depending on CM energy and the type of interaction.

We can write the non-perturbative amplitude corresponding to a GB exchanged between a  $q$  and a  $\bar{q}$ ; and we see that the Eikonal limit of this amplitude has  $E(\mathbf{b})$  defined in terms of  $\varphi(b)$ , and proportional to:  $\ln\{\varphi(b)\}$ . Here,  $E(\vec{b}) = E(b)$ , and  $V(\vec{r}) = V(r)$ .

Our method: Calculate the 2-D Fourier transform  $\tilde{E}(k_{\perp}^2)$  of  $E(b)$ .

Extend  $k_{\perp}^2 \rightarrow k_{\perp}^2 + k_L^2 = \vec{k}^2$ , so that we now have  $\tilde{E}(\vec{k}^2)$ ; and then calculate the 3-D transform of this  $\tilde{E}(\vec{k}^2)$ , which will yield  $V(r)$ .

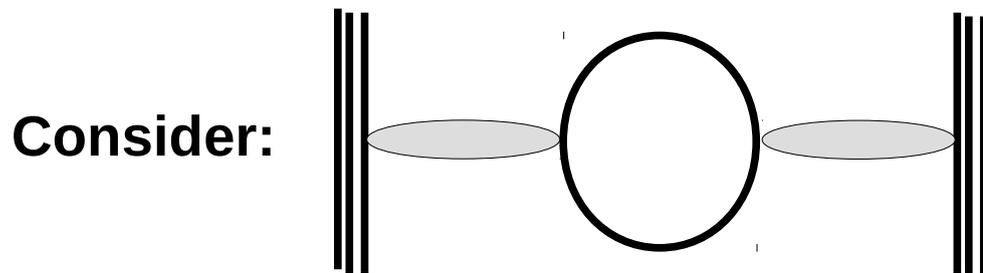
More on Eikonal in slide 26 on Scattering Amplitude

# Nuclear Physics from QCD

If you look up Nuclear forces on Wikipedia, you'll find the statement that there exists no derivation on the basis of QCD.

Here is the first (to our knowledge) example of nucleon binding (for a Model deuteron) from basic QCD. The Model neglects electrical charge, and nucleon spins (which can always be added in ); this is a Qualitative model, describing the essence of Nuclear Physics.

Question of Scale: Quark binding takes place for  $r_{ij} \sim m_{\pi}^{-1}$ , but nucleon binding takes place at 2, or 3, or 4 times that distance. How to achieve this?

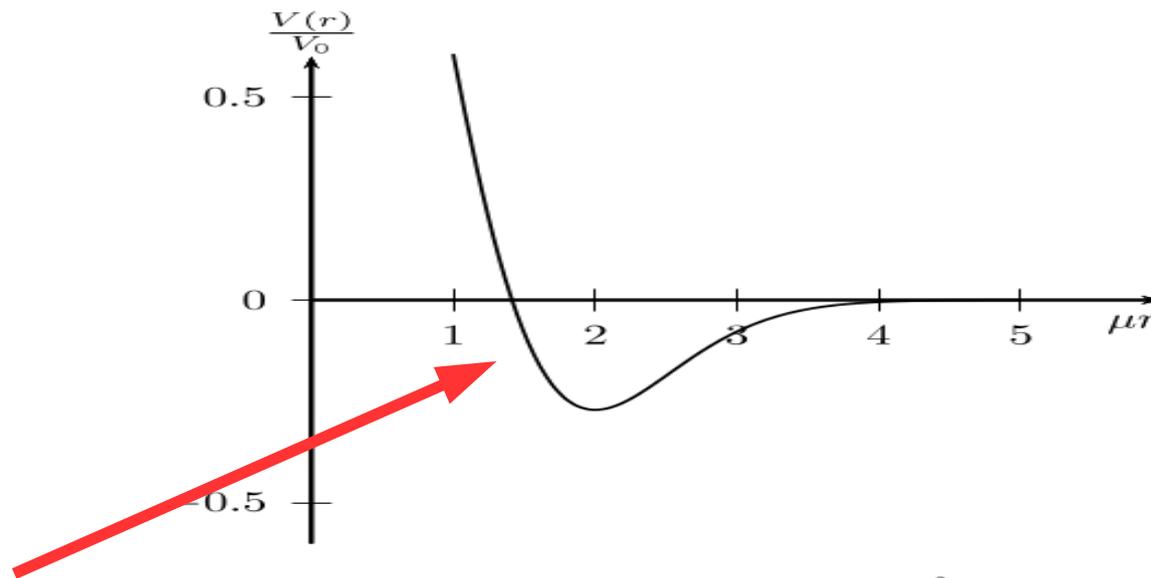


This requires extraction and regularization of the logarithmic UV divergence of the loop, which contributes two essential features:

a) It “stretches”, so that distances larger than  $m_{\pi}^{-1}$  can easily enter.

b) It provides a crucial change of sign for the effective n-n binding potential.

*H.M. Fried et al. / Annals of Physics 338 (2013) 107–122*



**Fig. 3.** Nucleon potential with  $V_0 = c(\frac{g_R^2}{4\pi})\mu$ .

This sign change can be the basis of nucleon-binding to form nuclei.

1104.4663

# Virtual Closed-Quark Loops, their interactions with GBs .

All the basic, “radiative correction” structure of non-perturbative Q comes from interacting closed-quark-loops with GBs. How can this efficiently described? I will try to do this in words, describing the functional operations that need to be performed.

A single 'dressed' quark has an amplitude proportional to

$$N \int d[\chi] e^{\frac{i\chi^2}{4}} (\det(gf \cdot \chi))^{-\frac{1}{2}} e^{\hat{\mathcal{D}}_A} \cdot G_c[A] e^{L[A]} \Big|_{A \rightarrow 0},$$

While two scattering quarks are described by

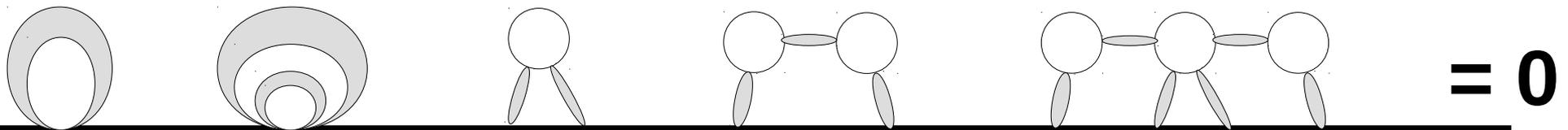
$$N \int d[\chi] e^{\frac{i\chi^2}{4}} (\det(gf \cdot \chi))^{-\frac{1}{2}} e^{\hat{\mathcal{D}}_A} G_c^{(1)}[A] G_c^{(2)}[A] e^{L[A]} \Big|_{A \rightarrow 0},$$

$$\hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} (gf \cdot \chi)^{-1} \frac{\partial}{\partial A}$$

And  $\chi(x)$  is the Halpern functional variable originally used to represent  $e^{\frac{i}{4} \int d^4x F^2(x)}$

Every GB exchanged is represented by the linkage operator connecting the two  $G_c[A]'_s$  to each other, and the  $G_c[A]'_s$  to  $L[A]$  .

And here the relative simplicity of non-perturbative QCD shows itself clearly, for all of its “self-energy” graphs vanish, either by the asymmetry of the  $(f \cdot \chi)^{-1}$  color and coordinate indices, or by explicit loop integration.



What is the  $Z_q$  of a (non-perturbative) quark?  $Z_q = 1$ .

Non-Perturbative QCD is far simpler than QED!

# QCD Renormalization

The 'radiative corrections' of QCD enter when there is momentum transfer between one quark and another quark; and the procedure may occur when the momentum transfer passes through intermediate GBs and/or closed quark loops.

For simplicity, let us suppress possible quark binding into hadrons, and just consider two quarks exchanging momentum transfer in their CM.

A useful technique is the exact Functional Cluster Expansion,

$$e^{\hat{\mathcal{D}}_A} \cdot e^{L[A]} = \exp\left[\sum_{n=1}^{\infty} \frac{Q_n}{n!}\right], \quad Q_n = e^{\hat{\mathcal{D}}_A} (L[A])^n |_{connected}$$

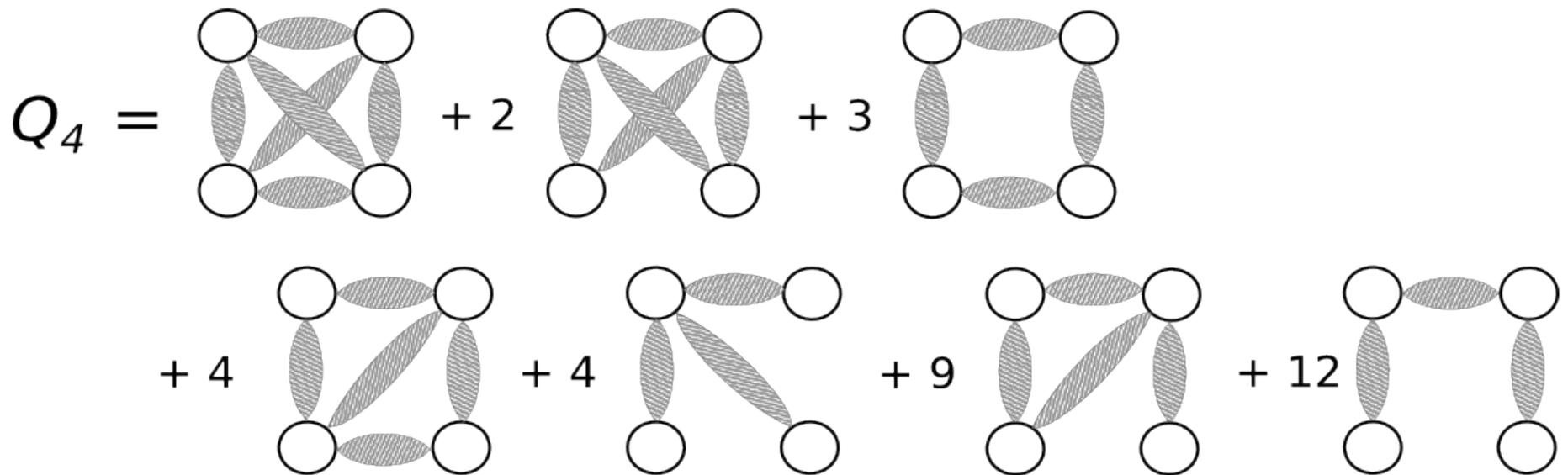
With linkage operator

$$\hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} \hat{K} \frac{\partial}{\partial A}, \quad (\hat{K})_{\mu\nu}^{ab} = (gf_{abc}\chi_{\mu\nu}^c)^{-1} \leftarrow GB$$

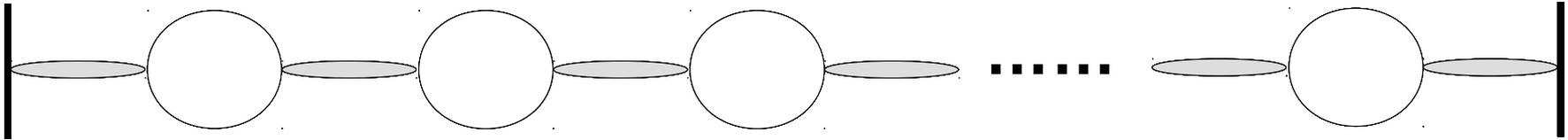
**For example,**  $Q_1 = e^{\hat{\mathcal{D}}^A} L[A] = \bigcirc + \bigcirc \text{---} \bigcirc + \bigcirc \text{---} \bigcirc + \dots = \bar{L}[A]$

**And**  $Q_2 = \bar{L}[A](e^{\hat{\mathcal{D}}^A} - 1)\bar{L}[A]$

**Things get complicated very quickly; e.g.  $Q_4$  is given by**



**Even functionally, it is a horrid mess. But, there exists one way of reducing this to an easily-calculated set of 'chain-graph-loops',**



