



# Some insights into the magnetic “QCD” phase diagram from the Sakai-Sugimoto model

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# Overview

- 1 Motivation
- 2 (Magnetic) holographic setup
- 3 The  $\rho$  meson mass in a magnetic field
- 4 Chiral transition in a magnetic field



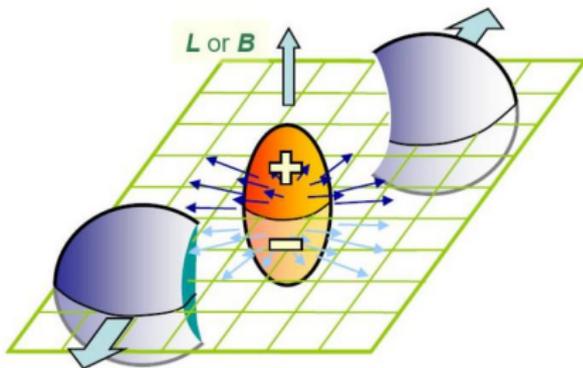
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## Why study strong magnetic fields?

- experimental relevance: appearance in QGP after a heavy ion collision (order  $eB \sim 1 - 15m_\pi^2$ ) (work of Skokov, Tuchin, Kharzeev, McLerran, Deng, Huang)



- lifetime<sub>constant B</sub>  $\sim 10$  fm McLerran, Skokov, arXiv:1305.0774; Tuchin, arXiv:1305.5806 .  
lifetime<sub>QGP</sub>  $\sim 1 - 10$  fm  $\rightarrow$  Incentive to take  $eB$  constant (ignoring “spatial decay” as well!)
- from a holographic viewpoint: interesting for comparison with recent lattice efforts



## Why study strong magnetic fields?

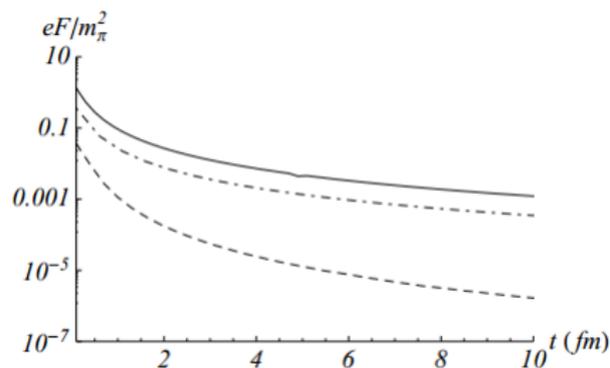
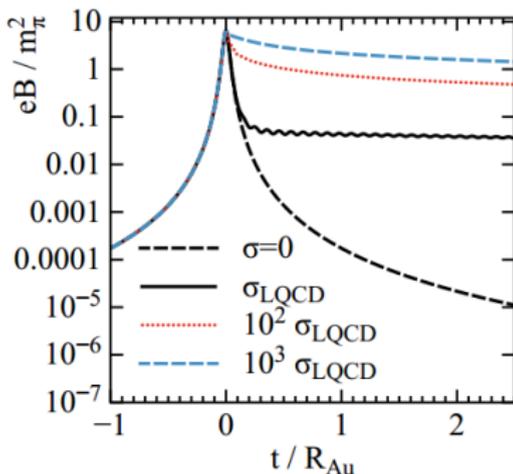


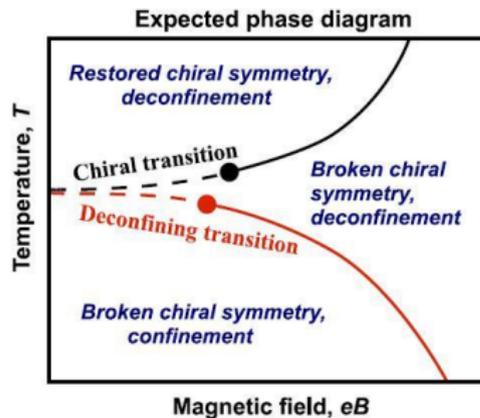
Figure : Tuchin, arXiv:1305.5806

Figure : McLerran, Skokov,  
arXiv:1305.0774

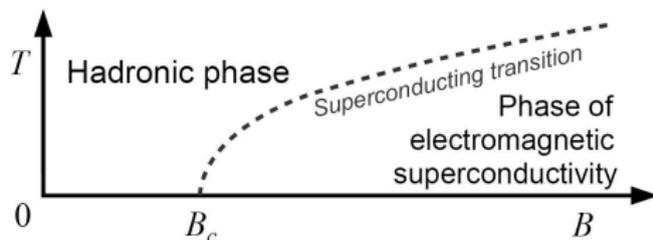


## Studied effects

- split between  $T_c(eB)$  and  $T_\chi(eB)$ ?



- $\rho$  meson condensation?





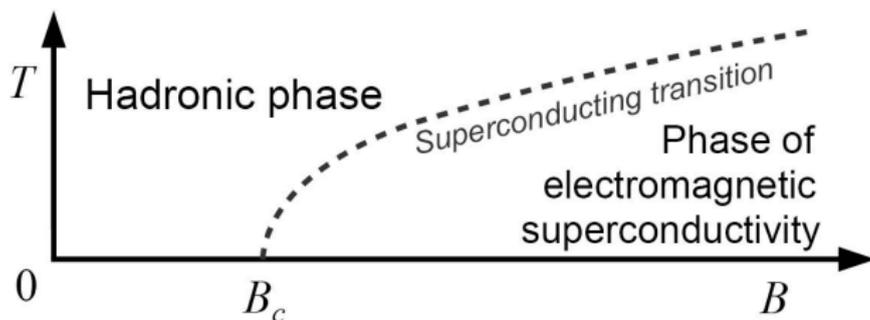
## $\rho$ meson condensation

Studied effect:  $\rho$  meson condensation in vacuum ( $T = 0$ ) (see: Chernodub, Van

Doorselaere, Verschelde; PRD85 (2012) 045002; Chernodub, PRL106 (2011) 142003, first suggestion made in Schramm,

Muller, Schramm, MPLA7 (1992) 973, inspiration from Ambjorn, Olesen on  $W$ -condensation (80ies) )

QCD vacuum instable towards forming a superconducting state of condensed charged  $\rho$  mesons at critical magnetic field  $eB_c$



Small note: academic exercise ("hubris") since by the time  $\rho$  enters,  $B$  might have already (long) left...



