Simulating quantum dynamics of lattice gauge theories

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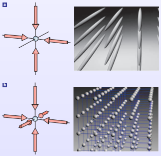
Outline

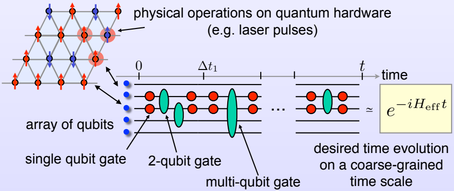
From optical lattices/trapped ions · · ·

Adjust parameters such that atoms in optical traps act as d.o.f



cold atoms in optical lattices realize Bosonic and Fermionic Hubbard type models.

Ions confined in ion-trap with interactions between individual ions can be controlled.



Advantage: Much more control over interactions; Challenge: Scalability.

Prepare the "quantum" system and let it evolve. Make measurements at times t_i on identically prepared systems. Achievement: observation of Mott-insulator (disordered) to superfluid (ordered) phase. Greiner et. al. (2002)

$\cdots$ to real time/finite density QCD

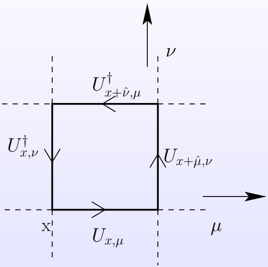
- Lattice calculations of static and finite temperature properties of QCD well controlled
- At finite μ_B, lattice methods fail due to the sign problem
- Questions about real-time dynamics also inaccessible:

$$\langle \Phi_0 | \mathcal{O}(t) \mathcal{O}(0) | \Phi_0 \rangle = \frac{1}{\mathcal{Z}} \sum_m |\langle \Phi_0 | \mathcal{O} | m \rangle|^2 \, e^{-i(E_m - E_0)t}$$

- What if the fermions themselves can be used as degrees of freedom in themselves in simulations? Feynman, 1982
- Exploit the advances made in optical lattices to set up systems which mimic Hamiltonians of interest to particle physics
- Need finite Hilbert spaces! $\rightarrow$ Quantum Links

Outline

The U(1) pure gauge theory



$$S[u] = -J \sum_{x,\mu>\nu} [u_\square + u^\dagger_\square]$$

$$u_\square = u_{x,\mu} u_{x+\mu,\nu} u^\dagger_{x+\nu,\mu} u^\dagger_{x,\nu}$$

$$u_{x,\mu} = \exp(i\phi_{x,\mu}); \quad v_x = \exp(i\alpha_x)$$

$$u'_{x,\mu} = v_x u_{x,\mu} v_{x+\mu}$$

Hamiltonian formulation:

$\hat u_{x,\mu}, \ \hat u^\dagger_{x,\mu} \ \in \ U(1)$ operators in an infinite dimensional Hilbert space. Dynamics of u: $\hat e = -i\partial_\phi$

$$[\hat e, \hat u] = \hat u; \ [\hat e, \hat u^\dagger] = -\hat u^\dagger; \ [\hat{\mathbf{u}}, \hat{\mathbf{u}}^\dagger] = \mathbf{0}$$

U(1) gauge transformations generated by Gauss Law:

$$G_x = \sum_i (\hat e_{x,i} - \hat e_{x-i,i}); \ [G_x, H] = 0$$

U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2} \sum_{x,i} \hat e^2_{x,i} - \frac{1}{2g^2} \sum_{x,i\neq j} (\hat u_\square + \hat u^\dagger_\square)$$

The U(1) Quantum Link Model

- $U = S^1 + iS^2$, $U^\dagger = S^1 - iS^2$ and $E = S^3 \Rightarrow$ finite Hilbert space of quantum spin S at each link
- **continuous** U(1) gauge invariance is exact, due to the commutation relations:

$$[E, U] = U; \; [E, U^\dagger] = -U^\dagger; \; [\mathbf{U}, \mathbf{U}^\dagger] = \mathbf{2E}$$