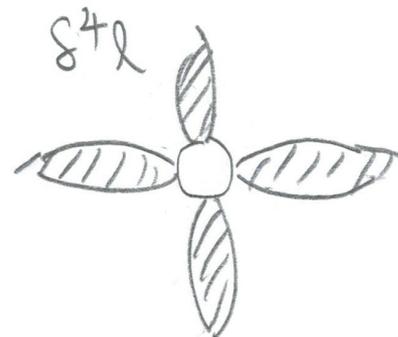
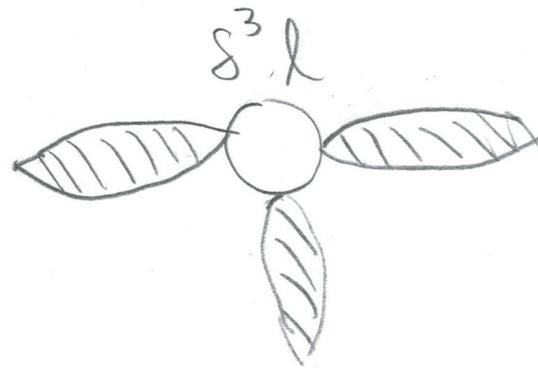
**Which form a geometric series that can be summed.**

**But this depends crucially on the definition of renormalization....**

**At this point, let's 'take stock' of where we stand. We began with a Theory of quarks and gluons; but the gluons have disappeared, and only their GB sums remain.**

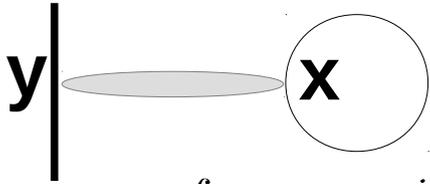
**What to do? Renormalize the GB's:**

# QCD Renormalization



...

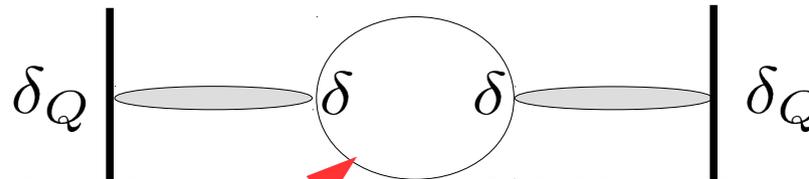
# QCD Renormalization



$$N \int d[\chi] \exp\left[\frac{i}{4} \int \chi^2 + ig \int_0^s ds' u'_\mu(s') \Omega_a(s') (f \cdot \chi(w))^{-1} \Big|_{\mu\alpha}^{ab} \int_0^t dt' v_\alpha(t') \hat{\Omega}_b(t') \cdot \int d^2 x'_\perp \int d^2 y'_\perp a(x_\perp - x'_\perp) \cdot a(y_\perp - y'_\perp) \cdot \delta^{(4)}(y' - u(s') - x' + v(t'))\right]$$

Because each quark represents the “physical particle” of QCD, we'll replace the  $\delta$  at each quark site by  $\delta_Q$ , a finite quantity. But where the  $\delta$  connects to the loop – which is a virtual and not a physical particle – it remains  $\delta$ , and (very shortly)  $\rightarrow 0$ .

In this one-loop, 2 GB drawing, there is a net  $\delta^2$  multiplying the loop.



But the loop is proportional to an expected UV log divergence, which we'll call

$$\ell, \quad \ell = \ln(\Lambda/m_\alpha)$$

This loop – as well as every such loop in a chain of such GB loops – produces a factor of

$$\delta^2 \cdot \ell \Big|_{\ell \rightarrow \infty, \delta \rightarrow 0} = \kappa$$

which we DEFINE to be real, finite, positive number (subsequently determined by experiment).

$$\delta^2 \cdot l |_{l \rightarrow \infty, \delta \rightarrow 0} = \kappa$$

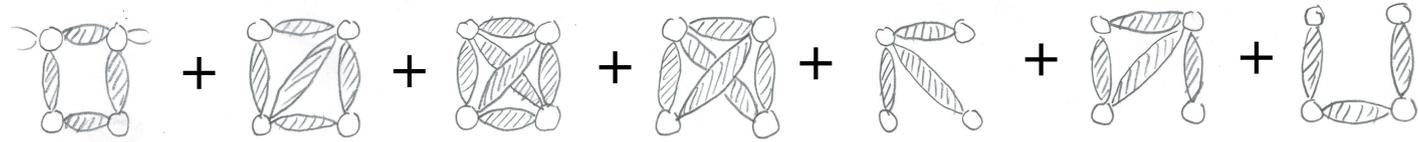
Q<sub>2</sub>



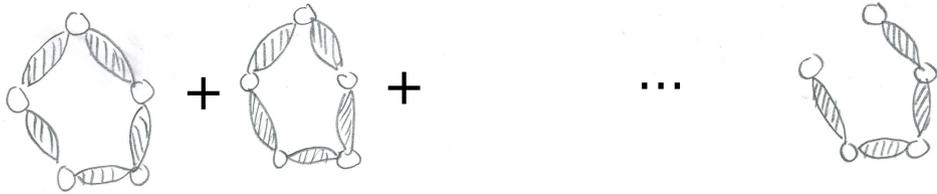
Q<sub>3</sub>



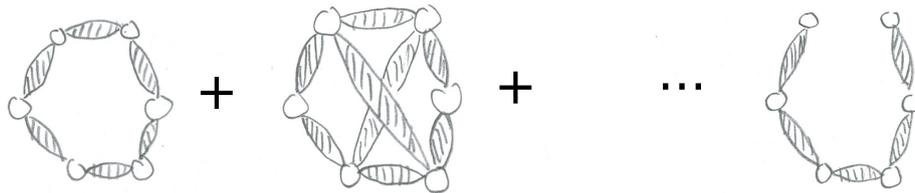
Q<sub>4</sub>



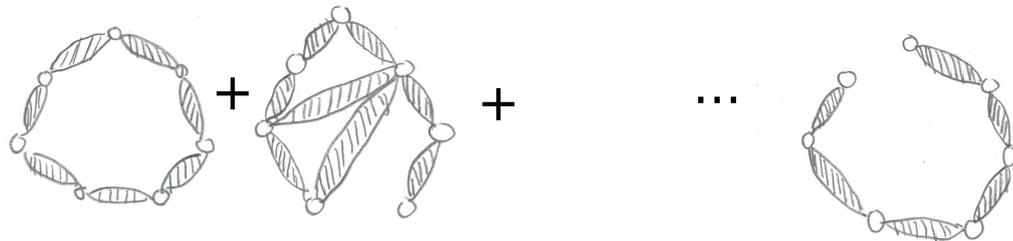
Q<sub>5</sub>



Q<sub>6</sub>



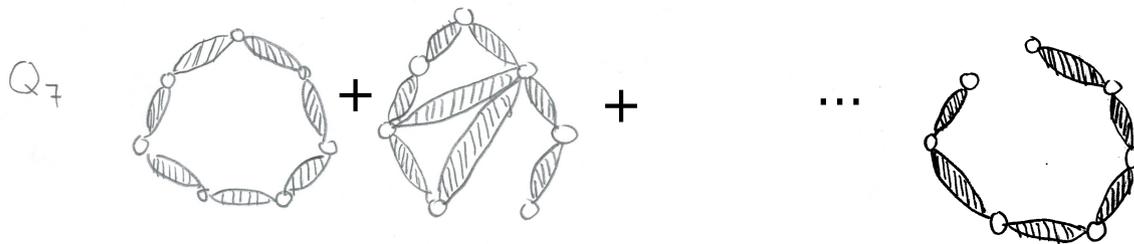
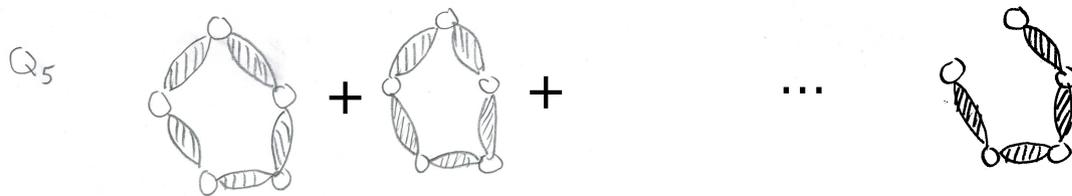
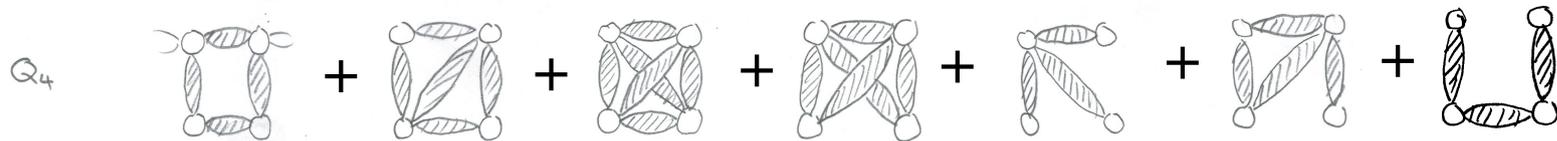
Q<sub>7</sub>



⋮

(1412.2072)

$$\delta^2 \cdot l |_{l \rightarrow \infty, \delta \rightarrow 0} = \kappa$$

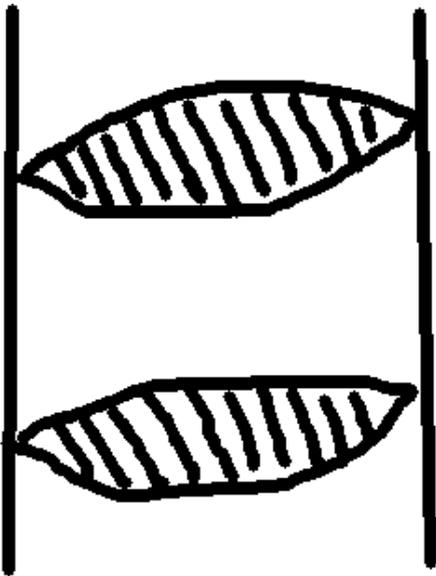


⋮

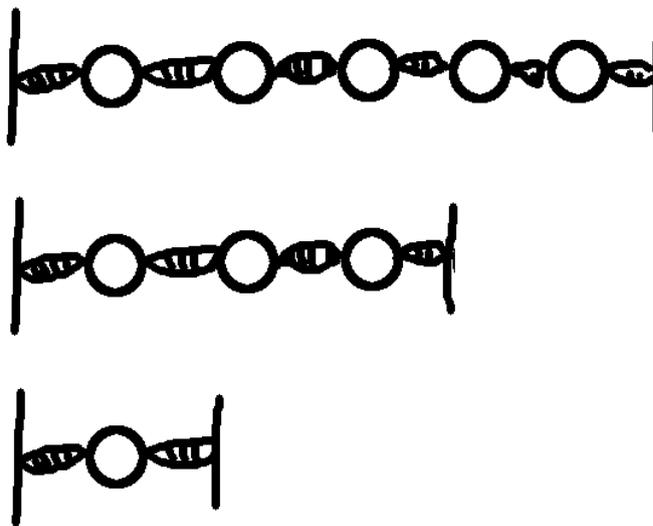
To calculate scattering problems:

- 1) Reduce full 6 body problem with 2 body scattering problem
- 2) Write  $f \cdot \chi$  as  $\sqrt{R}$  where R is real number.