## $\rho$ meson condensation: Landau levels

The energy levels  $\varepsilon$  of a free relativistic spin- $s$  particle moving in a background of the external magnetic field  $\vec{B} = B\vec{e}_z$  are the Landau levels

### Landau levels

$$\varepsilon_{n,s_z}^2(p_z) = p_z^2 + m^2 + (2n - 2s_z + 1)|eB|.$$

Appropriate polarization combinations (spin  $s_z = 1$  parallel to  $\vec{B}$ ) can condense, since in the lowest energy state ( $n = 0, p_z = 0$ ):

$$M_\rho^2(eB) = m_\rho^2 - eB,$$

→ tachyonic if the magnetic field is strong enough. **Important:**  
**gyromagnetic ratio = 2!**

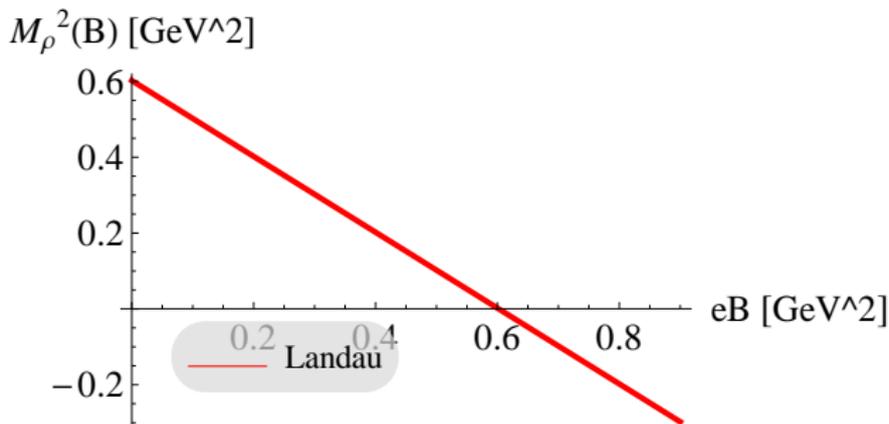


## $\rho$ meson condensation: Landau levels

$$M_\rho^2(eB) = m_\rho^2 - eB,$$

$\implies$  The fields  $\rho$  and  $\rho^\dagger$  should condense at the critical magnetic field

$$eB_c = m_\rho^2.$$





## $\rho$ meson condensation

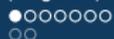
- phenomenological models:  $eB_c = m_\rho^2 = 0.6 \text{ GeV}^2$  (effective DSGS  $\rho$ -model, Chernodub, PRD82 (2010) 085011 ),  $eB_c \approx 1 \text{ GeV}^2$  (NJL) Chernodub, PRL106 (2011) 142003
- lattice simulation:  $eB_c \approx 0.9 \text{ GeV}^2$  Braguta et al, PLB718 (2012) 667
- $\rightsquigarrow$  holographic approach:
  - can the  $\rho$  meson condensation be modeled?
  - can this approach deliver new insights? e.g. taking into account strong magnetic effects on constituents ( $\rightarrow$   $\rho$ -substructure), effect on  $eB_c$

Callebaut, Dudal, Verschelde, JHEP 1303 (2013) 033; work in progress



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# Holographic QCD

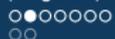
- **What is holographic QCD?**

“QCD”  $\stackrel{dual}{\equiv}$  (super)gravitation in a higher-dimensional background:  
 $4D$  “QCD” “lives” on the boundary of a  $5D$  space where the  
 (super)gravitation theory is defined

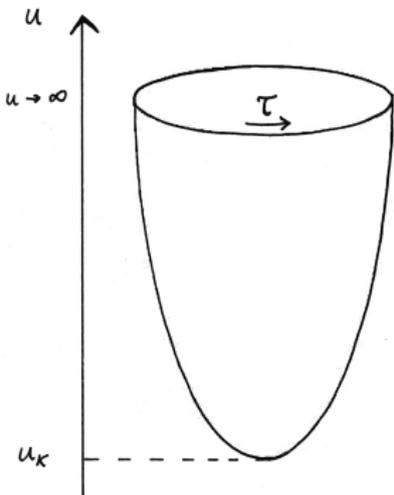
- **Origin of the QCD/gravitation duality idea?**

AdS/CFT-correspondence (Maldacena 1997):

supergravitation in  $AdS_5$  space  $\stackrel{dual}{\equiv}$  conformal  $\mathcal{N}=4$  SYM theory



# The D4-brane background

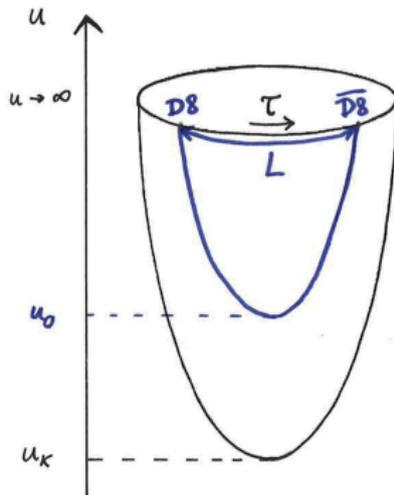


$$ds^2 = \left(\frac{u}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(u) d\tau^2) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \quad F_4 = \frac{N_c}{V_4} \varepsilon_4, \quad f(u) = 1 - \frac{u_K^3}{u^3},$$



# The Sakai-Sugimoto model



$$ds^2 = \left(\frac{u}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(u) dt^2) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \quad F_4 = \frac{N_c}{V_4} \varepsilon_4, \quad f(u) = 1 - \frac{u_K^3}{u^3},$$



## The Sakai-Sugimoto model

- Flavour  $\rightarrow N_f$  pairs of D8- $\overline{\text{D8}}$  flavour branes are added to the D4-brane background. Karch, Katz, JHEP 0206 (2002) 043
- Probe approximation  $N_f \ll N_c$ : backreaction of flavour branes on background is ignored  $\sim$  quenched “QCD”.
- Stack of  $N_f$  coinciding pairs of D8- $\overline{\text{D8}}$  flavour branes  $\rightarrow U(N_f)_L \times U(N_f)_R$  theory, to be interpreted as the chiral symmetry in QCD
- background geometry (U-shape) enforces “ $L = R$ ” (joining of flavour branes):  $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ .

