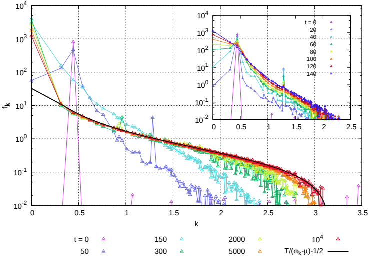
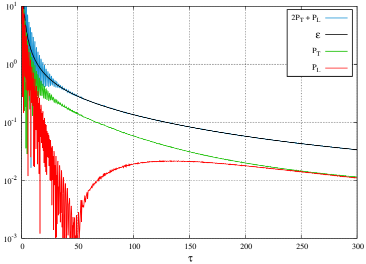
The early stages of heavy ion collisions





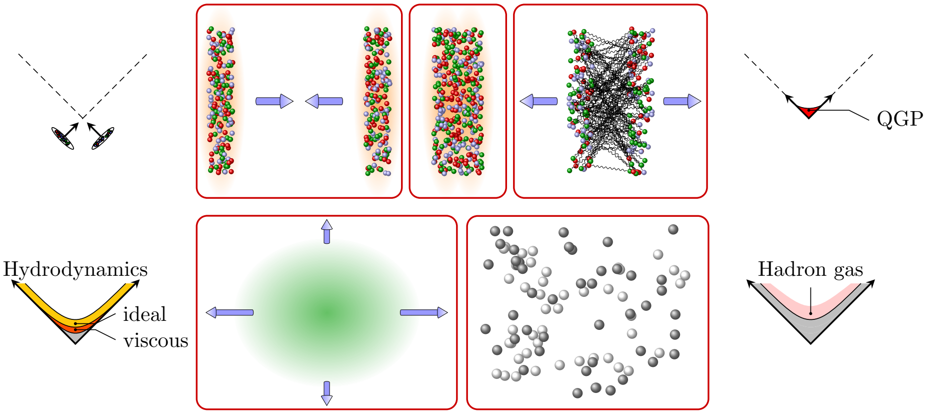
Paris, 11th June 2013

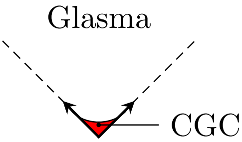
Thomas EPELBAUM

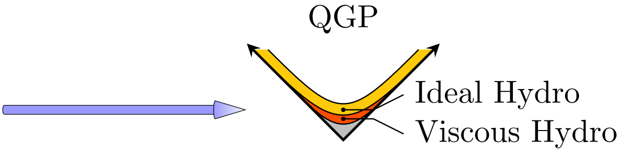
IPhT

Outline

THE GENERAL PICTURE







Out of thermal equilibrium:

No Equation of state (EOS)

Huge anisotropy

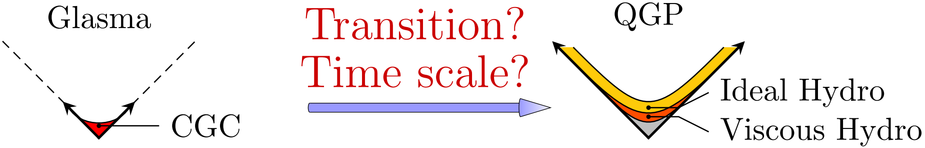
f_k far from being thermal

Close from thermal equilibrium:

EOS

Small anisotropy

f_k close from being thermal



Out of thermal equilibrium:

No Equation of state (EOS)

Huge anisotropy

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Close from thermal equilibrium:

EOS

Small anisotropy

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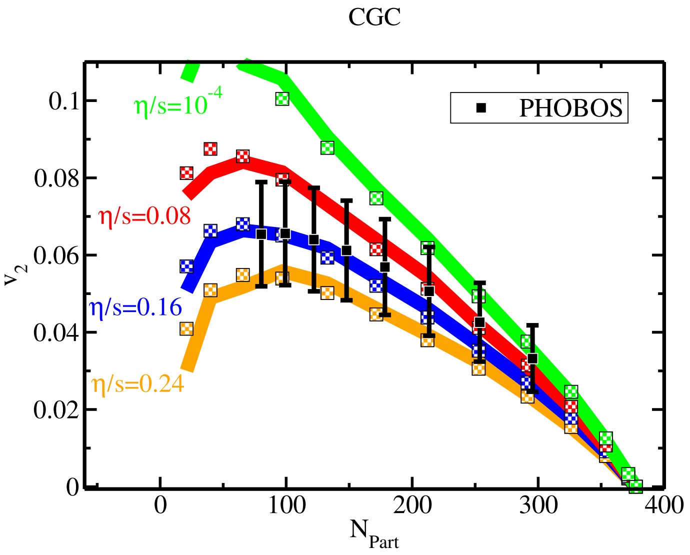
WHY DO WE TRUST HYDRODYNAMICS IN THE FIRST PLACE?

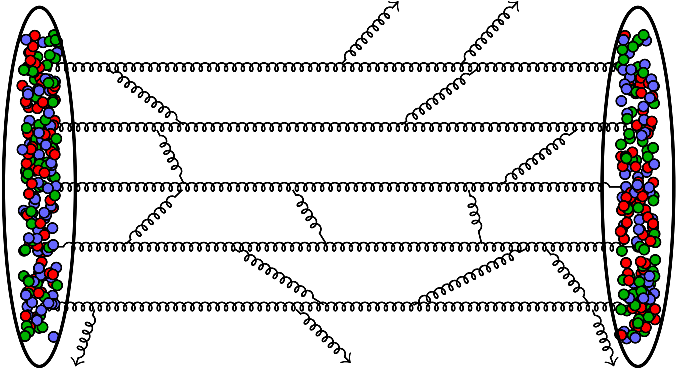


[LUZUM, ROMATSCHKE (2008)]

Dilute Regime

Dense regime:

$\alpha_s \ll 1$ **but** $f_{\text{gluon}} \sim \frac{Q_s^3}{\alpha_s}$**...**



How to study the transition then?

Theoretical framework

- Color Glass Condensate (CGC)
- JIMWLK Equation
- Resummation Scheme

Objectives:

Transitions?

- No (EOS), big anisotropy $\Rightarrow$ EOS, small anisotropy ?
- $f_{\boldsymbol{k}}$ far from Bose-Einstein $\Rightarrow f_{\boldsymbol{k}} \sim \frac{1}{e^{\beta\,\omega_{\boldsymbol{k}}}-1}$?
- Thermalization time?