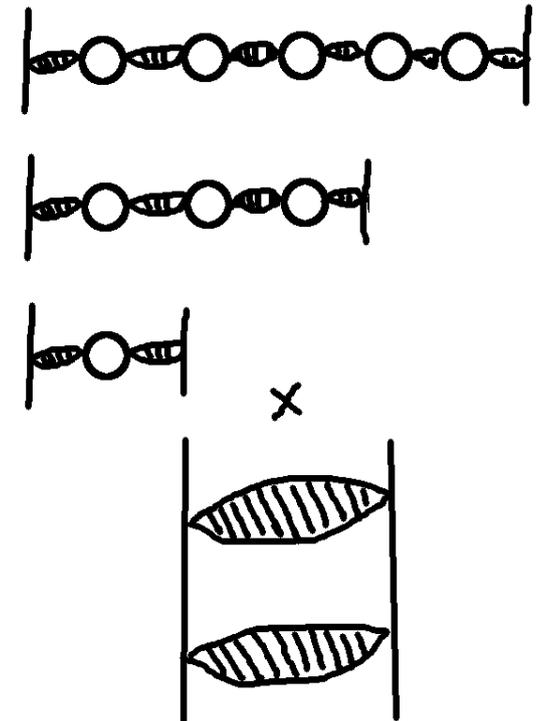
Gluon Bundles



Chain loops



Interference terms



## Energy Dependence

**Simplifying the full 6 body problem of 2 hadrons scattering, along with using  $\mathbf{f} \cdot \boldsymbol{\chi}$  as  $\sqrt{\mathbf{R}}$  where  $\mathbf{R}$  is real does not allow us to keep track of energy dependences.**

**2 body problem: [Fried, Grandou, Hofmann 2016]**

$$\pm \sum_{\text{monomials}} \left\langle \prod_{i=1}^N \left[ \frac{\sqrt{4iN_c} \sqrt{\hat{s}(\hat{s} - 4m^2)}}{m^2} \right] \frac{[(\mathcal{OT})_i]^{-2}}{g\varphi(b)} \right. \\ \left. \times G_{03}^{30} \left( \left[ \frac{g\varphi(b)}{\sqrt{32iN_c} \sqrt{\hat{s}(\hat{s} - 4m^2)}} \right]^2 \left[ (\mathcal{OT})_i \right]^4 \left| \frac{1}{2}, \frac{3 + 2q_i}{4}, 1 \right. \right) \right\rangle_{O_N(\mathbb{R})}$$

$$\pm \frac{DC_{2f}}{N} \mathbf{I}_{3 \times 3} \sum_{\text{monomials}} \left( \prod_1^N A_i^1 \right) \left( \sum_1^N \frac{A_i^3}{A_i^1} \right) \left\{ \frac{g\varphi(b)}{\sqrt{2iN_c}} \times \left( \frac{\mathbf{m}^2}{\hat{\mathbf{E}}^2} \right), \quad g \frac{\mu m}{|u'_0 \bar{u}'_3|} e^{-(\mu b)^2} \times \left( \frac{\mathbf{m}}{\hat{\mathbf{E}}} \right) \right\}$$

$$A_i^1 = \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{2q_i + 1}{4}\right), \quad A_i^2 = \Gamma\left(\frac{1 - 2q_i}{4}\right)\Gamma\left(\frac{-2q_i - 1}{4}\right), \quad A_i^3 = \Gamma\left(-\frac{1}{2}\right)\Gamma\left(\frac{2q_i - 1}{4}\right),$$

**$\mathbf{N} = \mathbf{D} (\mathbf{N}_c - 1)$  gives  $\mathbf{N}(\mathbf{N}-1)/2$  terms or  $2^{496}$  monomials.**

**By symmetry,  $2^{120}$**

**Simplier problem N = 4 gives**

$$\frac{DC_{2f}}{N} I_{3 \times 3} \left[ \frac{g\mu m}{|u'_0 \bar{u}'_3|} \sum_{res.mon.} \left( \prod_1^4 A_i^1 \right) \left( \sum_1^4 \frac{A_i^3}{A_i^1} \right) \right] \left( \frac{m}{E} \right) \longrightarrow \frac{DC_{2f}}{N} I_{3 \times 3} \left( \frac{m}{E} \right)^p,$$

**Gives  $p \sim 0.26$**

**(Fried, Grandou, Gabellini, Sheu, PT, to appear)**

# Parameters to learn from experiments

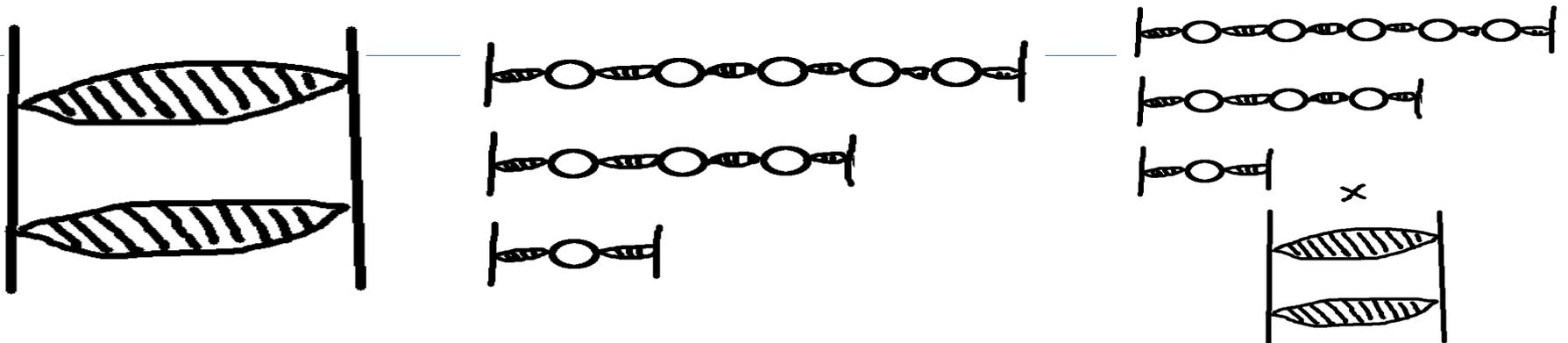
- **g coupling**
- $\kappa = \delta^2 \cdot \ell$     Loop renormalization constant
- Gluon Bundle  $m_{\text{ext}}$ , Loop Chain masses  $m_{\text{int}}$
- **$(m/E)^p$  though can be derived in principle**

## Differential Cross-Section

$$\frac{d\sigma}{dt} = \frac{m^4}{16\pi p^2 E^2} |T|^2$$

### • Calculations

$$\begin{aligned} \frac{d\sigma}{dt} = & k(-9 \cdot 3 \cdot 4) \frac{\beta g^2}{4\pi} \left(\frac{m_{ext}}{E}\right)^{4p} \exp\left[\frac{-q^2}{8m_{ext}^2}\right] \\ & + (9 \cdot 3 \cdot 6) \frac{\beta g^2}{1} \left(\frac{m_{ext}}{E}\right)^{2p} \kappa \frac{q^2}{m_{ext}^2} \exp\left[\frac{-q^2}{4m_{int}^2}\right] \frac{1}{1 + \left(\beta g \kappa \frac{q^2}{m_{int}^2} \exp\left[\frac{-q^2}{4m_{int}^2}\right]\right)^2} \\ & + (4374 \beta^2 g^4 \kappa \left(\frac{m_{ext}}{E}\right)^{6p} \left(\frac{2m_{ext}}{9\pi m_{ext}^2}\right) \sqrt{\frac{2\pi}{3}} (12m_{ext}^2 + \\ & q^2) \exp\left[\frac{-q^2}{12m_{ext}^2}\right] \frac{1}{1 + \left(\beta g \kappa \frac{q^2}{m_{int}^2} \exp\left[\frac{-q^2}{4m_{int}^2}\right]\right)^2} \end{aligned}$$



## Differential Cross-Section

$$\frac{d\sigma}{dt} = \frac{m^4}{16\pi p^2 E^2} |T|^2$$

- **Calculations**

$$\frac{d\sigma}{dt} = k \left[ -9 \times 3 \times 4 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} \left(\frac{m}{E}\right)^{2p} \exp\left(\frac{-1}{8} \frac{x}{m^2}\right) + 9 \times 3 \times 6 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} 4\pi \lambda \bar{\kappa} \frac{x}{m^2} \exp\left(\frac{-x}{2m^2}\right) \right]^2$$

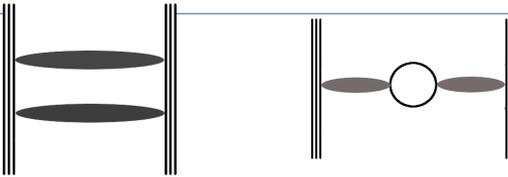
**Energy dependence of a single quark-quark subprocess yields:**