Sakai, Sugimoto, PTP113 (2005) 843; PTP114 (2005) 1083



## The flavour gauge field

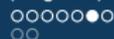
The  $U(N_f)$  **gauge field**  $A_\mu(x^\mu, u)$  that lives on the flavour branes describes a **tower of vector mesons**  $v_{\mu,n}(x^\mu)$  in the dual QCD-like theory:

### $U(N_f)$ gauge field

$$A_\mu(x^\mu, u) = \sum_{n \geq 1} v_{\mu,n}(x^\mu) \psi_n(u)$$

with  $v_{\mu,n}(x^\mu)$  a tower of vector mesons with masses  $m_n$ , and  $\{\psi_n(u)\}_{n \geq 1}$  a complete set of functions of  $u$ , satisfying the **eigenvalue equation**

$$u^{1/2} \gamma_B^{-1/2}(u) \partial_u \left[ u^{5/2} \gamma_B^{-1/2}(u) \partial_u \psi_n(u) \right] = -R^3 m_n^2 \psi_n(u),$$



## Why use the Sakai-Sugimoto model

- the way it works:

dynamics of the flavour D8/ $\overline{\text{D8}}$ -branes: 5D YM theory  $S_{DBI}[A_\mu] = \dots$ ,

$$A_\mu(x^\mu, u) = \sum_{n \geq 1} v_{\mu,n}(x^\mu) \psi_n(u)$$



integrate out the extra radial dimension  $u$

effective 4D meson theory for  $v_\mu^n(x^\mu)$

- ideal holographic QCD model to study low-energy QCD
  - confinement and chiral symmetry breaking
  - effective low-energy QCD models drop out: Skyrme ( $\pi$ , also: baryons as skyrmions), HLS ( $\pi, \rho$  coupling), VMD



## Approximations of the model

Duality is valid in the limit  $N_c \rightarrow \infty$  and large 't Hooft coupling  $\lambda = g_{YM}^2 N_c \gg 1$ , and at low energies (where redundant massive d.o.f. decouple).

Approximations (inherent to the model):

- quenched approximation ( $N_f \ll N_c$ )
- chiral limit ( $m_\pi = 0$ , bare quark masses zero)

Choices of parameters:

- $N_c = 3$
- $N_f = 2$  to model charged mesons



## How to turn on the magnetic field

A non-zero value of the flavour gauge field  $A_m(x^\mu, z)$  on the boundary,

$$A_m(x^\mu, u \rightarrow \infty) = \bar{A}_\mu,$$

corresponds to an external gauge field in the boundary field theory that couples to the quarks

$$\bar{\Psi} i \gamma_\mu D_\mu \Psi \quad \text{with} \quad D_\mu = \partial_\mu + \bar{A}_\mu.$$

To apply an external electromagnetic field  $A_\mu^{em}$ , put

$$A_\mu(u \rightarrow +\infty) = -iQ_{em} A_\mu^{em} = \bar{A}_\mu$$



## How to turn on the magnetic field

To apply a magnetic field along the  $x_3$ -axis,

$$A_2^{em} = x_1 B,$$

in the  $N_f = 2$  case,

$$Q_{em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1}{6} \mathbf{1}_2 + \frac{1}{2} \sigma_3,$$

we set

$$\bar{A}_\mu = -ieQ_{em}A_\mu^{em} = -ieQ_{em}x_1 B\delta_{\mu 2}$$



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# Plan

DBI action:

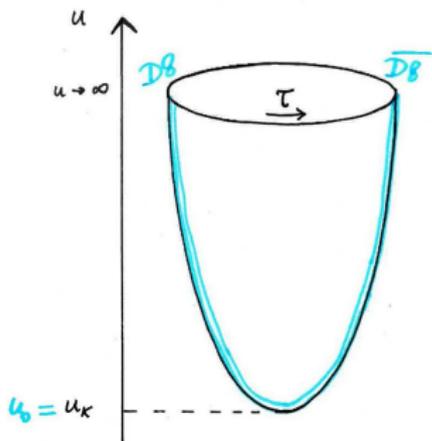
$$S_{DBI} = -T_8 \int d^4x \, 2 \int_{u_0}^{\infty} du \int \epsilon_4 e^{-\phi} \text{STr} \sqrt{-\det[g_{mn}^{D8} + (2\pi\alpha') iF_{mn}]},$$

with

- $\text{STr}(F_1 \cdots F_n) = \frac{1}{n!} \text{Tr}(F_1 \cdots F_n + \text{all permutations})$  the symmetrized trace,
- $g_{mn}^{D8} = g_{mn} + g_{\tau\tau} (D_m \tau)^2$  the induced metric on the D8-branes (with covariant derivative  $D_m \tau = \partial_m \tau + [A_m, \tau]$ )
- $\tau$  = the brane embedding (+ fluctuations)