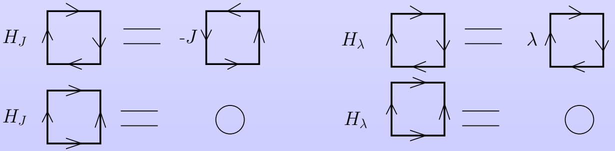
- Gauge theory with a 2-d Hilbert space at each link
  Horn (1981); Orland (1990); Chandrasekharan, Wiese (1996)

$$H = -J \sum_{\square} \left(U_\square + U_\square^\dagger \right) + \lambda \sum_{\square} \left(U_\square + U_\square^\dagger \right)^2$$

- The Gauss Law as before generates gauge transformations:

$$G_x = \sum_i \left(E_{x,i} - E_{x-i,i} \right); \; [G_x, H] = 0$$

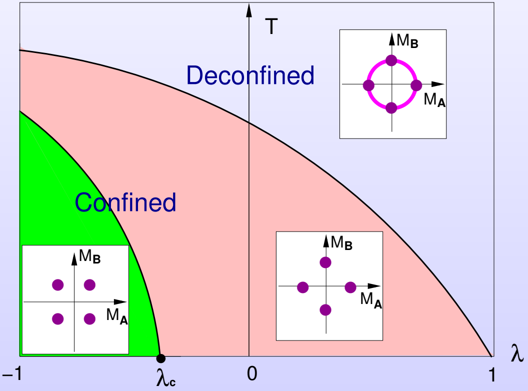
- Second term introduces non-trivial physics and interesting phase structure

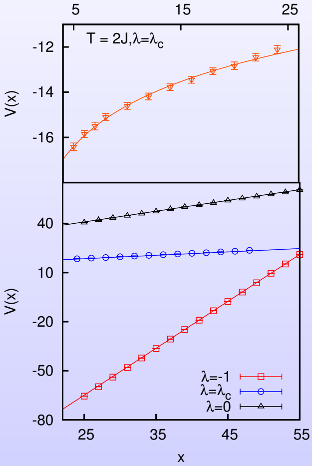


Phase diagram

Explored with exact diagonalization and a newly-developed efficient cluster alogrithm

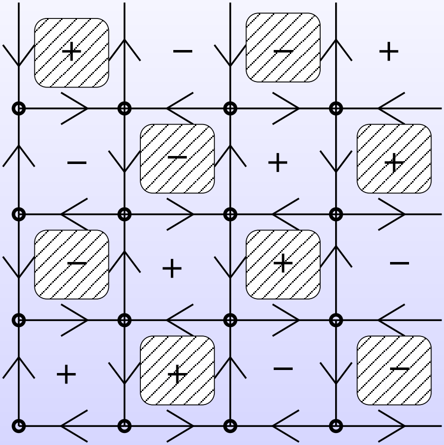
DB, Jiang, Widmer, Wiese (2013)





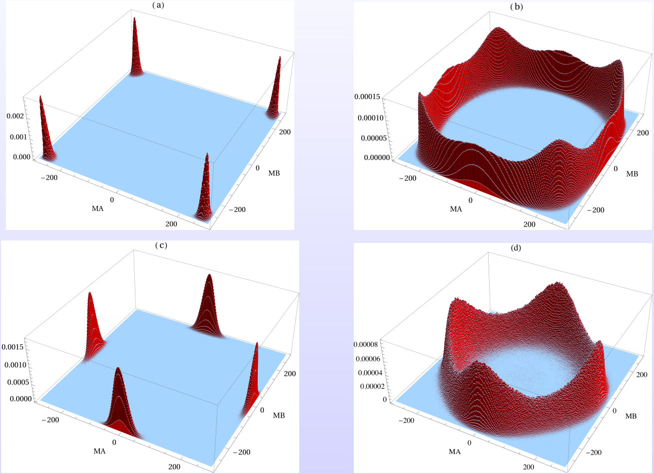
A *global SO(2) symmetry is "almost" emergent* at λ_c. However, description in terms of a low-energy *effective theory suggests weak 1st order transition*.

Order parameters



2-component order parameter constructed out of dual variables residing at the centre of plaquettes. The phase is related to which of the two sublattices can order at a given λ.