$$\left(\frac{m}{\sqrt{\hat{s}}}\right)^{2p}$$

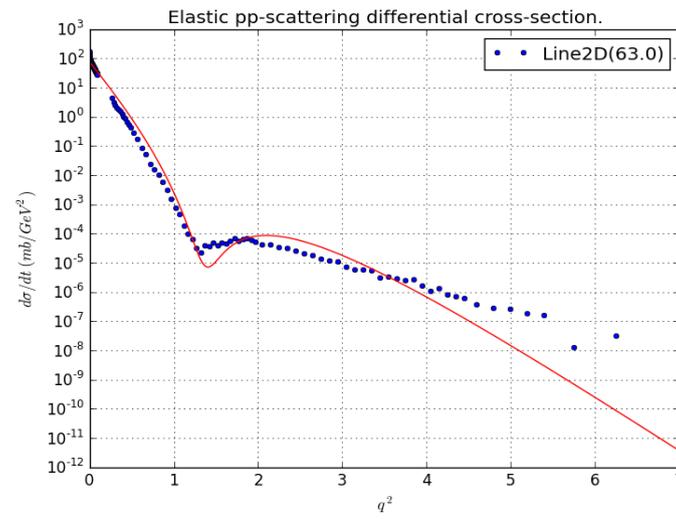
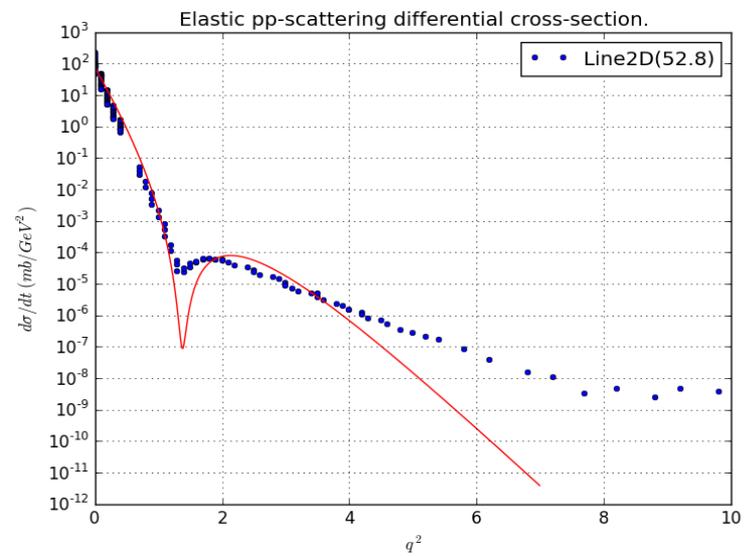
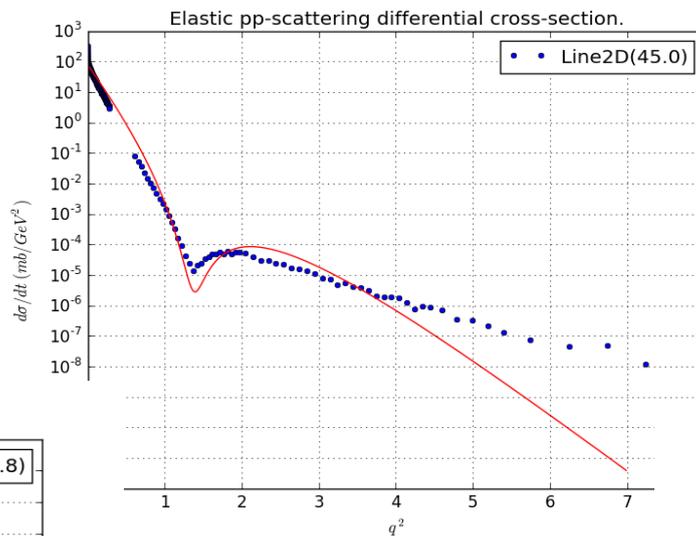
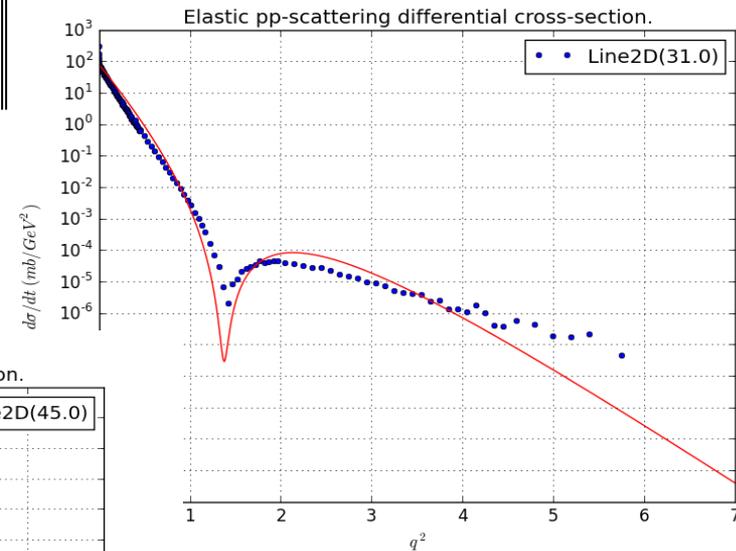
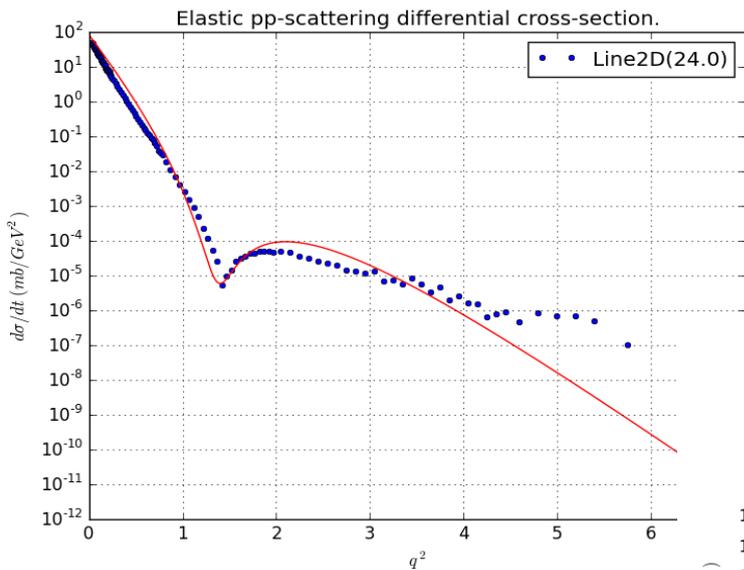
**(H.M.Fried, T.Grandou, Y.-M.Sheu Ann. Physics 2014)**

# Gluon Bundles + One loop

$$\frac{d\sigma}{dt} = k \left[ -12 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} \left(\frac{m}{E}\right)^{2p} \exp\left(\frac{-3}{8} \frac{x}{m^2}\right) + 9 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} 4\pi \lambda \bar{\kappa} \frac{x}{m^2} \exp\left(\frac{-x}{2m^2}\right) \right]^2$$



## ISR elastic pp data



$g=2.9$   
 $m=2m_\pi$   
 $p=0.01$   
 $\kappa=0.0296$

# Differential Cross-Section

$$\frac{d\sigma}{dt} = \frac{m^4}{16\pi p^2 E^2} |T|^2$$

## • Calculations

**one loop:**

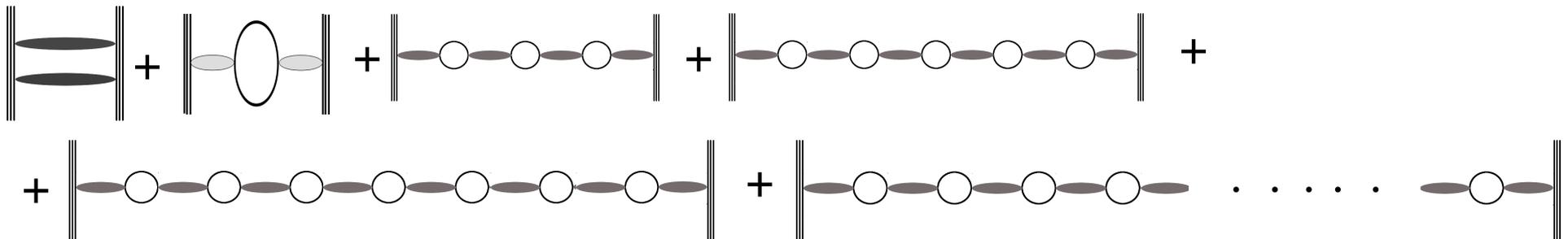
$$\frac{d\sigma}{dt} = k \left[ -12 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} \left(\frac{m}{E}\right)^{2p} \exp\left(\frac{-1}{8} \frac{q^2}{m^2}\right) + 9 \frac{\beta g^2}{4\pi} \left(\frac{m}{E}\right)^{2p} 4\pi \lambda \bar{\kappa} \frac{q^2}{m^2} \exp\left(\frac{-q^2}{2m^2}\right) \right]^2$$

## Infinite loops:

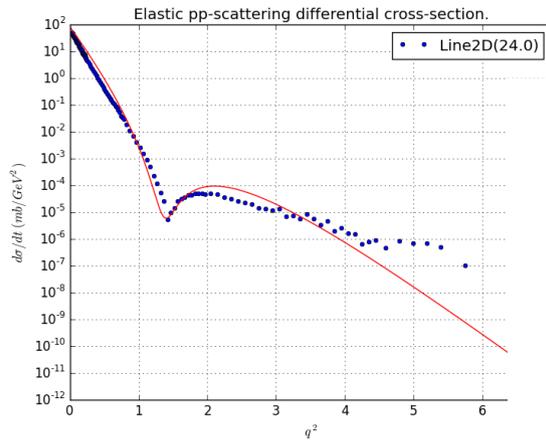
$$\frac{d\sigma}{dt}(E, q^2) = K \left[ g^2 \beta \left(\frac{m_{ext}}{E}\right)^{2p} \right]^2 \left[ \frac{1}{4\pi} (9 \times 3 \times 4) \left(\frac{m_{ext}}{E}\right)^{2p} e^{-(3q^2/8m_{ext}^2)} - \frac{(9 \times 3 \times 8) A_{ext}(q^2)}{1 + \beta^2 g^2 A_{int}^2(q^2)} \right]^2$$

## Infinite loops:

$$A(q^2) = \kappa (q^2/m^2) e^{-(q^2/4m^2)}$$

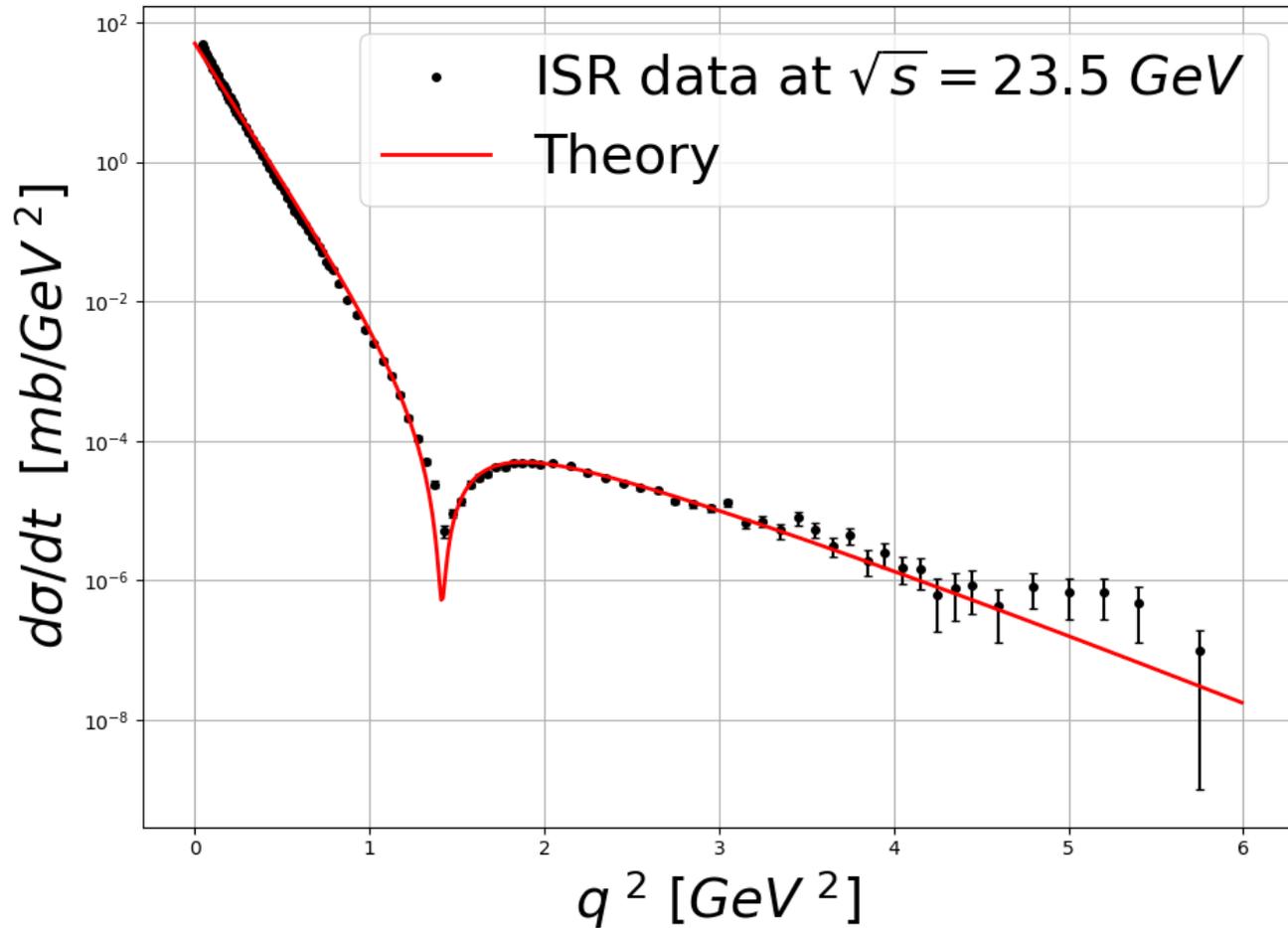


# Gluon bundles + infinite loops summed

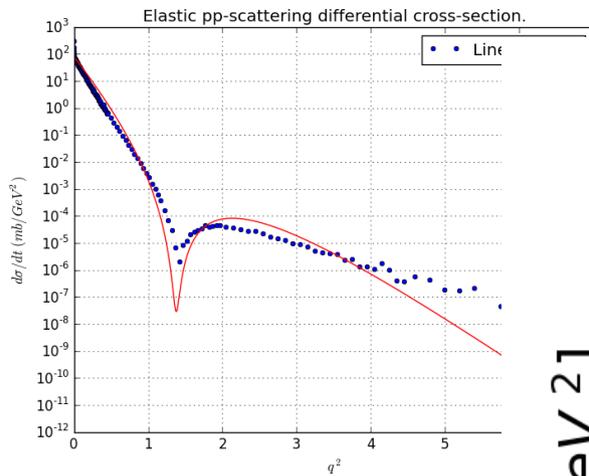


**GB+one loop**

**$g=6.0$**   
 **$m_{\text{ext}} \approx 1m_{\pi}$**     **$m_{\text{int}} \approx 3m_{\pi}$**   
 **$p=0.10$**   
 **$\kappa=4 \cdot 10^{-6}$**