Hydro prerequisites?

- Actual value of ϵ, P_T, P_L?
- Actual value of the transport coefficients? $\frac{\eta}{s}$...

2 Theoretical framework

CGC

JIMWLK

Resummation Scheme

The Color Glass Condensate (CGC) [McLerran, Venugopalan (1994)]

Fast and slow partons are not considered in the same way

The main assumptions

- Fast partons ($k > \Lambda$) $\Rightarrow$ static color sources.

$$J^\mu(x^+, x^-, \boldsymbol{x}_\perp) = \delta^{\mu +}\delta(x^-)\rho^1(\boldsymbol{x}_\perp) + \delta^{\mu -}\delta(x^+)\rho^2(\boldsymbol{x}_\perp)$$

- Small x $\Rightarrow$ Gluon saturation $\Rightarrow$ $J \propto g^{-1}$.
- Probabilistic knowledge of the ρ $\Rightarrow$ $W_\Lambda[\rho]$.
- Slow partons $\Rightarrow$ $\mathcal{A}^\mu$.
- System boost-invariant $\Rightarrow$ $\mathcal{A}^\mu$ rapidity independant.

THE COLOR GLASS CONDENSATE (CGC)
[MCLERRAN, VENUGOPALAN (1994)]

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- System boost-invariant $\Rightarrow \mathcal{A}^\mu$ rapidity independant.

Langrangian of theory reads

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu\mathcal{A}^\mu$$

The Color Glass Condensate (CGC) [McLerran, Venugopalan (1994)]

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- System boost-invariant $\Rightarrow \mathcal{A}^\mu$ rapidity independant.

Perturbative expansion of observables

$$T^{\mu\nu}[\rho^1, \rho^2] = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 ...\right]$$

2 THEORETICAL FRAMEWORK

- CGC
- JIMWLK
- Resummation Scheme

JIMWLK equation [Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997)]

Schematical vision of the degrees of freedom

Cutoff dependance

Theory Λ dependant $\rightarrow \log(\Lambda)$ appears at NLO

JIMWLK equation [Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997)]

Schematical vision of the degrees of freedom

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Theory Λ dependant $\rightarrow \log(\Lambda)$ appears at NLO

Renormalization group equation

$$\Lambda \frac{\partial}{\partial \Lambda} W_\Lambda[\rho] = \mathcal{H} W_\Lambda[\rho]$$

JIMWLK equation
[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997)]

Schematical vision of the degrees of freedom

Cutoff dependance

Theory Λ dependant $\rightarrow \log(\Lambda)$ appears at NLO

Renormalization group equation

$$\Lambda \frac{\partial}{\partial \Lambda} W_\Lambda[\rho] = \mathcal{H} W_\Lambda[\rho]$$

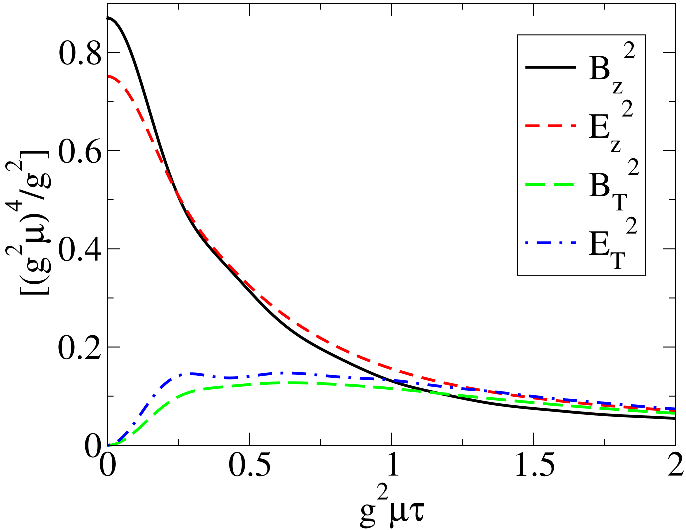
Issues:

- Very Anisotropic system at $\tau = 0^+$
- Secular divergences.

ANISOTROPY

$$\epsilon = E_T^2 + B_T^2 + E_L^2 + B_L^2$$
$$P_T = E_L^2 + B_L^2$$
$$P_L = E_T^2 + B_T^2 - E_L^2 - B_L^2$$

ANISOTROPY



[LAPPI, MCLERRAN (2006)]

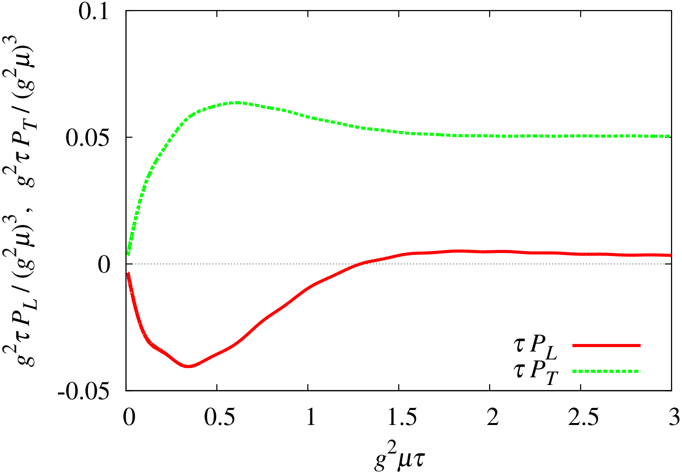
$$\epsilon = \underbrace{E_T^2}_{0} + \underbrace{B_T^2}_{0} + E_L^2 + B_L^2$$

$$P_T = E_L^2 + B_L^2$$

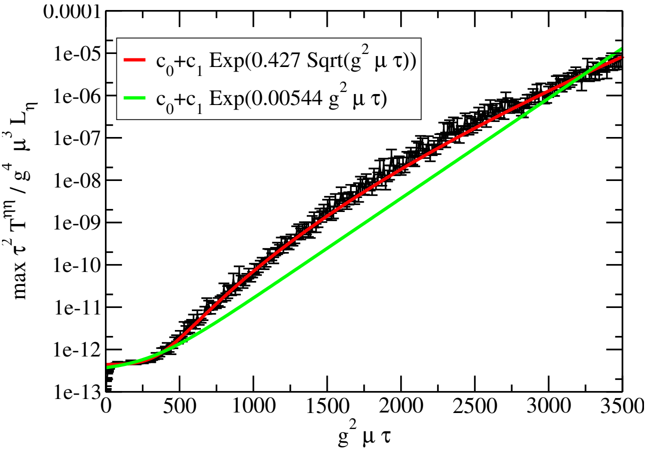
$$P_L = \underbrace{E_T^2}_{0} + \underbrace{B_T^2}_{0} - E_L^2 - B_L^2$$

Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)$!