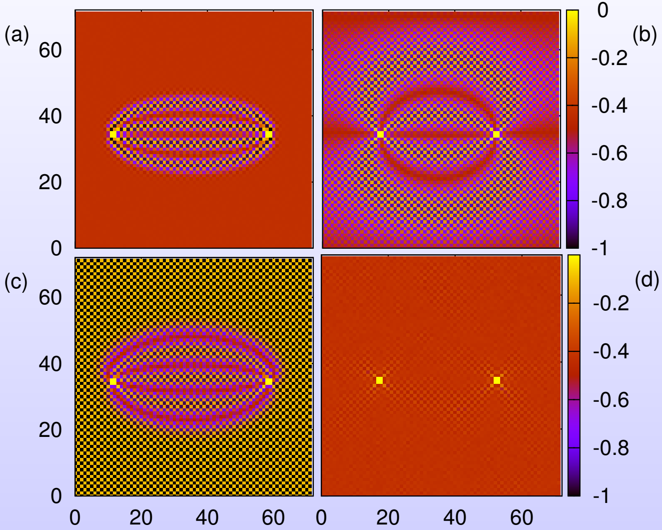
Probability distributions of order parameters



Clockwise from top: (a) $\lambda = -1$ both sublattices order; (b) $\simeq \lambda_c$ “almost“ emergent $SO(2)$ symmetry

(c) $\lambda_c < \lambda < 0$ (d) $\lambda = 0$ either sublattice orders

Crystalline confinement



Energy density $\langle H_J \rangle$ of two charges $Q = \pm\, 2$ placed in along the axis in $L = 72$ lattice

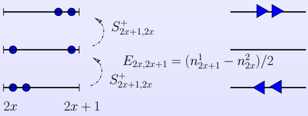
Outline

Schwinger model with quantum links and staggered fermions

$$H = -t\sum_x \left[\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_x E_{x,x+1}^2$$

⋆ Use the bosonic *rishon* representation

$$U_{x,x+1} = S^+ = b_x b_{x+1}^\dagger; \quad E_{x,x+1} = S^z = \frac{1}{2}(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x)$$



Rishon for spin $S = 1; \mathcal{N} = 2$

$$n_x + n_{x+1} = 2S = 2$$

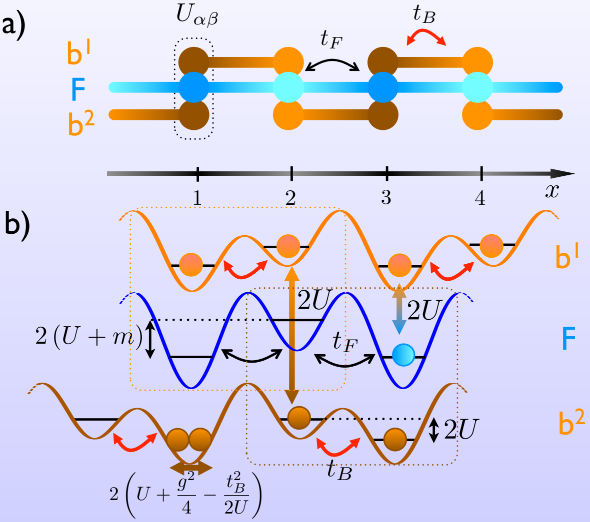
⋆ Gauss Law: $\tilde{G}_x = \nabla\cdot E - \rho = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}[(-1)^x - 1]$

⋆ In optical lattices, realized using a microscopic Hubbard-type Hamiltonian

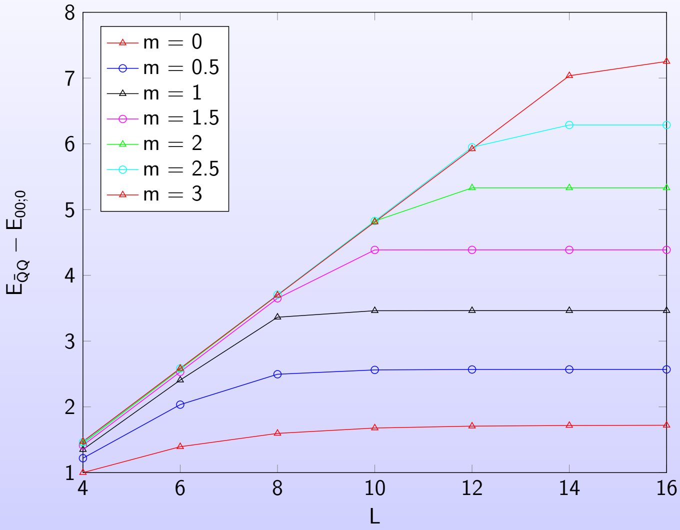
$$\begin{aligned}
\tilde{H} &= \sum_x h_B^{x,x+1} + \sum_x h_F^{x,x+1} + m\sum_x (-1)^x n_x^F + U\sum_x \tilde{G}_x^2 \\
&= -t_B \sum_{x\in\text{odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x\in\text{odd}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.} \\
&+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha
\end{aligned}$$