## Simplest embedding to start: $u_0 = u_K$



- 1 Embedding trivial:  $\partial_u \bar{\tau} = 0$  for all values of the magnetic field
- 2 Determine EOM for  $\rho_\mu$ :
  - STr reduces to regular Tr (because of coincident branes)
  - $\tilde{A}_m$  and  $\tilde{\tau}$  automatically decouple
  - $\tilde{A}_\mu = \rho_\mu(x)\psi(u)$  (retain only lowest meson of the tower, most likely to condense)

$$\mathcal{L}_{5D} = \int d^4x \int du \left\{ -\frac{1}{4} f_1 (F_{\mu\nu}^a)^2 - \frac{1}{2} f_2 (F_{\mu u}^a)^2 - \frac{1}{2} f_3 \sum_{\mu, \nu=1}^2 \bar{F}_{\mu\nu}^3 \epsilon_{3ab} \tilde{A}_\mu^a \tilde{A}_\nu^b \right\}$$

EOM for  $\rho$  for  $u_0 = u_K$ 

$$\mathcal{L}_{5D} = \int d^4x \int du \left\{ -\frac{1}{4} f_1 \underbrace{(\mathcal{F}_{\mu\nu}^a)^2}_{(\mathcal{F}_{\mu\nu}^a)^2 \Psi^2} - \frac{1}{2} f_2 \underbrace{(F_{\mu\nu}^a)^2}_{(\rho_\mu^a)^2 (\partial_u \Psi)^2} - \frac{1}{2} f_3 \sum_{\mu, \nu=1}^2 \bar{F}_{\mu\nu}^3 \epsilon_{3ab} \underbrace{\tilde{A}_\mu^a \tilde{A}_\nu^b}_{\rho_\mu^a \rho_\nu^b \Psi^2} \right\}$$

demand  $\int du f_1 \Psi^2 = 1$  and  $\int du f_2 (\partial_u \Psi)^2 = m_\rho^2$ , then  $\int du f_3 \Psi^2 = k$

$$\Rightarrow \mathcal{L}_{4D} = \int d^4x \left\{ -\frac{1}{4} (\mathcal{F}_{\mu\nu}^a)^2 - \frac{1}{2} m_\rho^2 (\rho_\mu^a)^2 - \frac{1}{2} k \sum_{\mu, \nu=1}^2 \bar{F}_{\mu\nu}^3 \epsilon_{3ab} \rho_\mu^a \rho_\nu^b \right\}$$

(with  $\mathcal{F}_{\mu\nu}^a = D_\mu \rho_\nu^a - D_\nu \rho_\mu^a$ )

standard 4D Lagrangian for a vector field in an external EM field



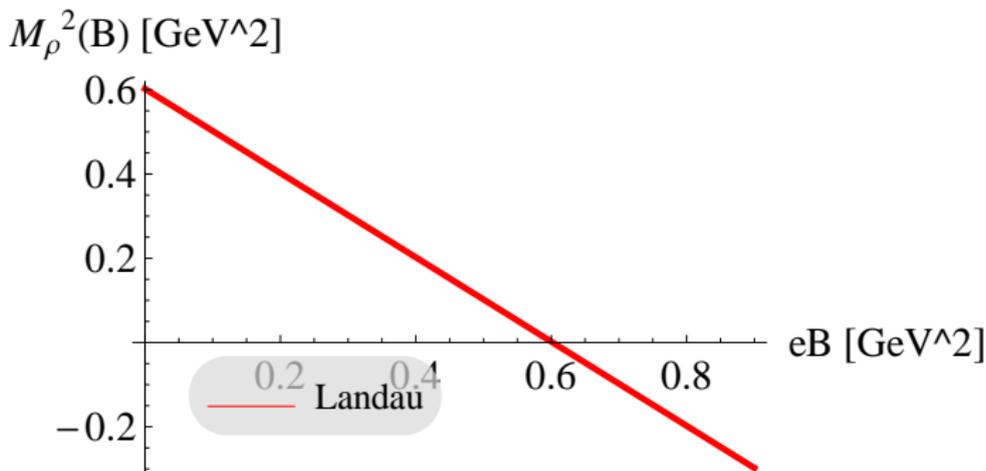
## Landau levels for Sakai-Sugimoto $u_0 = u_K$

Standard 4D Lagrangian for a vector field in an external EM field with

$$k = 1 (\Leftarrow f_3 = f_1)$$

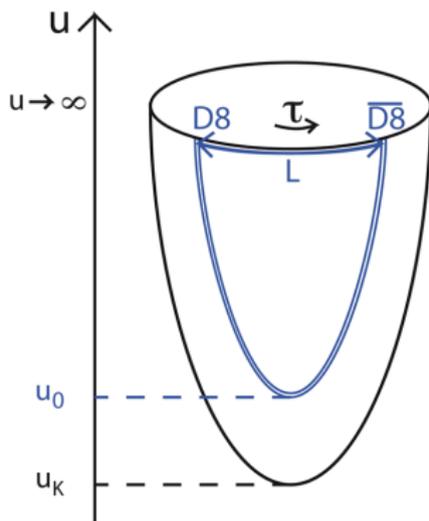
$\rightsquigarrow$  Landau levels and

$$M_\rho^2(eB) = m_\rho^2 - eB$$





## General embedding $u_0 > u_K$



$u_0 > u_K$  to model non-zero constituent quark mass which is related to the distance between  $u_0$  and  $u_K$ .

Aharony, Sonnenschein, Yankielowicz, Ann.Phys.322 (2007) 1420



## Numerical fixing of holographic parameters

There are three unknown free parameters ( $u_K$ ,  $u_0$  and  $\kappa(\sim \lambda N_c)$ ). In order to get results in physical units, we fix the free parameters by matching to

- the constituent quark mass  $m_q = 0.310$  GeV,
- the pion decay constant  $f_\pi = 0.093$  GeV and
- the  $\rho$  meson mass in absence of magnetic field  $m_\rho = 0.776$  GeV.