ANISOTROPY

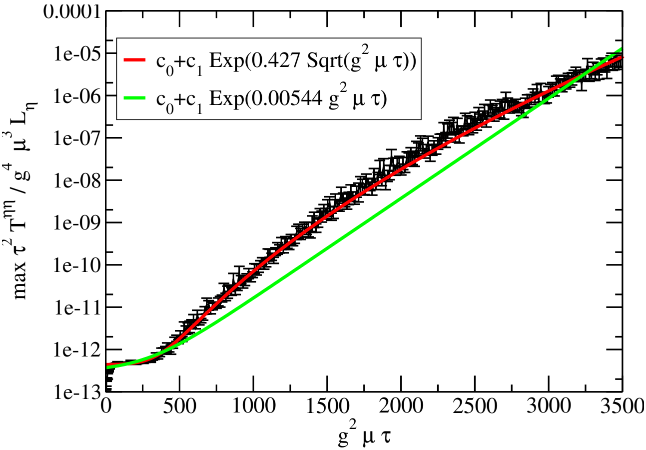


[FUKUSHIMA, GELIS (2012)]



[ROMATSCHKE, VENUGOPALAN (2006)]

Fluctuations grows like $e^{\sqrt{\mu\tau}}$!



[ROMATSCHKE, VENUGOPALAN (2006)]

when $ge^{\sqrt{\mu\tau}} \approx 1$, perturbative expansion breaks down!

❷ THEORETICAL FRAMEWORK

Resummation formula

Resummation

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}}[\varphi_0] + T^{\mu\nu}_{\text{NLO}}[\varphi_0] + \dots$$

RESUMMATION

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}}[\varphi_0] + T^{\mu\nu}_{\text{NLO}}[\varphi_0] + \ldots$$

INITIAL VALUE PROBLEM

$$\phi(t_0, \boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \text{Re} \int_{\boldsymbol{k}} c_{\boldsymbol{k}}\, a_{\boldsymbol{k}}(t, x)$$

$$\Box \phi(t, \boldsymbol{x}) + V'[\phi(t, \boldsymbol{x})] = 0$$

LAGRANGIAN OF THE THEORY

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

WHY DO WE USE THIS MODEL?

- Scale invariance in $3+1$ dimensions
- Parametric resonance
- A lot simpler!

INITIAL CONDITION OF THE EOM

$$\phi_{\mathrm{i}}(t_0, \boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \sum_{\boldsymbol{k}} \mathrm{Re}\left[c_k \, a_{\boldsymbol{k}}\right]$$

- All the Quantum information in the initial condition
- Purely classical evolution

INITIAL CONDITION OF THE EOM

$$\phi_{\mathrm{i}}(t_0, \boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \sum_{\boldsymbol{k}} \mathrm{Re}\left[c_{\boldsymbol{k}}\, e^{i\omega_k t_0}\, v_{\boldsymbol{k}}(\boldsymbol{x})\right]$$

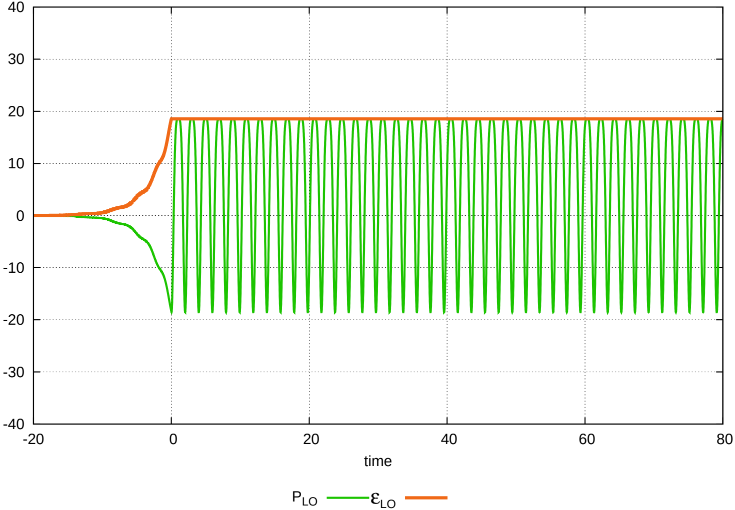
$$[-\Delta + V''(\varphi_0)]\, v_{\boldsymbol{k}}(\boldsymbol{x}) = \omega_{\boldsymbol{k}}^2 v_{\boldsymbol{k}}(\boldsymbol{x})$$

$$\langle c_{\boldsymbol{k}}\; c_{\boldsymbol{l}}^* \rangle = \delta(\boldsymbol{k} - \boldsymbol{l})$$

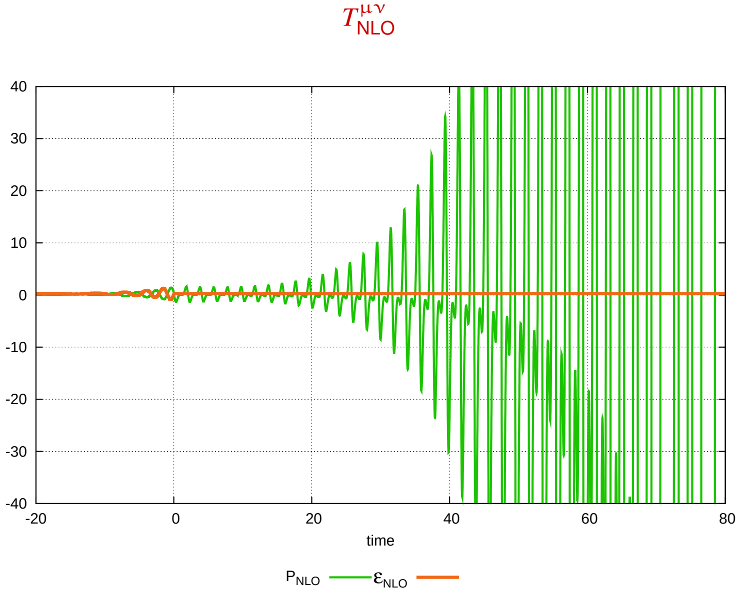
- All the Quantum information in the initial condition
- Purely classical evolution