DB, Dalmonte, Müller,Rico Ortega, Stebler, Wiese, Zoller (2012)

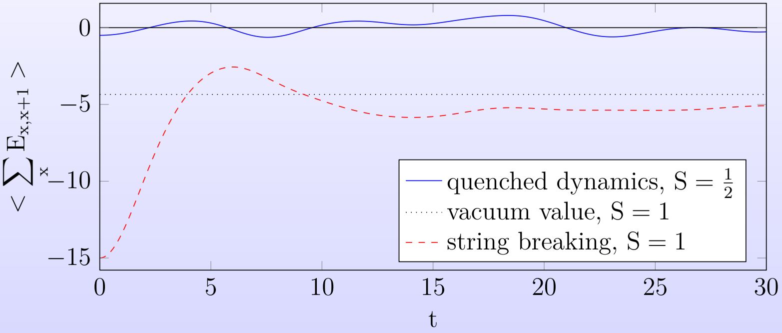
Optical lattice setup



Static and Real-time physics



Static and Real-time physics



Outline

Non-Abelian quantum link models

For QCD with quantum links and domain-wall fermions Brower, Chandrasekharan, Wiese (1999)
The Hamiltonian with staggered fermions are given by:

$$H = -t \sum_{\langle xy \rangle} \left(s_{xy} \psi_x^{i\dagger} U_{xy}^{ij} \psi_y^j + \mathrm{h.c.} \right) + m \sum_x s_x \psi_x^{i\dagger} \psi_x^i + V \sum_x (\psi_x^{i\dagger} \psi_x^i)^2$$

where $s_x = (-1)^{x_1 + \cdots + x_d}$ and $s_{xy} = (-1)^{x_1 + \cdots + x_{k-1}}$, with $y = x + \hat{k}$.

DB, Bögli, Dalmonte, Rico Ortega, Stebler, Wiese, Zoller (2012)

The non-Abelian Gauss law:

$$G_x^a = \psi_x^{i\dagger} \lambda_{ij}^a \psi_x^j + \sum_k \left(L_{x,x+\hat{k}}^a + R_{x-\hat{k},x}^a \right), \quad G_x = \psi_x^{i\dagger} \psi_x^i - \sum_k \left(E_{x,x+\hat{k}} - E_{x-\hat{k},x} \right),$$

λ^a: Gell-Mann matrices; L_{xy}^a, R_{xy}^a : $SU(N)$ electric fluxes, E_{xy}: Abelian $U(1)$ flux.

Other possible terms in the Hamiltonian: $\frac{g^2}{2} \sum_{\langle xy \rangle} \left(L_{xy}^a L_{xy}^a + R_{xy}^a R_{xy}^a \right)$, $\frac{g'^2}{2} \sum_{\langle xy \rangle} E_{xy}^2$, $\frac{1}{4g^2} \sum_{\square} \left(U_{\square} + \mathrm{h.c.} \right)$. Not included in our study.