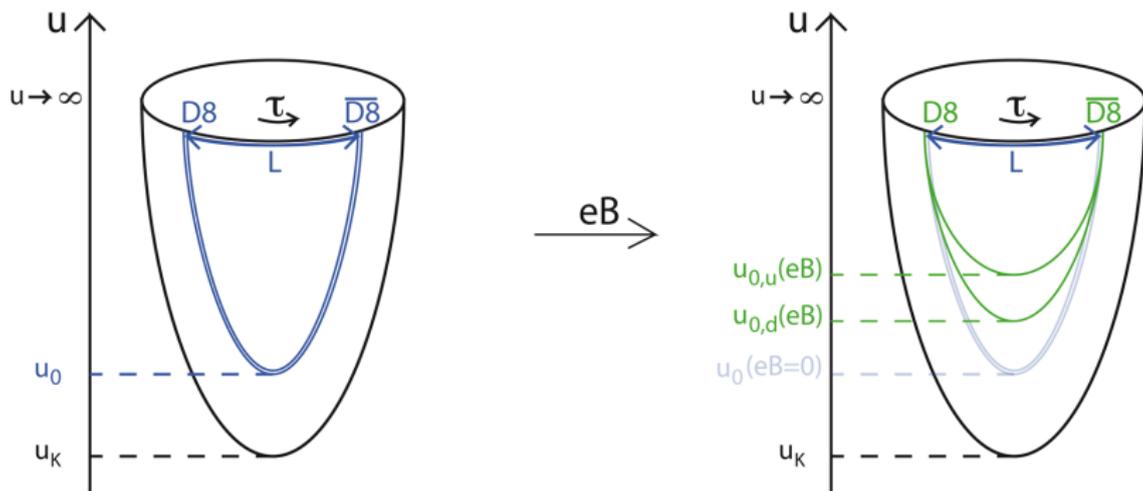
Results:

$$u_K = 1.39 \text{ GeV}^{-1}, \quad u_0 = 1.92 \text{ GeV}^{-1} \quad \text{and} \quad \kappa = 0.00678$$

- cross-check: “QCD” string tension  $\sigma \approx 0.18 \text{ GeV}^2$  (= standard lattice estimate)



## $eB$ -dependent embedding for $u_0 > u_K$



Keep  $L$  fixed:  $u_0(eB)$  rises with  $eB$ . This models **magnetic catalysis of chiral symmetry breaking** Johnson, Kundu JHEP0812 (2008) 053; in general: Miransky et al.

Non-Abelian:  $u_{0,u}(eB) > u_{0,d}(eB)$ !  $U(2) \rightarrow U(1)_u \times U(1)_d$



## $eB$ -dependent embedding for $u_0 > u_K$

Change in embedding models:

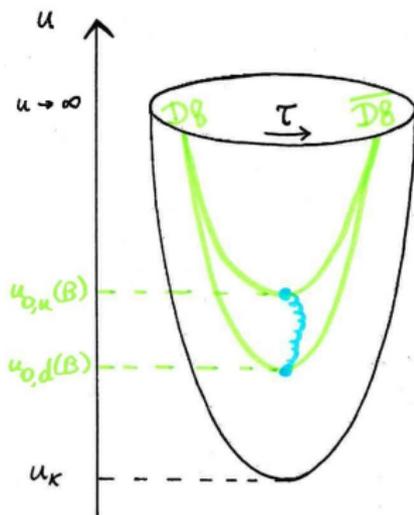
- chiral magnetic catalysis  $\Rightarrow m_u(eB)$  and  $m_d(eB) \nearrow$
- $eB$  explicitly breaks global  $U(2) \rightarrow U(1)_u \times U(1)_d$

Effect on  $\rho$  mass?

- expect  $m_\rho(eB) \nearrow$  as constituents get heavier
- split between branes generates other mass mechanism: 5D gauge field gains mass through **holographic Higgs mechanism**



## $eB$ -induced Higgs mechanism



The string associated with a charged  $\rho$  meson ( $\bar{u}d, \bar{d}u$ ) stretches between the now separated up- and down brane  $\Rightarrow$  because a string has tension it contributes to the mass.



## EOM for $\rho$ for $u_0 > u_K$ ?

Non-trivial embedding

$$\bar{\tau}(u) = \begin{pmatrix} \bar{\tau}_u(u)\theta(u - u_{0,u}) & 0 \\ 0 & \bar{\tau}_d(u)\theta(u - u_{0,d}) \end{pmatrix} \neq \mathbf{1},$$

describing the splitting of the branes, **utterly complicates** the analysis  
(**but necessary for a realistic modeling!**).

$$\begin{aligned} \mathcal{L}_{5D} = \text{STr} \{ & .. ([\tilde{A}_m, \bar{\tau}] + D_m \tilde{\tau})^2 + .. (F_{\mu\nu})^2 + .. (F_{\mu u})^2 + .. \bar{F}_{\mu\nu} [\tilde{A}_\mu, \tilde{A}_\nu] \\ & + .. (\partial_u \bar{\tau}) \bar{F} ([\tilde{A}, \bar{\tau}] + D \tilde{\tau}) F \} \end{aligned}$$

with all the .. different functions  $\mathcal{H}(\partial_u \bar{\tau}, \bar{F}; u)$  of the background fields  $\partial_u \bar{\tau}, \bar{F}$ .



## STr-prescription

Myers, JHEP 9912 (1999) 022; Deneff, Sevrin, Troost, Nucl.Phys. B581 (2000) 135

- STr = symmetric average over all orderings of  $F_{ab}$ ,  $D_a\phi^i$ ,  $[\phi^i, \phi^j]$  and the individual non-Abelian scalars  $\phi^k$  appearing in the non-Abelian Taylor expansions of the background fields.

$$STr(\mathcal{H}(\bar{F})\tilde{F}^2) = -\frac{1}{2} \sum_{a=1}^2 \tilde{F}_a^2 I(\mathcal{H}) + \sum_{a=0,3} \dots$$

with

$$I(\mathcal{H}) = \frac{\int_0^1 d\alpha \mathcal{H}(F_0 + \alpha F_3) + \int_0^1 d\alpha \mathcal{H}(F_0 - \alpha F_3)}{2}$$

- The integral functions  $I(\mathcal{H})$  are complicated functions of  $eB$  and  $u$ , even discontinuous in  $u$  (at  $u = u_{0,u}$ ).