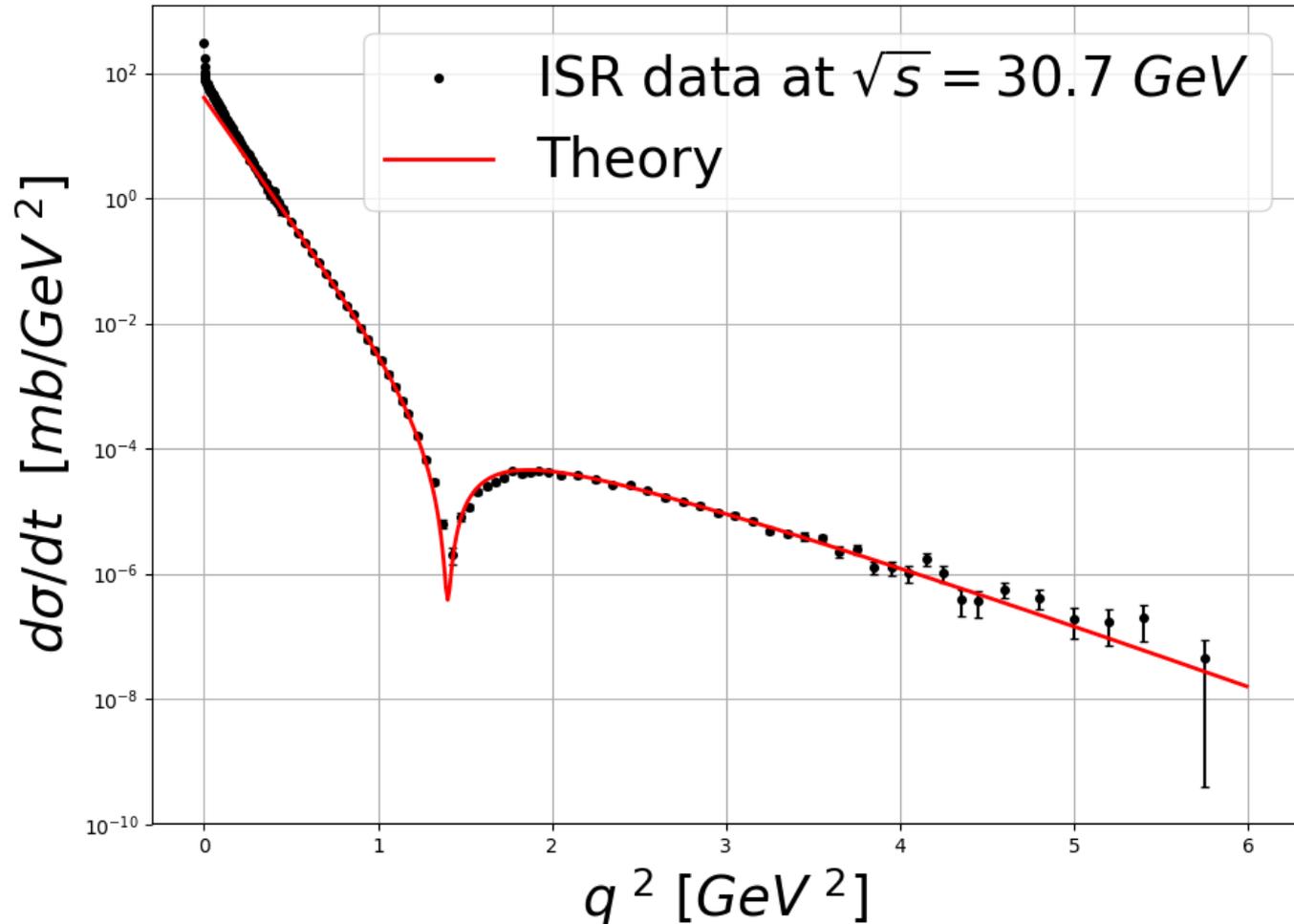


# Gluon bundles + infinite loops summed



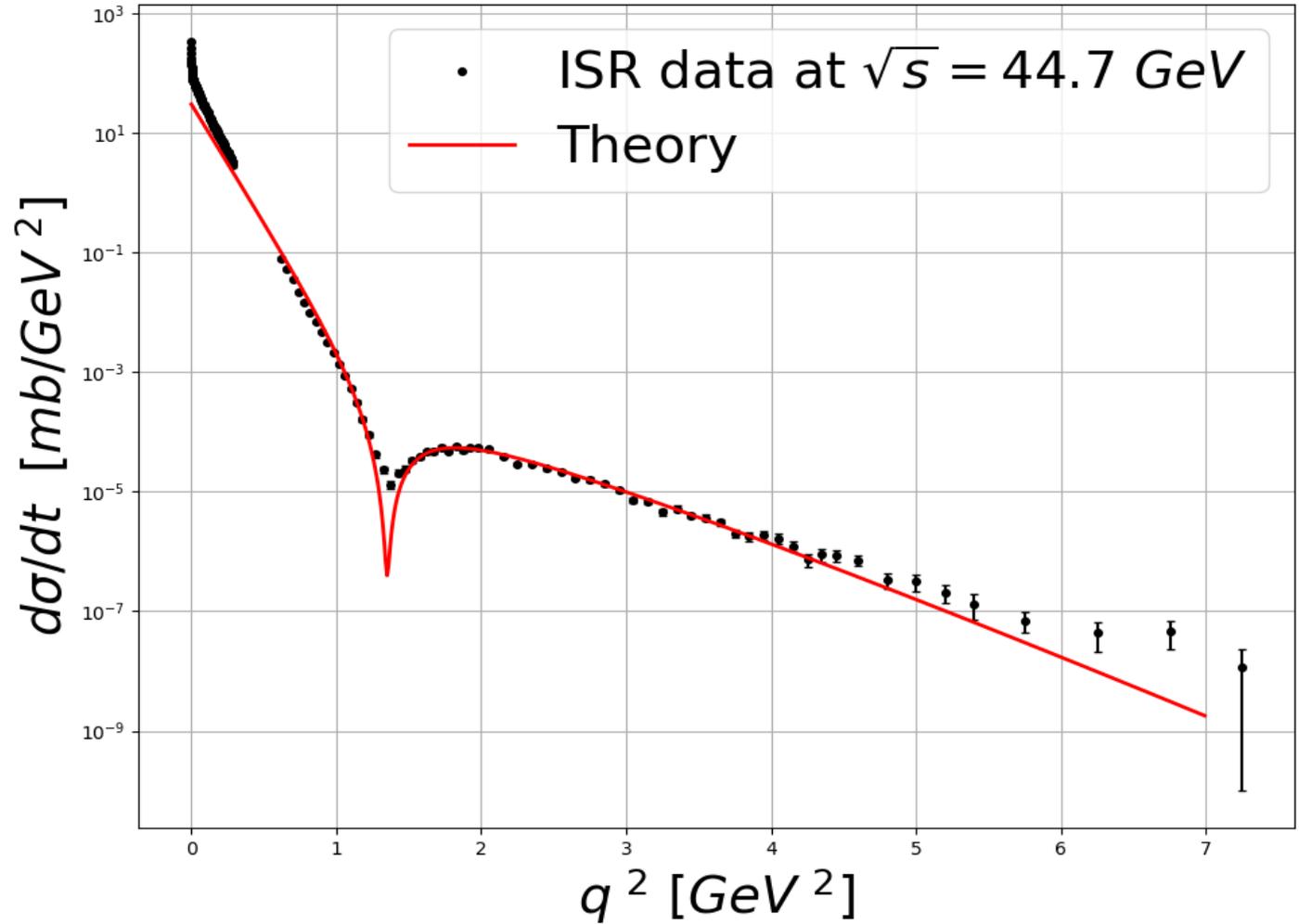
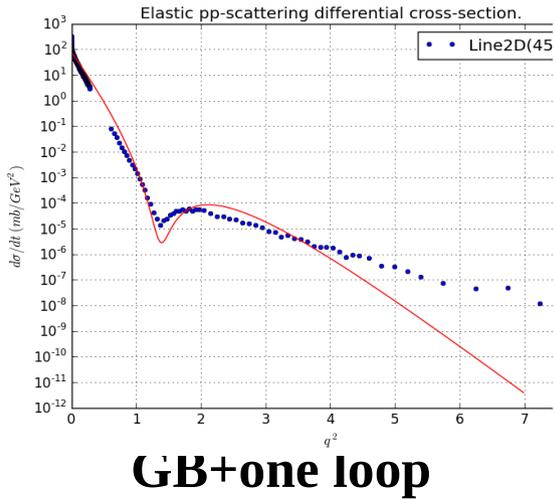
**GB+one loop**

**$g=6.0$**   
 **$m_{\text{ext}} \approx 1m_{\pi}$**      **$m_{\text{int}} \approx 3m_{\pi}$**   
 **$p=0.1$**   
 **$\kappa=4 \cdot 10^{-6}$**



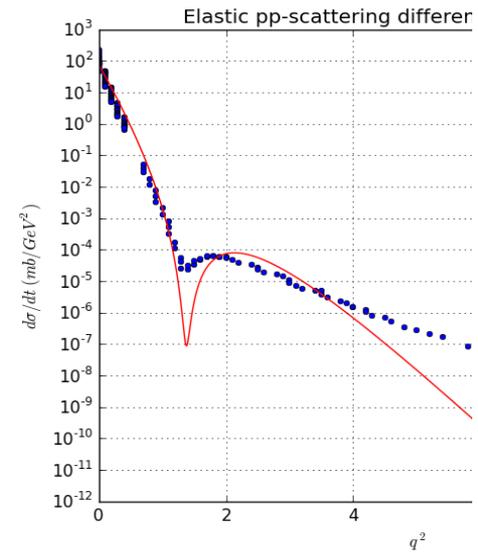
**Note the dip moves to the left with increasing Energy. This can be obtained with the exact 6 body problem. Here we input the energy dependence As  $(E/m)^{(2p)}$**