$T^{\mu\nu}_{\rm LO}$ [Dusling, TE, Gelis, Venugopalan (2010)]

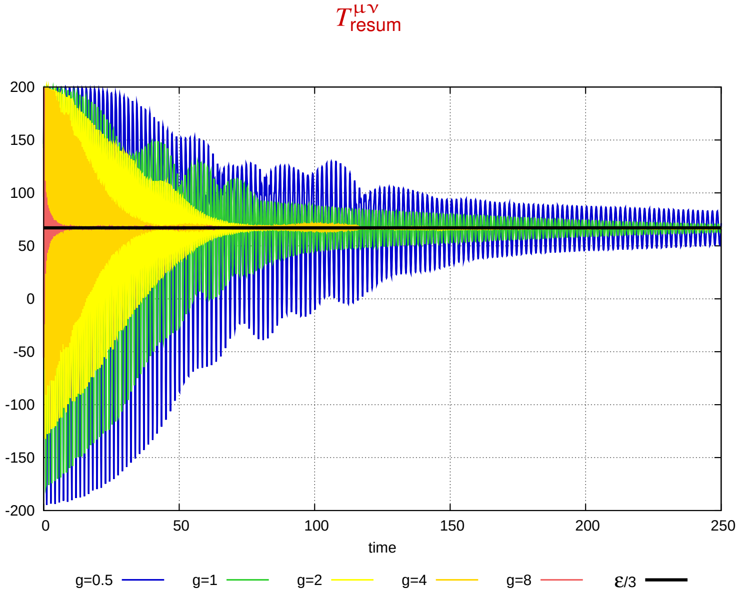
$T^{\mu\nu}_{\rm LO}$



$T^{\mu\nu}_{\text{NLO}}$: SECULAR DIVERGENCES

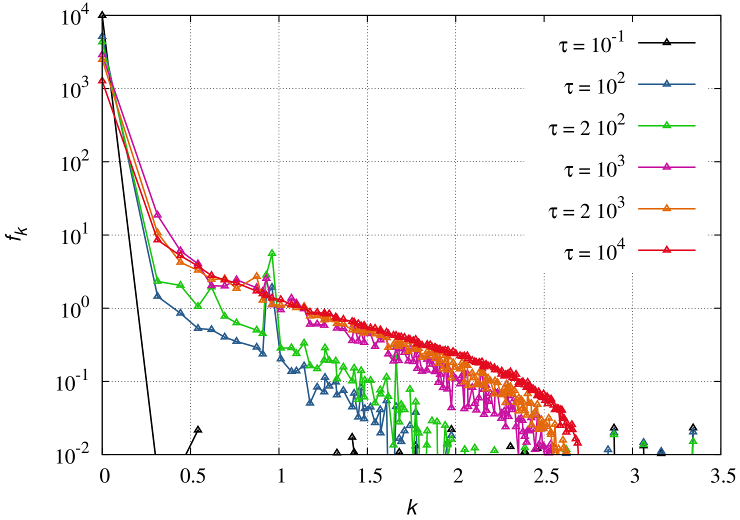


$T^{\mu\nu}_{\text{RESUM}}$: PRESSURE EQUILIBRATION

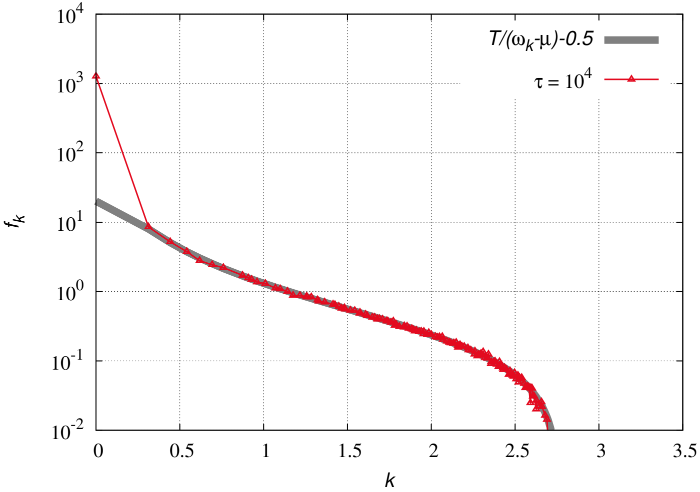


3 FIXED VOLUME

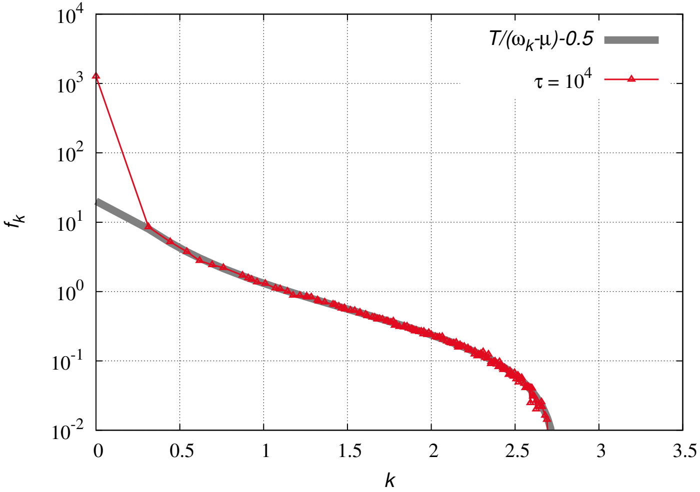
Time evolution of the occupation number [TE, Gelis (2011)]



"CLASSICAL" EQUILIBRIUM DISTRIBUTION



- [BERGES, SEXTY(2012)]
- [MICHA, TKACHEV(2004)]