$U(N)$ gauge invariance requires:

$$[L^a, L^b] = 2i f_{abc} L^c, [R^a, R^b] = 2i f_{abc} R^c, \quad [L^a, R^b] = [E, L^a] = [E, R^a] = 0,$$
$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U \lambda^a, \ [E, U] = U$$

To study SU(N) theories, we must include the term $\gamma \sum_{\langle xy \rangle} (\det U_{xy} + \mathrm{h.c.})$

Rishons: the magic of the QLMs

Non-Abelian link fields can be represented by a finite-dimensional fermionic representation:

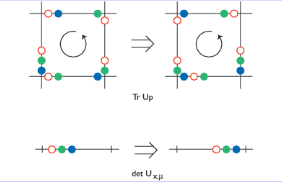
$$L^a_{xy} = c^{i\dagger}_{x,+}\lambda^a_{ij}c^j_{x,+},\ R^a_{xy} = c^{i\dagger}_{y,-}\lambda^a_{ij}c^j_{y,-},\ E_{xy} = \frac{1}{2}(c^{i\dagger}_{y,-}c^i_{y,-} - c^{i\dagger}_{x,+}c^i_{x,+}),\ U^{ij}_{x,y} = c^i_{x,+}c^{j\dagger}_{y,-}.$$

$c^i_{x,\pm k}$, $c^{i\dagger}_{x,\pm k}$ with color index $i \in \{1, 2, ..., N\}$ are rishon operators. They anti-commute:

$$\{c^i_{x,\pm k}, c^{j\dagger}_{y,\pm l}\} = \delta_{xy}\delta_{\pm k,\pm l}\delta_{ij},\ \{c^i_{x,\pm k}, c^j_{y,\pm l}\} = \{c^{i\dagger}_{x,\pm k}, c^{j\dagger}_{y,\pm l}\} = 0.$$

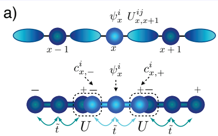
By fixing the no of rishons on a link, the Hilbert space can be truncated in a completely gauge-invariant manner: $\mathcal{N}_{xy} = c^{i\dagger}_{y,-}c^i_{y,-} + c^{i\dagger}_{x,+}c^i_{x,+}$; $[\mathcal{N}_{xy}, H] = 0$

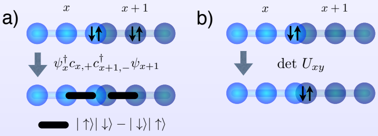
Action of the plaquette and the determinant on a SU(3) theory with $\mathcal{N}_{xy} = 3$ rishons per link.



Implementation of the non-Abelian models

Lattice with quark and rishon sites as a physical optical lattice for fermionic atoms.





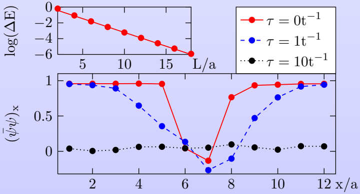
- Force the rishon number constraint per link by the term: $U\sum_{\langle xy\rangle}(\mathcal{N}_{xy}-n)^2$.
- Hopping is induced perturbatively with a Hubbard-type Hamiltonian.
- Color d.o.f are encoded in the internal states (the $2I+1$ Zeeman levels of the electronic ground state ${}^1S_0$) of fermionic alkaline-earth atoms.
- Remarkable property: scattering is almost exactly $2I+1$ symmetric.
- Since the hopping process between quarks and rishon sites is gauge invariant, the induced effective theory is also gauge invariant.

[Quantum simulator constructions also by Reznick, Zohar, Cirac (Tel-Aviv, Münich) and Tagliacozzo, Celi, Zamora, Lewenstein (Barcelona)]

Chiral Dynamics

dimension	group	$\mathcal{N}$	C	flavor	baryon	phenomena
$(1+1)$D	$U(2)$	1	no	no	no	χSB, $T_c = 0$
$(2+1)$D	$U(2)$	2	yes	$\mathbb{Z}(2)$	no	χSB, $T_c > 0$
$(2+1)$D	$SU(2)$	2	yes	$\mathbb{Z}(2)$	$U(1)$ boson	χSB, $T_c > 0$ χSR, $n_B > 0$
$(3+1)$D	$SU(3)$	3	yes	$\mathbb{Z}(2)^2$	$U(1)$ fermion	χSB, $T_c > 0$ χSR, $n_B > 0$