## STr-prescription

After many pages of  
computations (analytical +  
numerical)



EOM for  $\rho$  for  $u_0 > u_K$

$$\mathcal{L}_{5D} = \int d^4x \int du \left\{ -\frac{1}{4} f_1(\mathbf{eB}) \underbrace{(F_{\mu\nu}^a)^2}_{(\mathcal{F}_{\mu\nu}^a)^2 \Psi^2} - \frac{1}{2} f_2(\mathbf{eB}) \underbrace{(F_{\mu u}^a)^2}_{(\rho_\mu^a)^2 (\partial_u \Psi)^2} \right. \\ \left. - \frac{1}{2} f_3(\mathbf{eB}) \sum_{\mu, \nu=1}^2 \bar{F}_{\mu\nu}^3 \epsilon_{3ab} \underbrace{\tilde{A}_\mu^a \tilde{A}_\nu^b}_{\rho_\mu^a \rho_\nu^b \Psi^2} - \frac{1}{2} f_4(\mathbf{eB}) \underbrace{(\tilde{\mathcal{T}}_\mu^a)^2}_{(\bar{\tau}^3)^2 \Psi^2} \right\}$$

demand  $\int du f_1(\mathbf{eB}) \Psi^2 = 1$  and  $\int du f_2(\mathbf{eB}) (\partial_u \Psi)^2 + f_4(\mathbf{eB}) (\bar{\tau}^3)^2 \Psi^2 = m_\rho^2(\mathbf{eB})$ , then  $\int du f_3(\mathbf{eB}) \Psi^2 = k(\mathbf{eB}) \neq 1$  ( $\Leftarrow f_3(\mathbf{eB}) \neq f_1(\mathbf{eB})$ )

$$\Rightarrow \mathcal{L}_{4D} = \int d^4x \left\{ -\frac{1}{4} (\mathcal{F}_{\mu\nu}^a)^2 - \frac{1}{2} m_\rho^2(\mathbf{eB}) (\rho_\mu^a)^2 - \frac{1}{2} k(\mathbf{eB}) \sum_{\mu, \nu=1}^2 \bar{F}_{\mu\nu}^3 \epsilon_{3ab} \rho_\mu^a \rho_\nu^b \right\}$$

modified 4D Lagrangian for a vector field in an external EM field, with  $\mathbf{eB}$ -dependent gyromagnetic coupling!



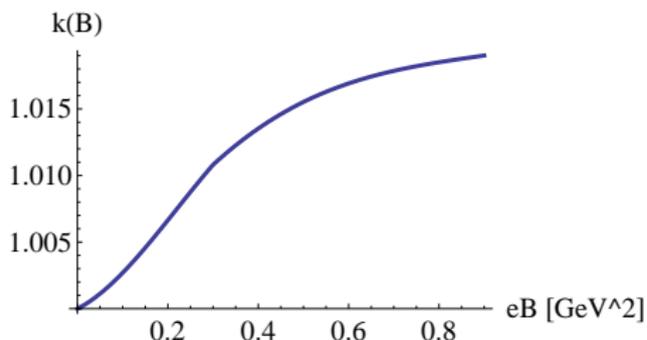
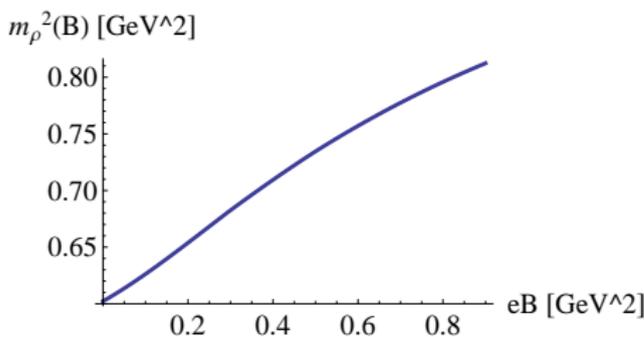
## Solve the eigenvalue problem

The normalization condition and mass condition on the  $\psi$  combine to the eigenvalue equation

$$f_1^{-1} \partial_u (f_2 \partial_u \psi) - f_1^{-1} f_4 (\bar{\tau}_3)^2 \psi = -m_\rho^2 \psi$$

with b.c.  $\psi(x = \pm\pi/2) = 0, \psi'(x = 0) = 0$

which we solve with a numerical shooting method to obtain  $m_\rho^2(eB)$ .



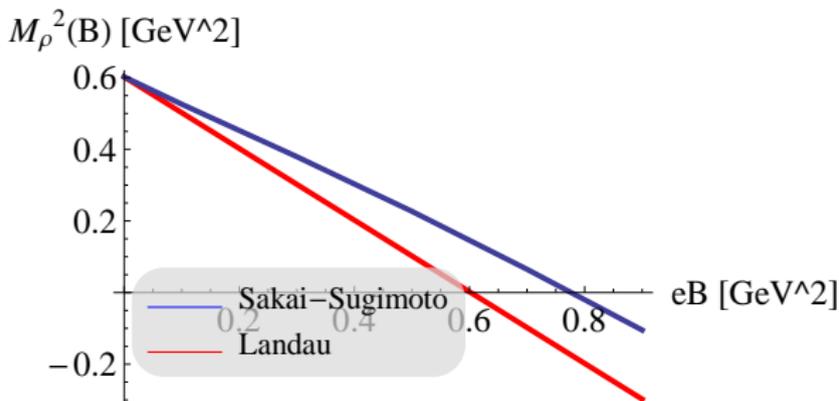


## Landau vs Sakai-Sugimoto $u_0 > u_K$

Modified 4D Lagrangian for a vector field in an external EM field with  $k(eB) \neq 1 (\Leftrightarrow f_3(eB) \neq f_1(eB))$