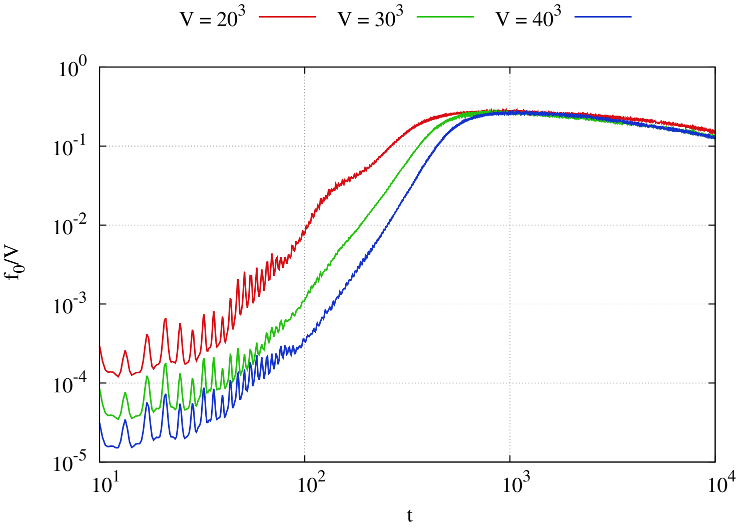


$$f_{\boldsymbol{k}} = \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2} + n_0 \delta(\boldsymbol{k})$$

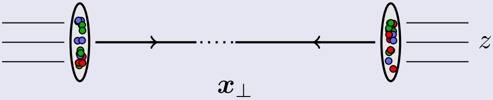
implies

$$\frac{f_0}{V} = \text{cte}$$

EVOLUTION OF THE CONDENSATE



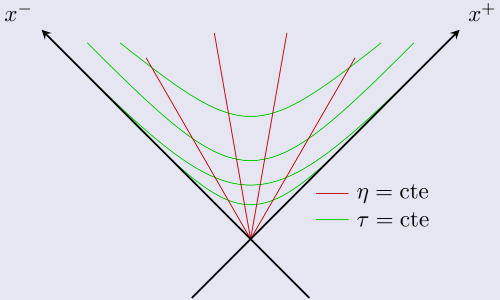
ADAPTED COORDINATE SYSTEM TO DESCRIBE A HEAVY ION COLLISION?



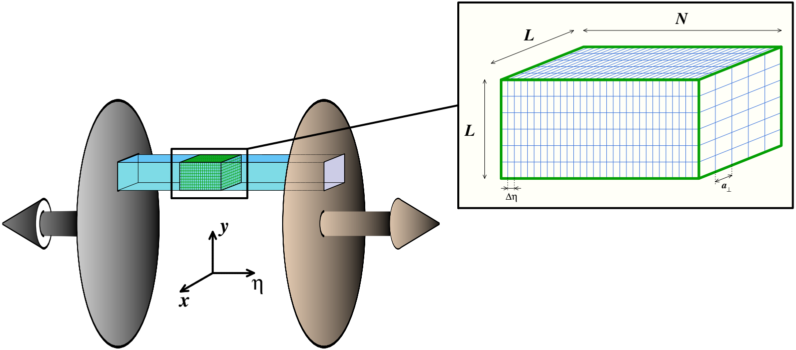
System boost invariant in z direction

Scalar field theory

Proper time/rapidity coordinate system



Scalar field theory



INITIAL CONDITION OF THE EOM

$$\phi_{\mathrm{i}}(\tau_0, \boldsymbol{x}_\perp, \eta) = \varphi_0(\boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} \mathrm{Re}\left[c_{\nu \boldsymbol{k}_\perp}\, a_{\nu \boldsymbol{k}_\perp}(\tau, \boldsymbol{x}_\perp, \eta)\right]$$

- All the Quantum information in the initial condition
- Purely classical evolution

Initial condition of the EOM

$$\phi_\mathrm{i}(\tau_0, \boldsymbol{x}_\perp, \eta) = \varphi_0(\boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} \mathrm{Re}\left[c_{\nu \boldsymbol{k}_\perp} H^{(2)}_{i\nu}(\omega_{\boldsymbol{k}_\perp} \tau_0)\, e^{i\nu\eta}\, v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp) \right]$$

$$\left[-\Delta_\perp + V''(\varphi_0) \right] v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp) = \omega^2_{\boldsymbol{k}_\perp} v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp)$$

$$\left\langle c_{\nu \boldsymbol{k}_\perp}\, c^*_{\mu \boldsymbol{l}_\perp} \right\rangle = \delta(\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)\delta(\nu - \mu)$$

- All the Quantum information in the initial condition
- Purely classical evolution