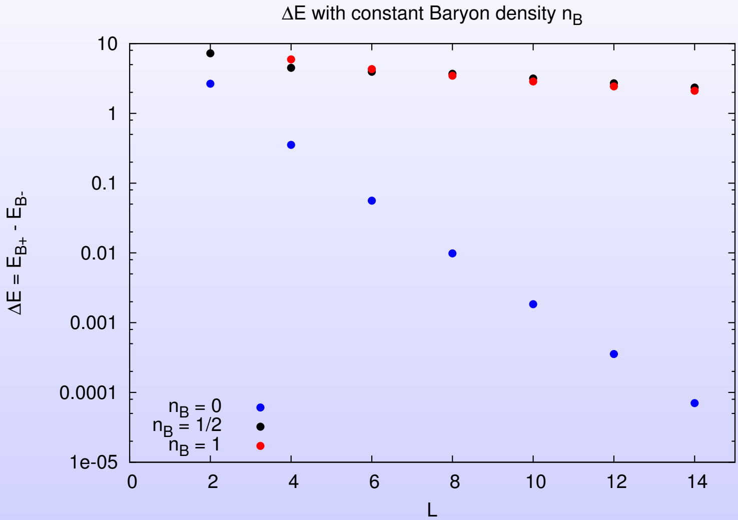
Table: Symmetries and phenomena in some QLMs.



Top: Chiral symmetry breaking in a $U(2)$ QLM with $m = 0$ and $V = -6t$.

Bottom: Real-time evolution of the order parameter profile $(\overline{\psi}\psi)_x(\tau) = s_x \langle \psi_x^{i\dagger} \psi_x^i - \frac{N}{2} \rangle$ for $L = 12$, mimicking the expansion of a hot quark-gluon plasma.

Chiral symmetry restoration at finite density



Outline

Conclusions

- Although quantum simulating QCD is still far away, many of the simpler models have similar physical phenomena. Very useful for insight into the physics of QCD.
- Realization of a quantum simulator for the Schwinger model would be quite remarkable achievement. Most of the tools needed in setting it up is available separately.
- Quantum simulators need to be validated by efficient classical simulations. Development of new efficient algorithms.
- In toy systems, this would allow quantum simulation of real-time evolution of string breaking and the study of ”nuclear” physics and dense “quark“ matter
- More interesting models may allow investigation of chiral symmetry restoration, baryon superfluidity, color superconductivity at high densities and ”nuclear“ collisions
- Every development brings the promise of interesting physics along with it!

Backup: An example of real-time evolution

Use the Trotter-Suzuki decomposition

$$e^{-i\mathcal{H}t} \simeq e^{-i\mathcal{H}_1 t} e^{-i\mathcal{H}_2 t} e^{[\mathcal{H}_1,\mathcal{H}_2]t^2/2}$$

to study the real time evolution of

2-quantum spins

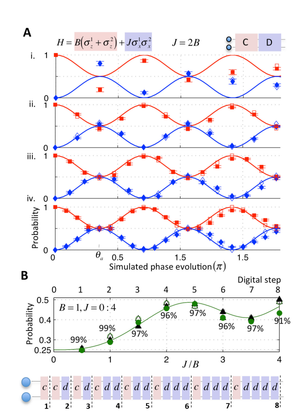
Time-dependent variation of parameters possible

Trotter errors known and bounded;

gate errors under control;

Implementation with upto 6

ions/spins Lanyon et. al. 2011



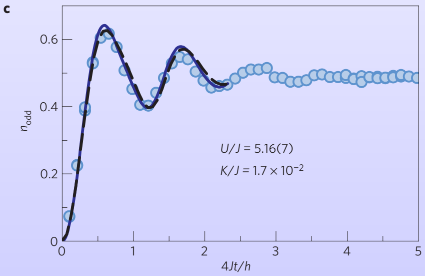
Backup: Classical vs Quantum Simulation

Example of a quantum quench in a strongly correlated Bose gas.

S. Trotzky et. al., Nature Physics (2012).

$$H = \sum_j \left[-J(a_j^\dagger a_{j+1} + \text{h.c.}) + \frac{U}{2} n_j(n_j - 1) + \frac{K}{2} n_j j^2 \right]$$

Start the system in the state $|\psi(t=0)\rangle = |\cdots, 1, 0, 1, 0, 1, \cdots\rangle$ and then study the evolution by the Hamiltonian



Measured: no of bosons on odd lattices. Solid curves are from DMRG results.