$\rightsquigarrow$  modified Landau levels and, with  $\xi = \frac{eB}{m_\rho^2(eB)}$ , Tsai, Yildiz PRD4 (1971) 3643; Obukhov et al, Theor.Math.Phys. 55 (1983) 536

$$M_\rho^2(eB) = m_\rho^2(eB) - eB + (1 - k(eB))m_\rho^2(eB) \left( \frac{\xi^2}{2} + \xi \sqrt{1 - \xi + \frac{\xi^2}{4}} \right)$$





## Summary $\rho$ meson

Studied effect: possibility for  $\rho$  meson condensation

- phenomenological models:  $eB_c = m_\rho^2 = 0.6 \text{ GeV}^2$
- lattice simulation: slightly higher value of  $eB_c \approx 0.9 \text{ GeV}^2$
- $\rightsquigarrow$  holographic approach:
  - can the  $\rho$  meson condensation be modeled? **yes**
  - can this approach deliver new insights? e.g. taking into account constituents, effect on  $eB_c$

Up and down quark constituents of the  $\rho$  meson can be modeled as separate branes, each responding to the magnetic field by changing their embedding. This is a modeling of the chiral magnetic catalysis effect. We take this into account and find also a string effect on the mass, leading to a  $eB_c \approx 0.78 \text{ GeV}^2$



# Overview

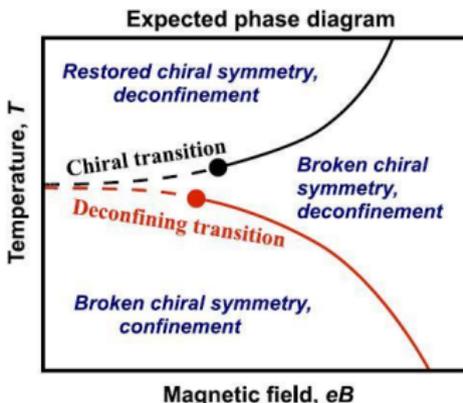
- 1 Motivation
- 2 (Magnetic) holographic setup
- 3 The  $\rho$  meson mass in a magnetic field
- 4 Chiral transition in a magnetic field**



# Chiral temperature

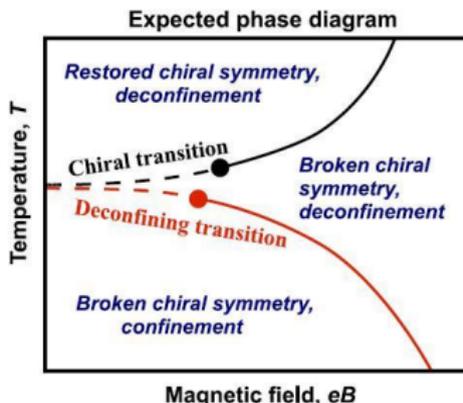
$T_\chi$  = temperature at which chiral symmetry is restored (chiral limit is understood)

Studied effect: possible split between  $T_c(eB)$  and  $T_\chi(eB)$





## Split between $T_C$ and $T_\chi$



Expected behaviour (Fig from '08):

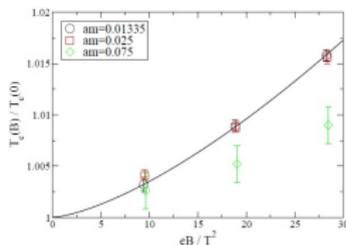
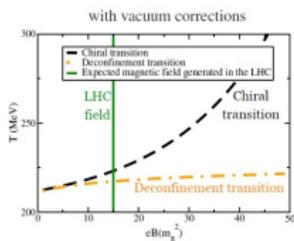
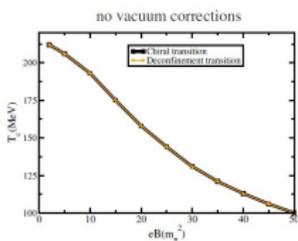
- $T_\chi(eB) \nearrow$ : “chiral magnetic catalysis” seen in chirally driven models (e.g. NJL) Miransky, Shovkovy, PRD66 (2002) 045006
- $T_C(eB) \searrow$ : quark gas thermodynamically favoured over pion gas (e.g. MIT bag) Agasian, Fedorov, PLB663 (2008) 445; Fraga, Palhares, PRD86 (2012) 016008; Fukushima,

Hidaka, PRL110 (2013) 031601

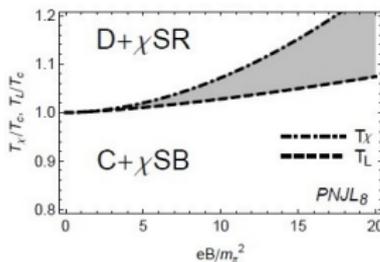
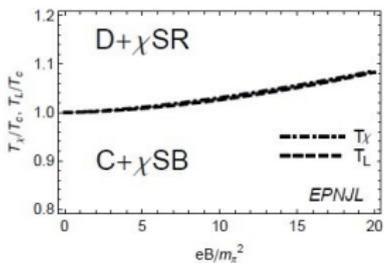


## Some results in different models

PLSM<sub>q</sub> model Mizher, Chernodub, Fraga, PRD82 (2010) 105016 Lattice D'Elia, Mukherjee, Sanfilippo, PRD82 (2010) 051501

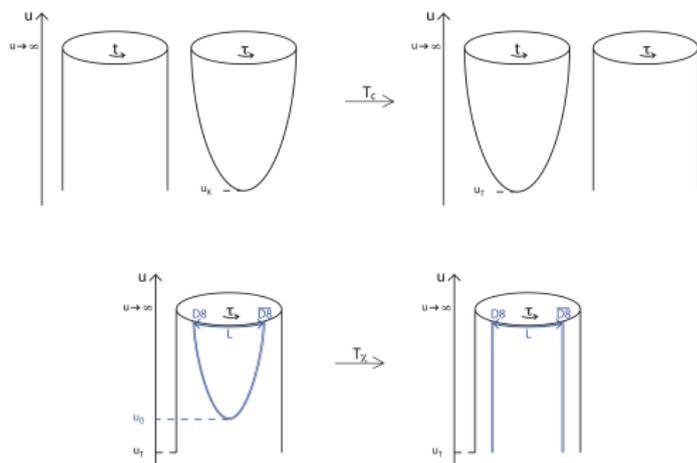


Different PNJL models Gatto, Ruggieri, PRD82 (2010) 054027; PRD83 (2011) 034016





## Sakai-Sugimoto at finite temperature



“Black D4-brane background”