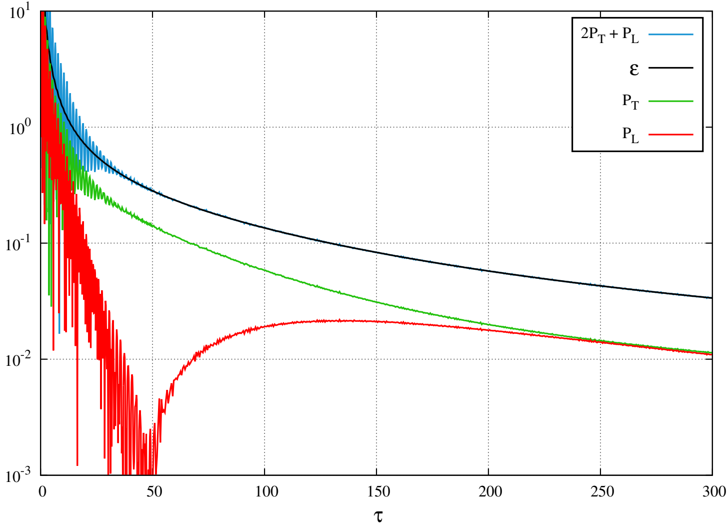
4 EXPANDING VOLUME

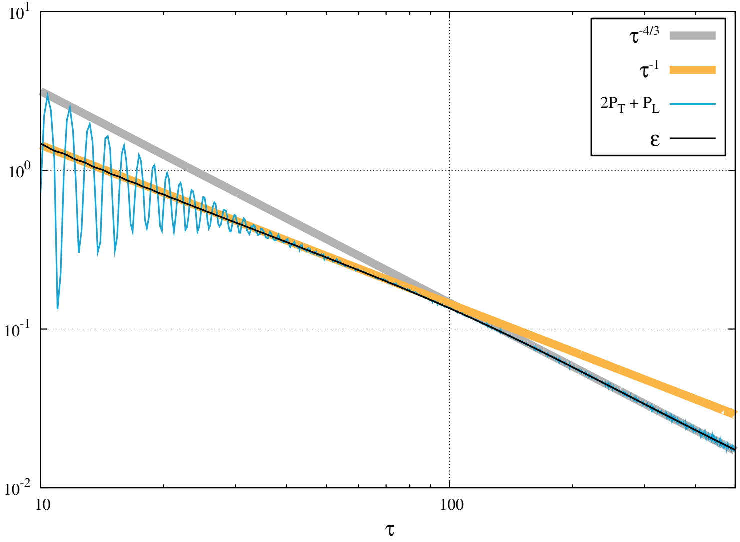
The model

Isotropization

Comparison with Hydro

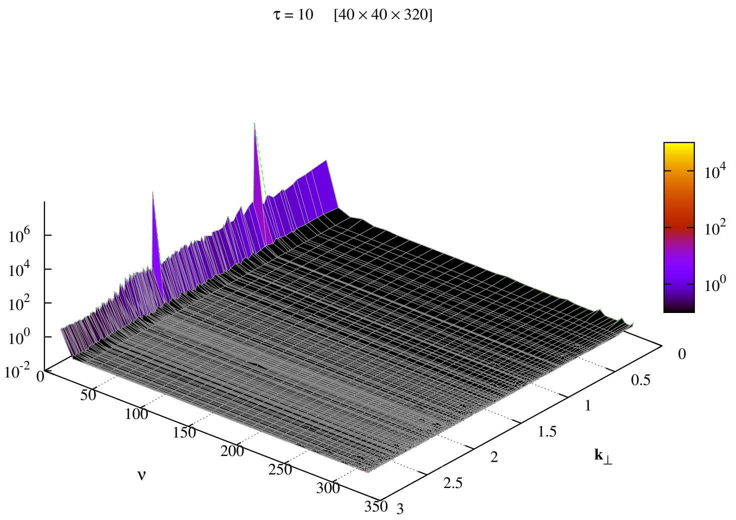
$T^{\mu\nu}_{\text{RESUM}}$ [Dusling, TE, Gelis, Venugopalan (2012]



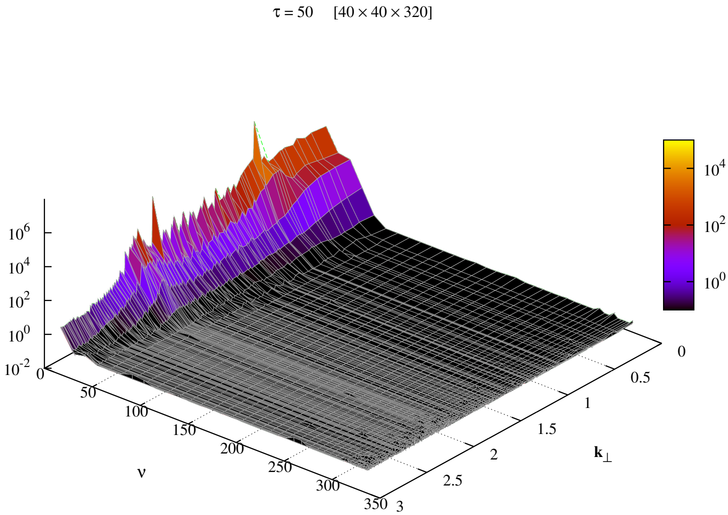


Bjorken Law: $\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$

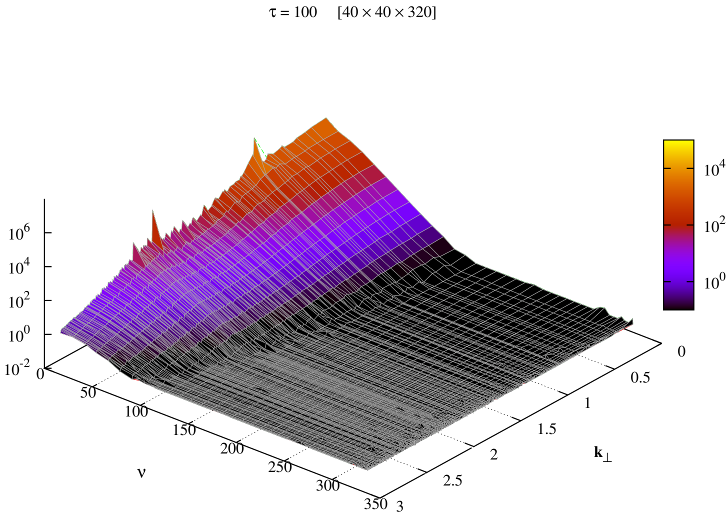
EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$



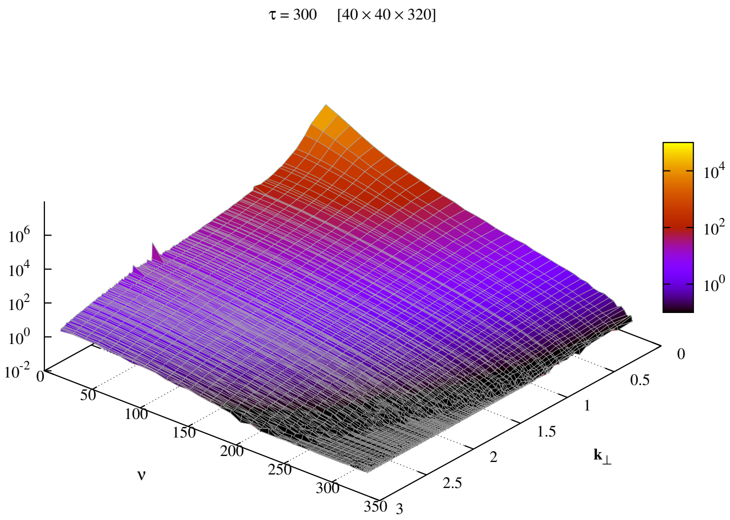
EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$