# Gluon bundles + infinite loops summed



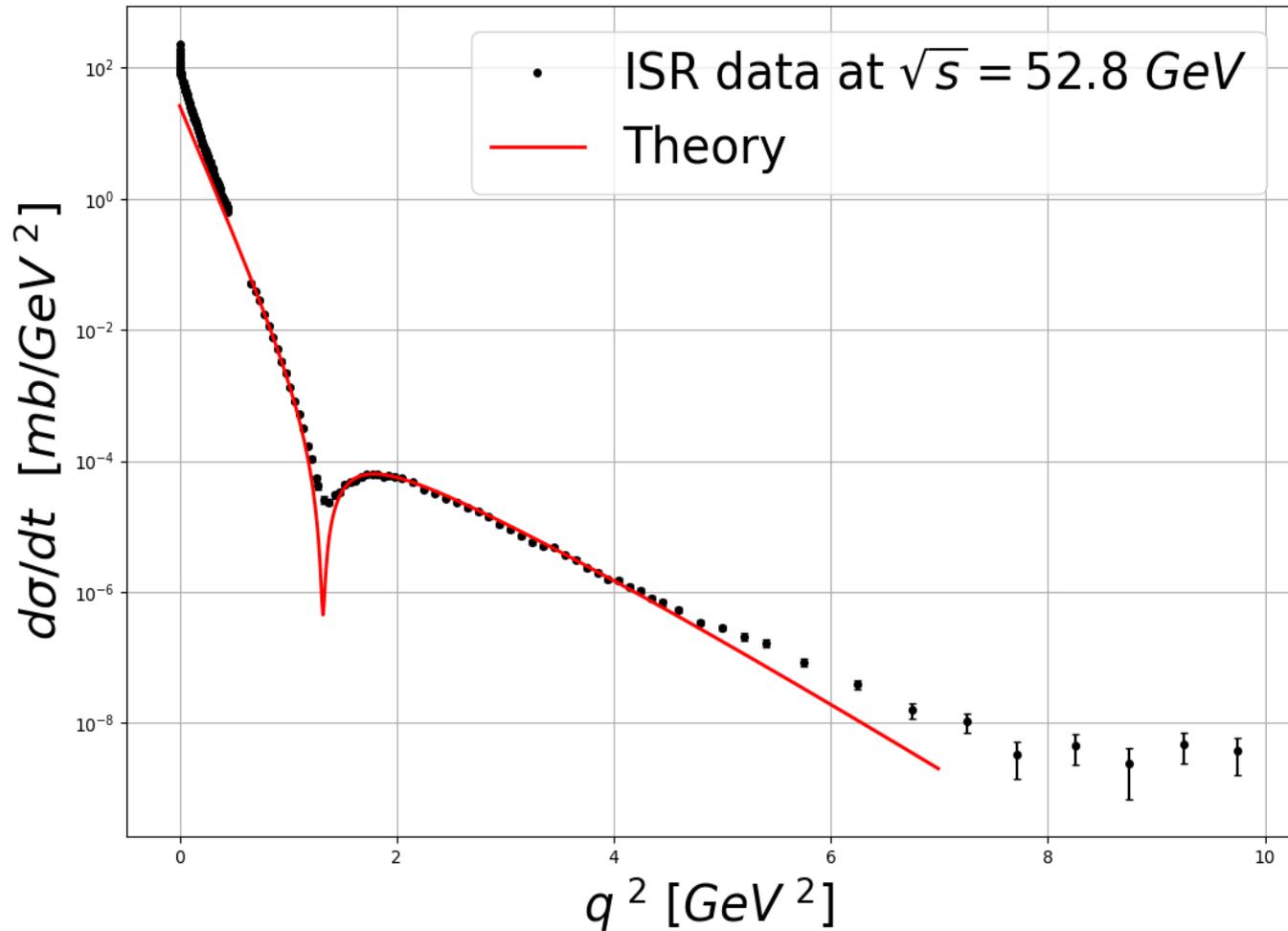
$g=6.0$   
 $m_{\text{ext}} \approx 1m_{\pi}$     $m_{\text{int}} \approx 3m_{\pi}$   
 $p=0.1$   
 $K=4.5 \cdot 10^{-6}$

# Gluon bundles + infinite loops summed

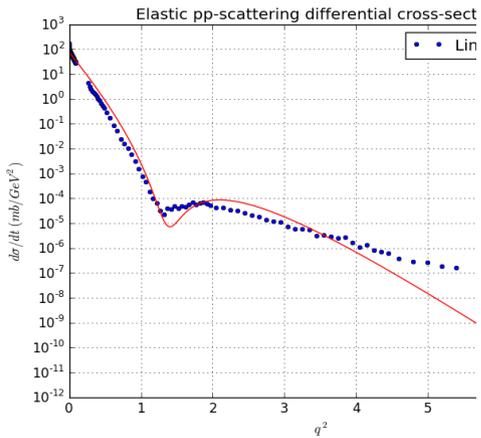


**GB+one loop**

**$g=6.0$**   
 **$m_{\text{ext}} \approx 1m_{\pi}$     $m_{\text{int}} \approx 3m_{\pi}$**   
 **$p=0.1$**   
 **$K=4.9 \cdot 10^{-6}$**

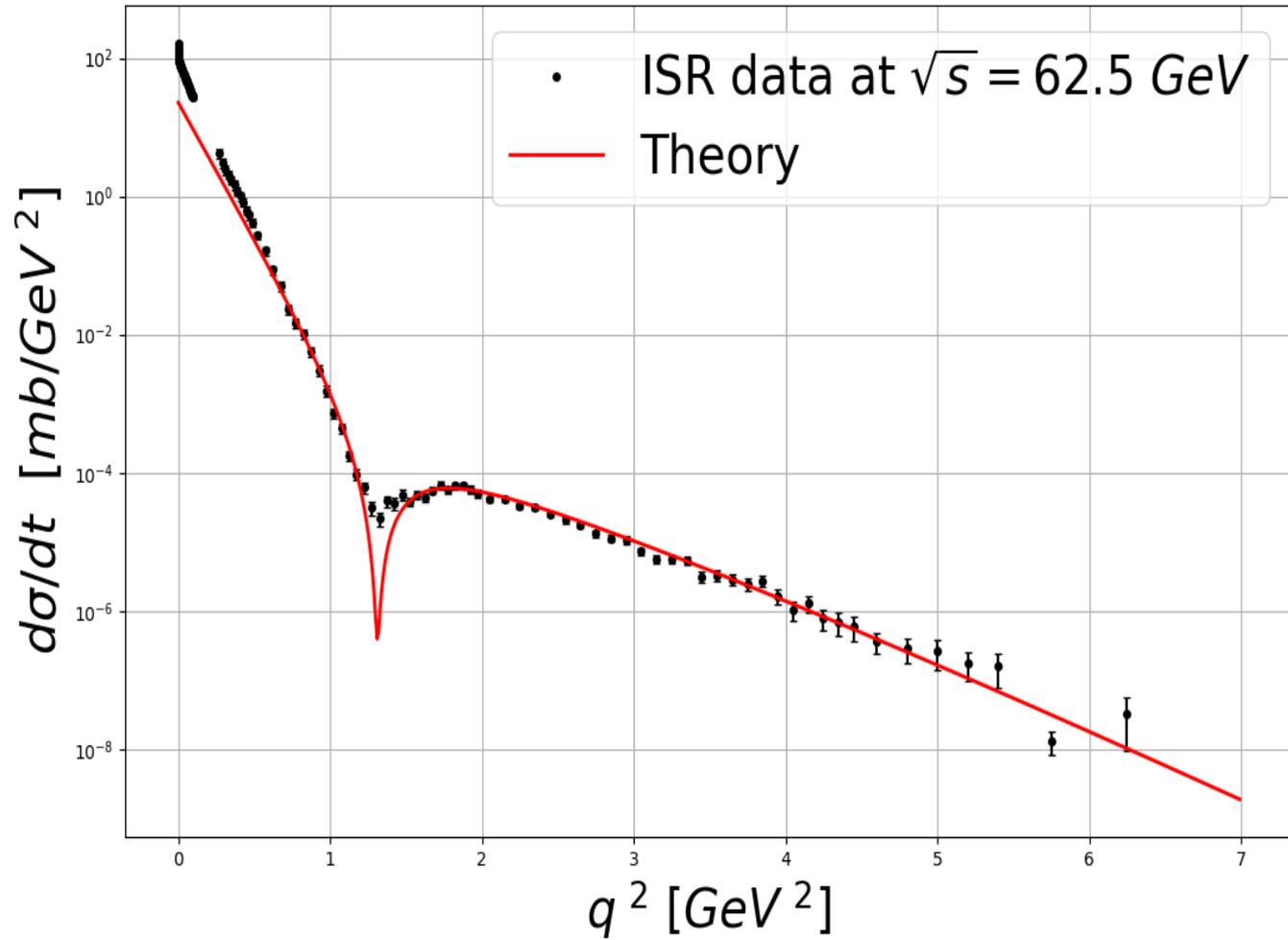


# Gluon bundles + infinite loops summed



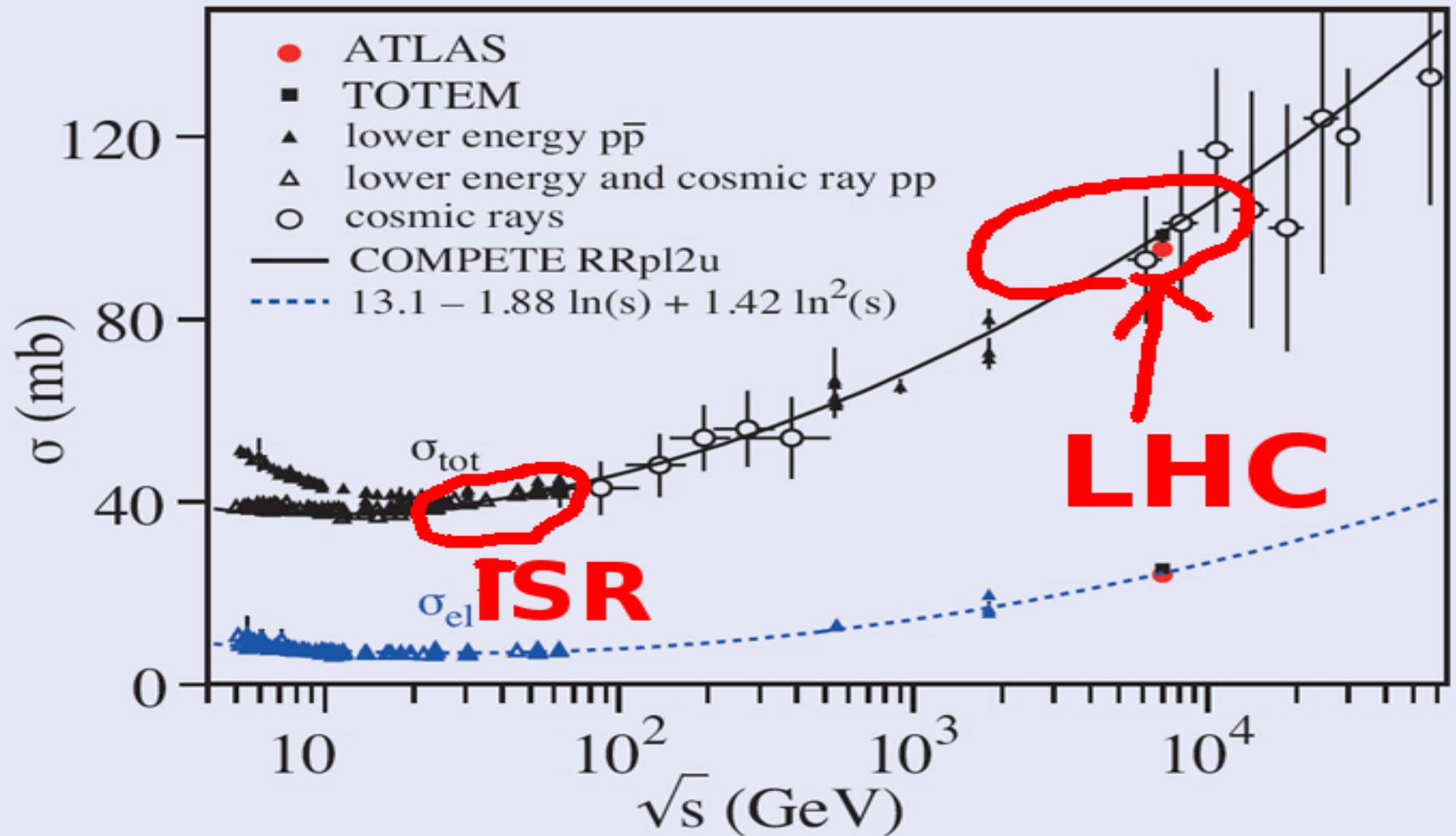
**GB+one loop**

**$g=6.0$**   
 **$m_{\text{ext}} \approx 1m_{\pi}$**      **$m_{\text{int}} \approx 3m_{\pi}$**   
 **$p=0.1$**   
 **$\kappa=4.9 \cdot 10^{-6}$**