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} (\hat{f}(u) dt^2 + \delta_{ij} dx^i dx^j + d\tau^2) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{\hat{f}(u)} + u^2 d\Omega_4^2\right)$$

$$\hat{f}(u) = 1 - \frac{u_T^3}{u^3}, \quad u_T \sim T^2$$



## Numerical fixing of holographic parameters

Input parameters at  $eB = 0$   $f_\pi = 0.093$  GeV and  $m_\rho = 0.776$  GeV fix all holographic parameters except “brane separation”  $L$ .

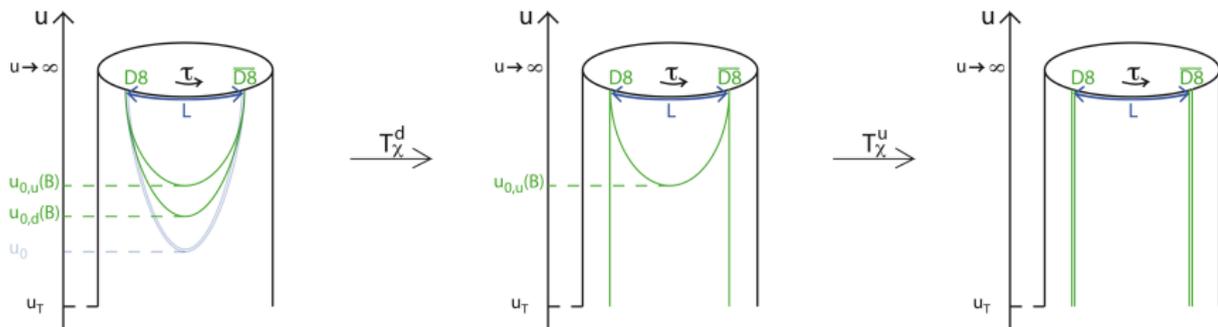
Choice of  $L$  a priori free, determines a kind of choice of holographic theory:

- $L$  small  $\sim$  NJL-type boundary field theory Antonyan, Harvey, Jensen, Kutasov, hep-th/0604017
- $L = \delta\tau/2$  maximal  $\sim$  maximal probing of the gluon background (original antipodal Sakai-Sugimoto)



## Sakai-Sugimoto at finite $T$ and $eB$

- no backreaction  $\Rightarrow T_c$  independent of  $eB$
- $eB$ -dependent embedding of flavour branes  $\Rightarrow T_\chi(eB)$ :

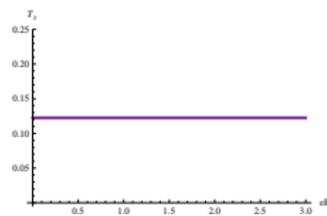
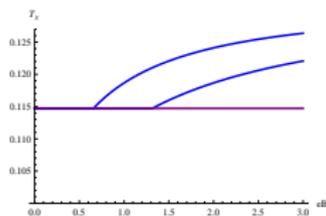
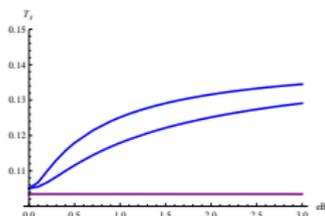


$$S_{\text{merged}} - S_{\text{separated}} = 0 \quad \Rightarrow \quad T_\chi$$



## Conclusion on $T_\chi(eB)$

The appearance of a split between  $T_\chi$  (GeV) (blue) and  $T_C$  (GeV) (purple) depends on the choice of  $L$ !



Plots for fixed  $L$  (from small to large) respectively corresponding to  $m_q(eB=0) = 0.357, 0.310$  and  $0.272$  GeV and  $T_C = 0.103, 0.115$  and  $0.123$  GeV Callebaut, Dudal, PRD87 (2013)106002

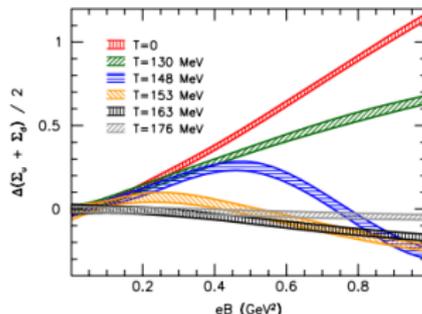
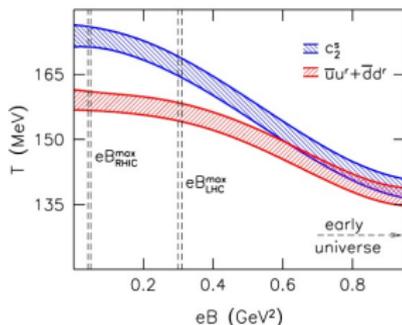
- Left: split for  $L$  small enough  $\sim$  NJL results
- Middle and right: split only at large  $eB$  or no split at all for parameter values that match best to QCD  $\sim SU(2)$ ,  $N_f = 2$  (chirally extrapolated) lattice data of Ilgenfritz et al, PRD85 (2012) 114504 (no split)



(Locally) Inverse magnetic catalysis

**BIG BUT**

Latest lattice data disagree with most previous results:  $T_\chi(eB) \searrow$



→ quenched vs. true QCD

Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg, Schafer, Szabo, JHEP 1202 (2012) 044; Phys.Rev. D86 (2012) 071502

Important task for future holographs: construct a “magnetized bulk geometry” that is sufficiently QCD-like



# Fin



# Merci!