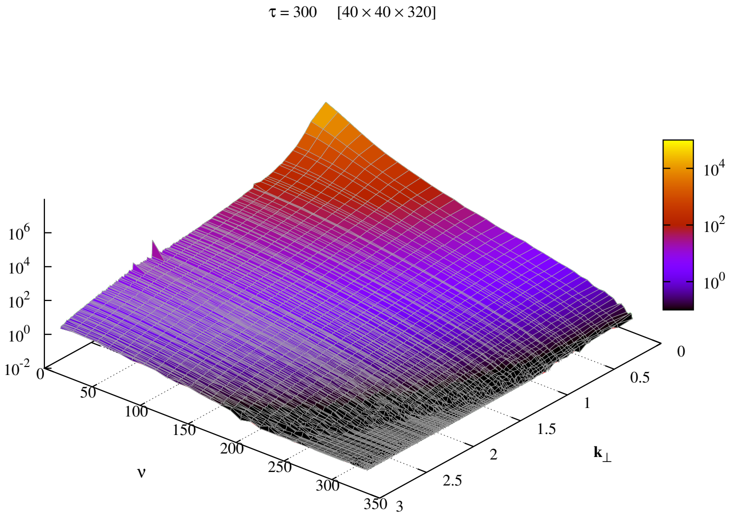
EVOLUTION OF $f_{\nu k_\perp}$



EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$



EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$

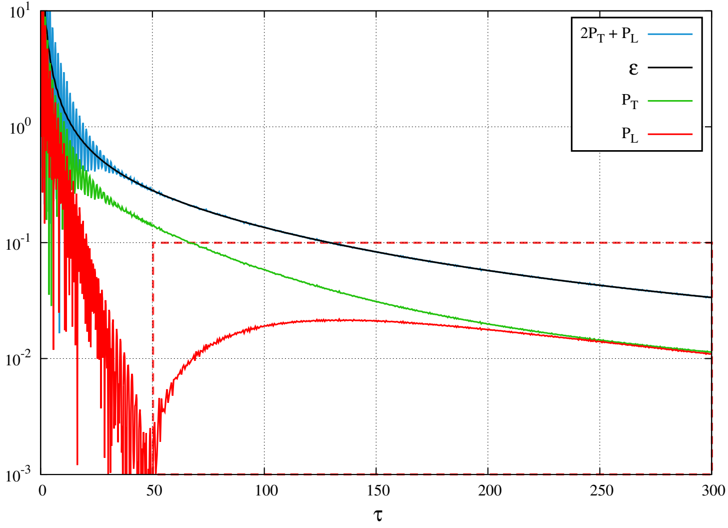


$$\epsilon \sim T^4 \sim \tau^{-4/3}$$

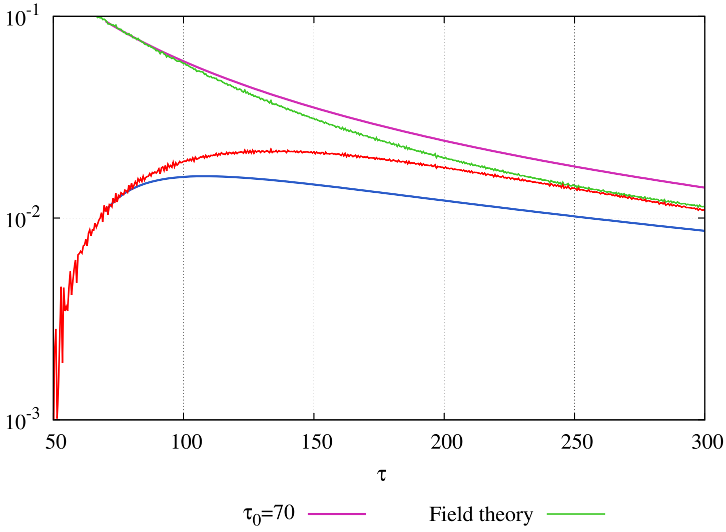
$$\nu \sim k_z \tau \sim \tau^{2/3}$$

4 Expanding volume

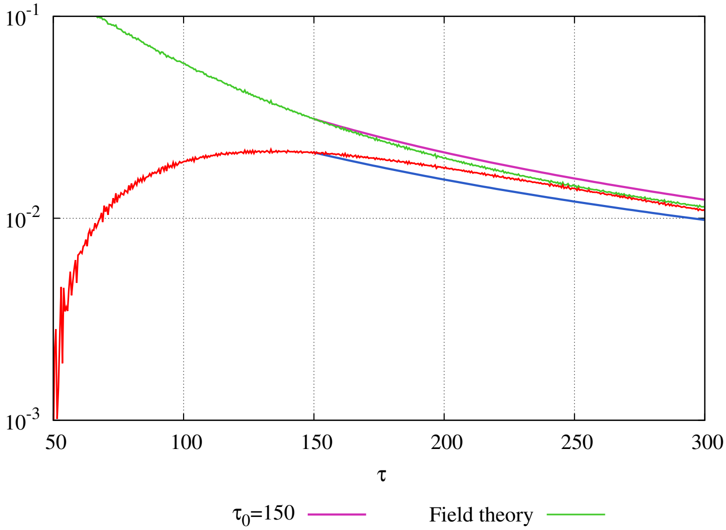
Comparison with Hydro: isotropization

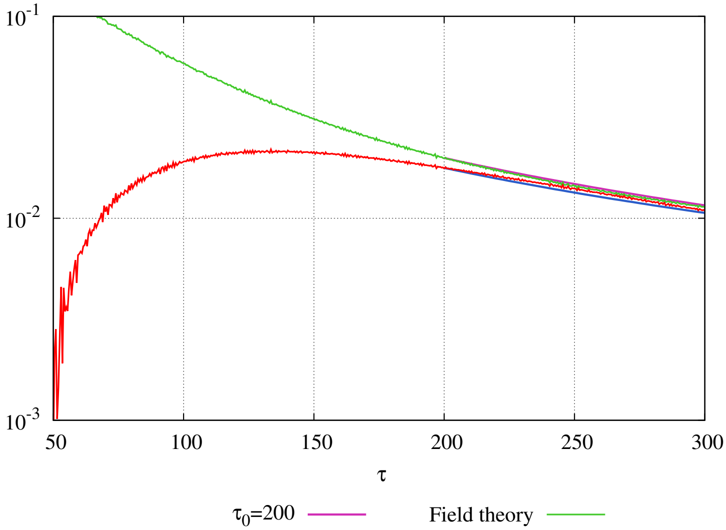


Comparison with Hydro: isotropization



Comparison with Hydro: isotropization

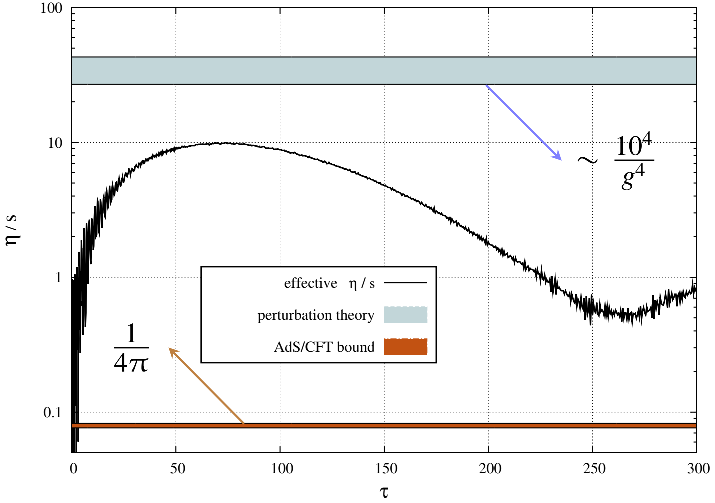




10^-1
10^-2
10^-3
50
100
150
200
250
300
τ
τ0=200
Field theory

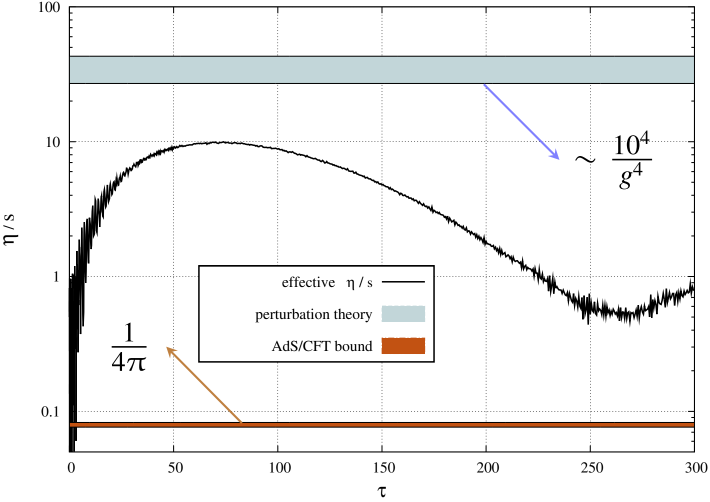
Comparison with Hydro: viscosity

$$P_T - P_L = \frac{2\eta}{\tau}$$



COMPARISON WITH HYDRO: VISCOSITY

see also [ASAKAWA, BASS, MULLER (2006-07)]

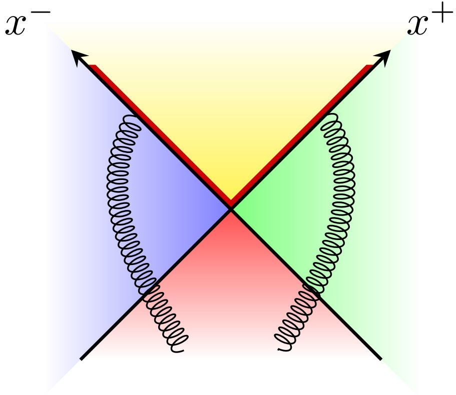


5 GAUGE CASE

Spectrum of fluctuations

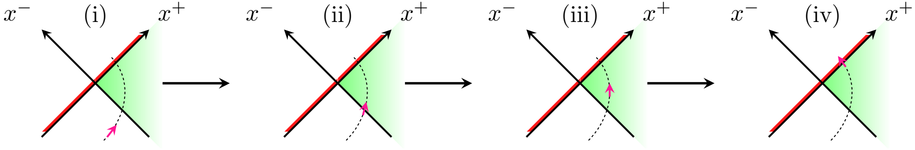
How to construct the Spectrum of fluctuations?

[TE,Gelis (in preparation)]



$$\begin{cases} \mathcal{A}^\mu = 0 \\ a^a_{\boldsymbol{k}\mu\lambda} = \epsilon^a_{\mu\boldsymbol{k}\lambda} e^{ikx} \end{cases}$$

The whole evolution



initial $A^a_\mu \to$ known

$$A^{\mu a}(\tau_0, \eta, \boldsymbol{x}_\perp) = \mathcal{A}^{\mu a}(\boldsymbol{x}_\perp) + \mathrm{Re} \sum_{\lambda, c} \int_{\boldsymbol{k}} c_{\boldsymbol{k}\lambda c}\, a^a_{\mu \boldsymbol{k}\lambda c}(\tau_0, \eta, \boldsymbol{x}_\perp)$$

Instabilities

Fixed Volume

Instabilities

$\downarrow$

Decoherence

FIXED VOLUME

Instabilities

$\downarrow$

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

Fixed Volume