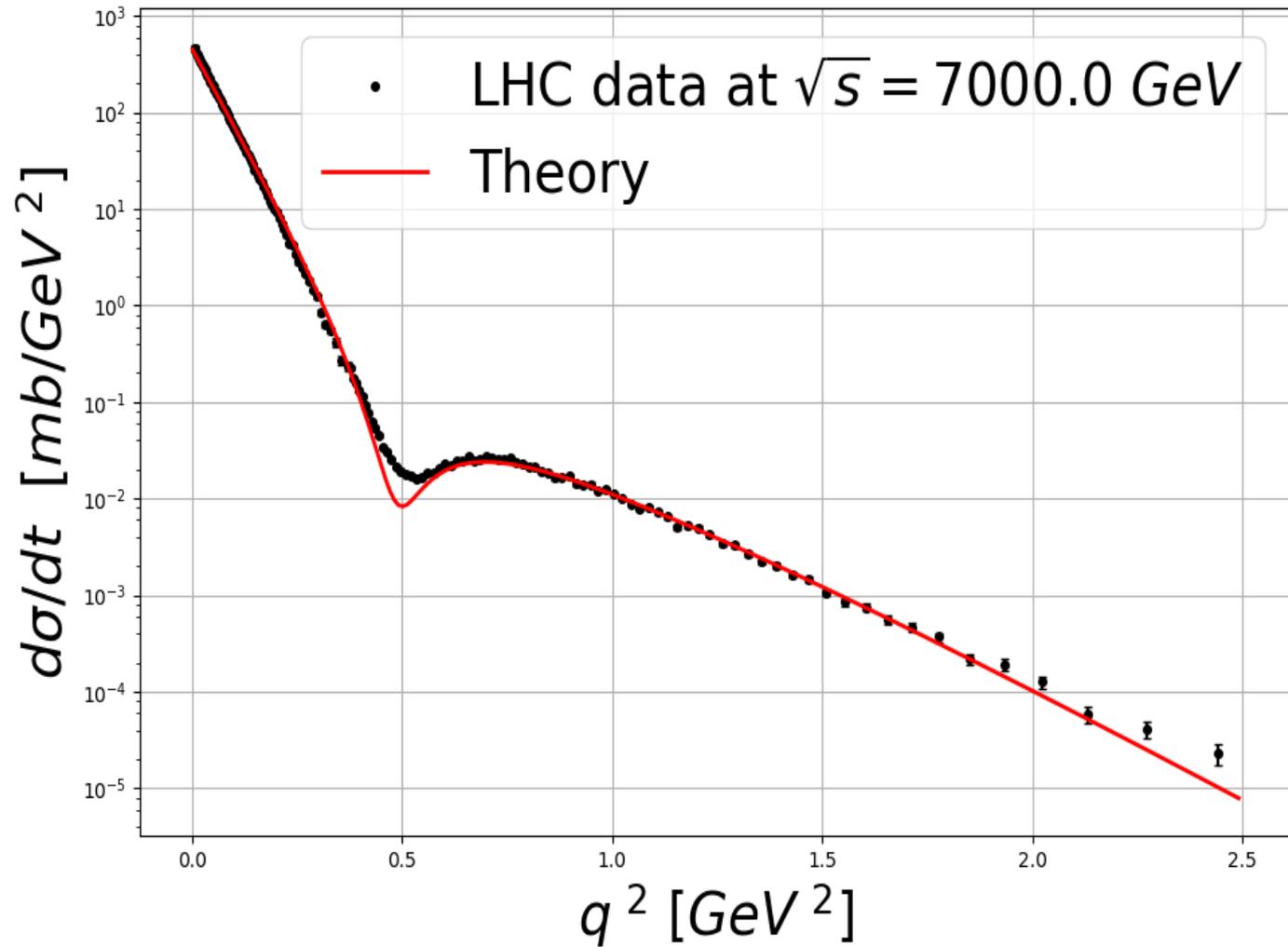
# Extension to LHC

$$\sigma = \frac{4\pi}{k} \Im[T(0)]$$



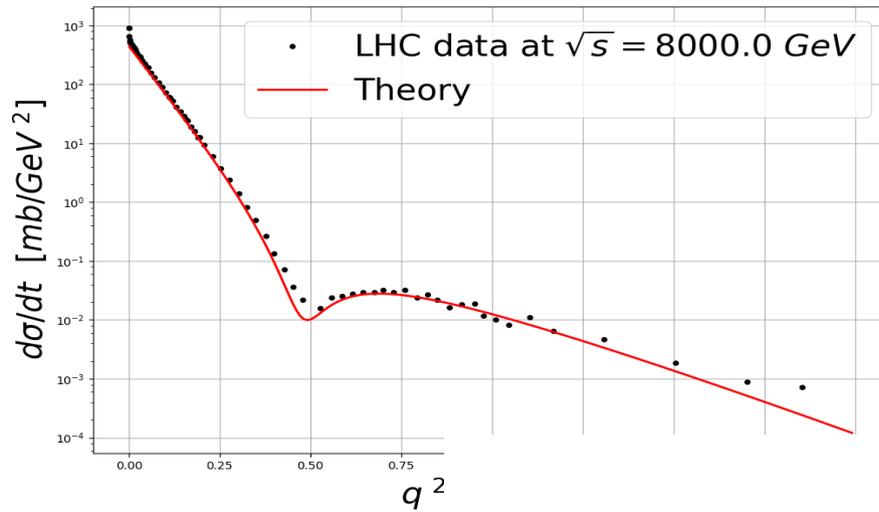
# LHC data compared with our QCD formalism

$g=6.0$   
 $m_{\text{ext}} \approx 1m_{\pi}$     $m_{\text{int}} \approx 2m_{\pi}$   
 $p=0.02$   
 $\kappa=5 \cdot 10^{-5}$

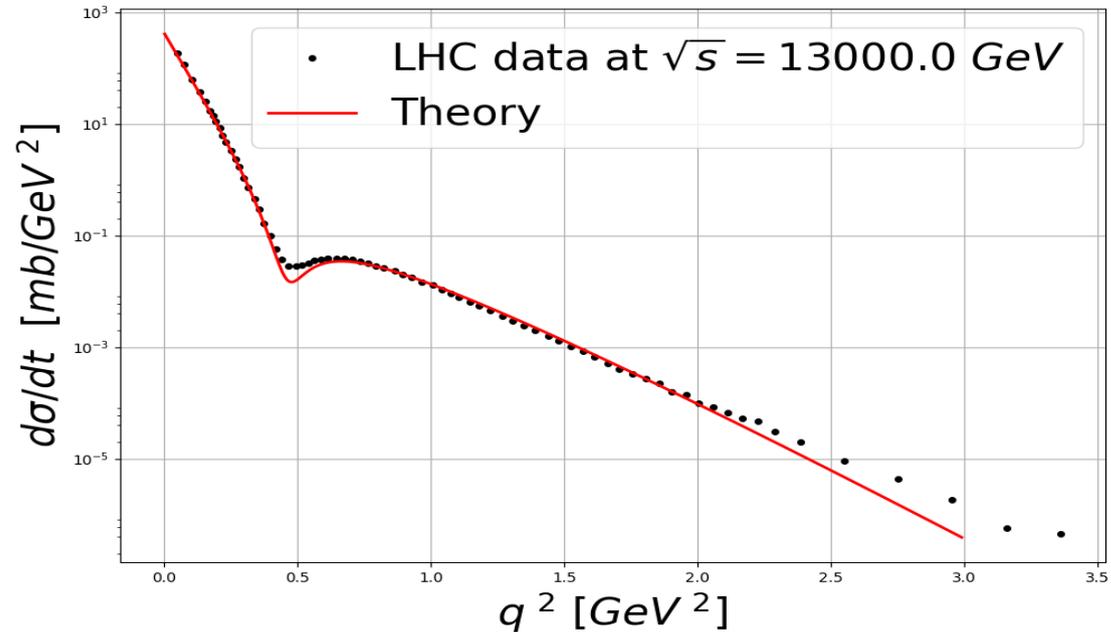


# LHC data compared with our QCD formalism

## Very preliminary 8 TeV data



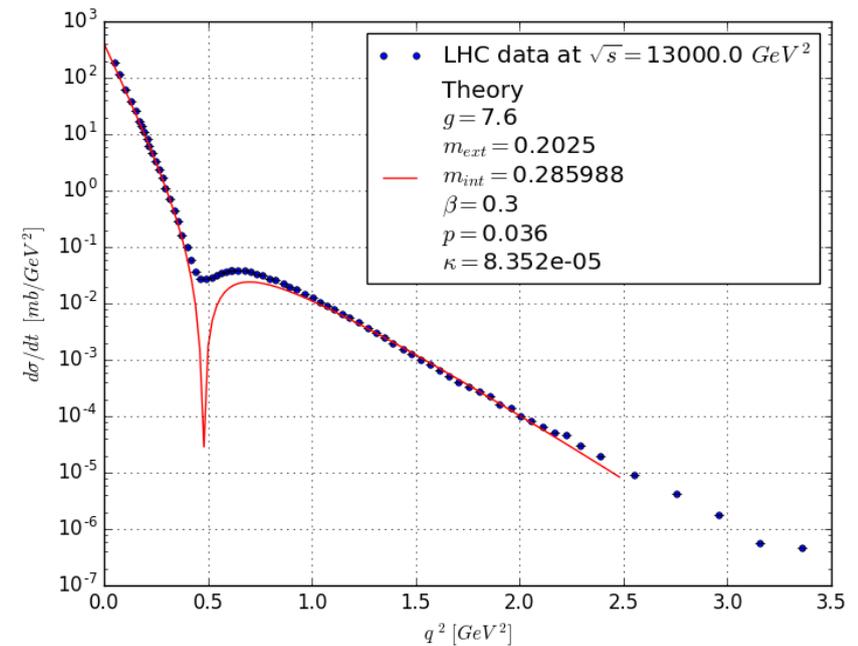
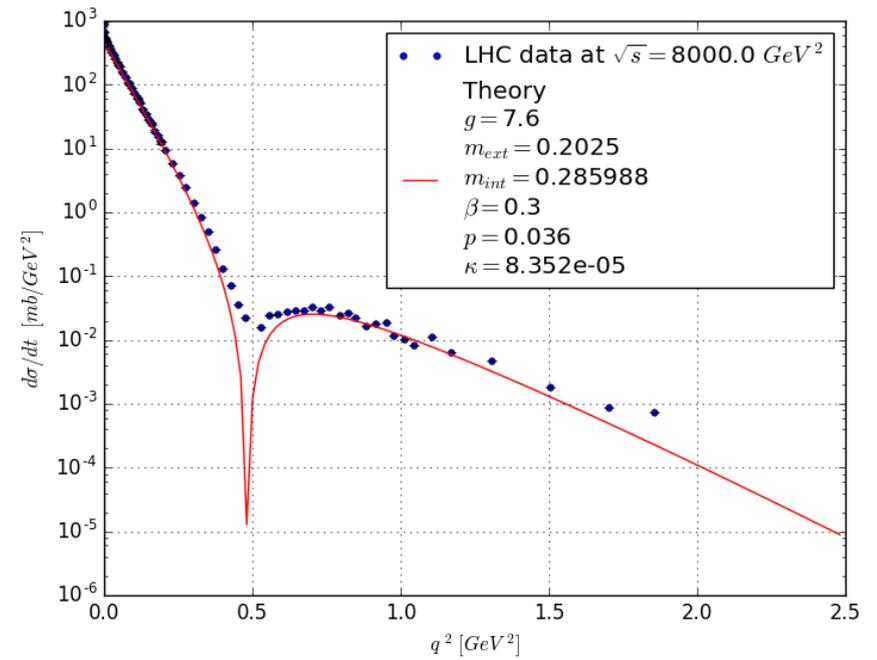
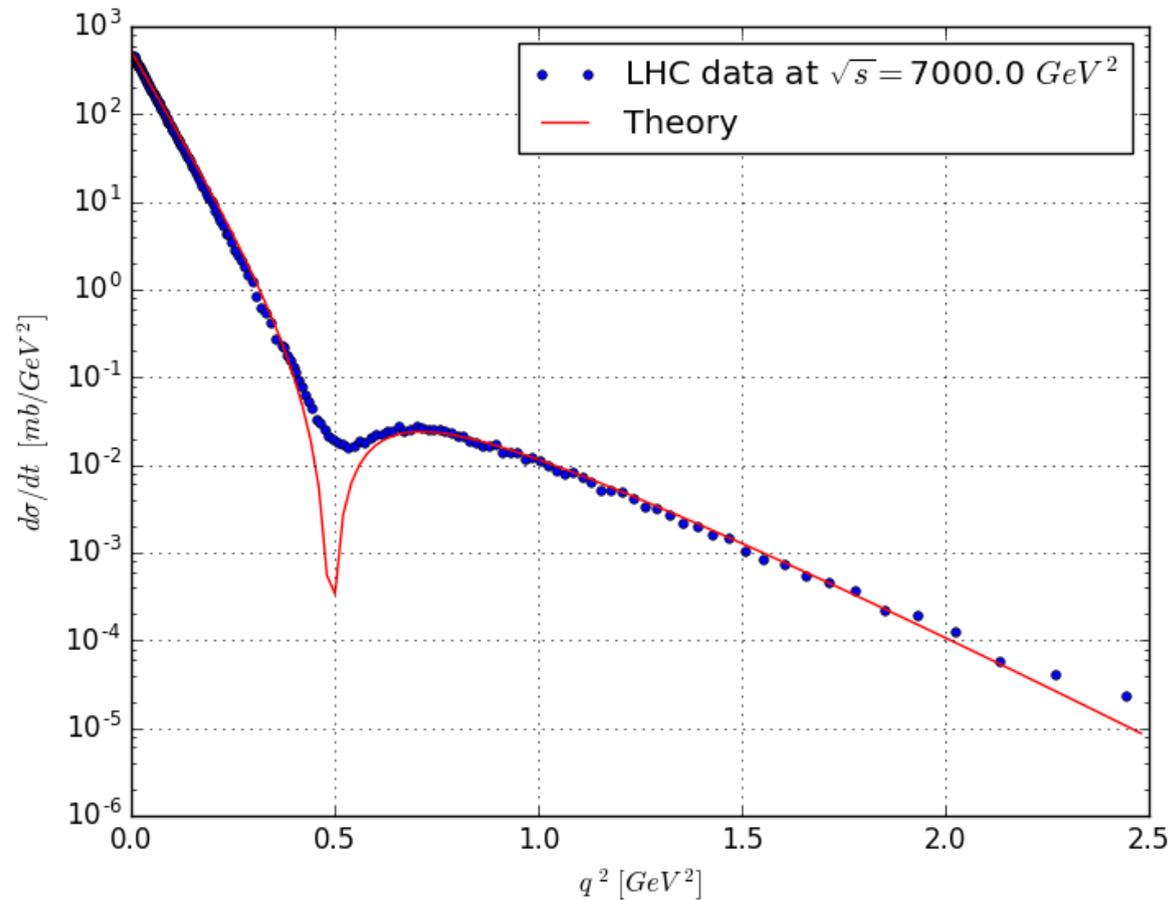
## Preliminary 13 TeV Data