Instabilities

$\downarrow$

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

$\downarrow$

Bose-Einstein condensate

FIXED VOLUME

Instabilities

$\downarrow$

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

$\downarrow$

BOSE-EINSTEIN condensate

$\downarrow$

$$f_{\boldsymbol{k}} \propto \frac{T}{\omega_{\boldsymbol{k}} - m} - \frac{1}{2}$$

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

$\downarrow$

Decoherence

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

$\downarrow$

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

↓

Decoherence

↓

Equation of state $\epsilon = 2P_T + P_L$

↓

Isotropization

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

$\downarrow$

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

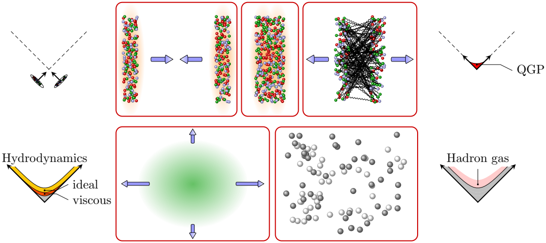
$\downarrow$

Isotropization

$\downarrow$? Thermal distribution

$\downarrow$? BOSE-EINSTEIN condensate?

Thank You!



Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

VISCOUS HYDRO

Anisotropic system

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \eta\, \pi^{\mu\nu}$$

In our case

$$P_T = \frac{\epsilon}{3} + \frac{2\eta}{3\tau} \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \;\rightarrow \partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}$$

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assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

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At a given time, knowing ϵ, P_T, P_L and assuming

- an EOS
- STEFAN-BOLTZMANN entropy
- $\eta = \frac{\eta_0}{\tau}$
- $\frac{\eta}{s} =$ cte

gives a very simple hydro model.

NON-ZERO INITIAL MODE: $\varphi_0 \sim \cos \boldsymbol{k}.\boldsymbol{x}$