$g=6.0$   
 $m_{\text{ext}} \approx 1m_{\pi}$     $m_{\text{int}} \approx 3m_{\pi}$   
 $p=0.02$   
 $\kappa=5.4 \cdot 10^{-5}$

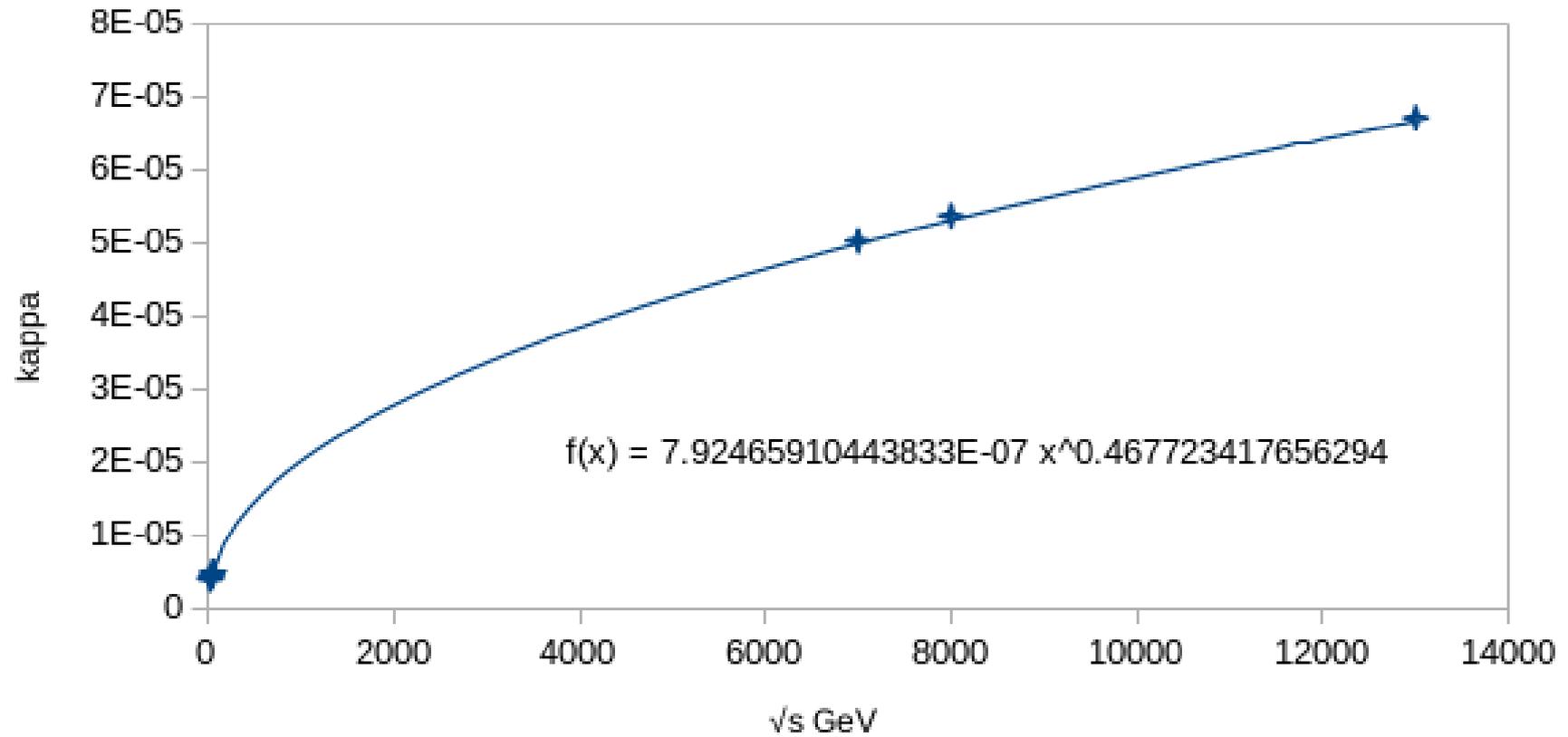
**Without interference terms.**

**Interference terms account for smoothing out of the dip.**



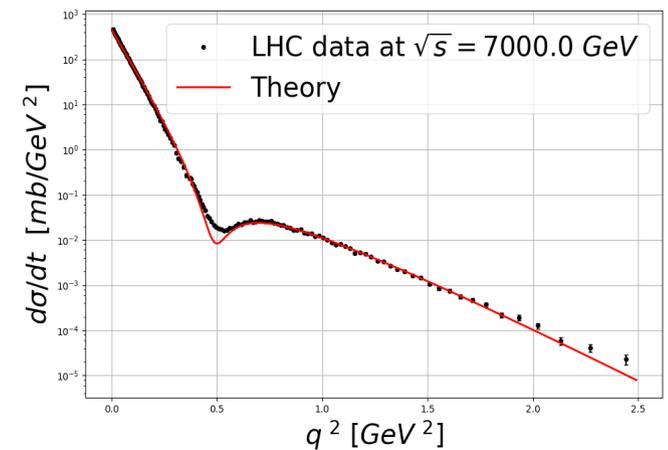
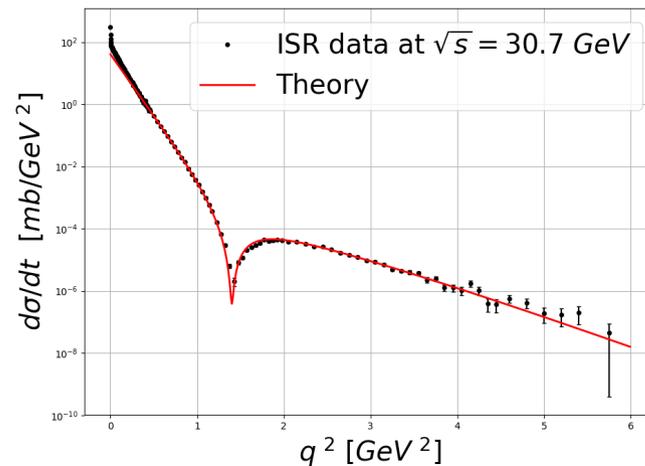
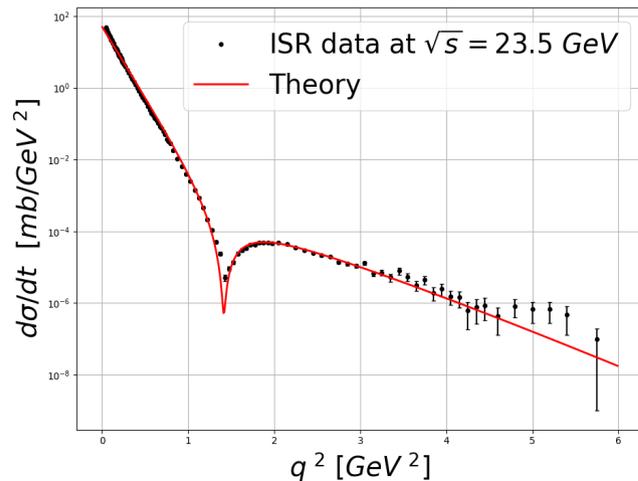
$\kappa$  is proportional to  $\sqrt{E}$

kappa vs  $\sqrt{s}$



Perhaps it is time to end this talk. What we have done is we began with the Schwinger Action, writing down the QCD Lagrangian, introduced transverse fluctuations for quarks. Gauge Invariance is insured by Gauge Independence. Preserve both Casimir,  $C_2$  and  $C_3$  invariants. New property in Effective Locality Individual gluons have disappeared giving the entire sum as a Gluon Bundle.

- all radiative corrections in QCD are formed from GB and Loops
- Quark Binding
- Nucleon Binding
- Pion mass
- Summed infinite loops graphs after making a particular (re)normalization
- Compared with ISR and LHC data



## Scattering Amplitude

$$T_{eikonal}(s, t; g^2) = \frac{iS}{2m^2} \int d^2b e^{iqb} [1 - e^{i\chi_0(s, b)}]$$

- Having a relation between  $\tilde{\Phi}(b)$  and the Eikonal  $\chi$

$$e^{i\chi(s, b)} = N' \int_0^\infty dR R^3 e^{iR^2/4 + ig\tilde{\Phi}(b)/R}$$

**The energy,  $s$ , dependence will come from our renormalization in slide to come.**

**We use approximation  $\sqrt{\chi^2} = R^2$**

**Now we can calculate the elastic cross-section, with differential cross-section as**

$$\frac{d\sigma}{dt} = \frac{m^4}{16\pi p^2 E^2} |T_{summed}|^2$$

