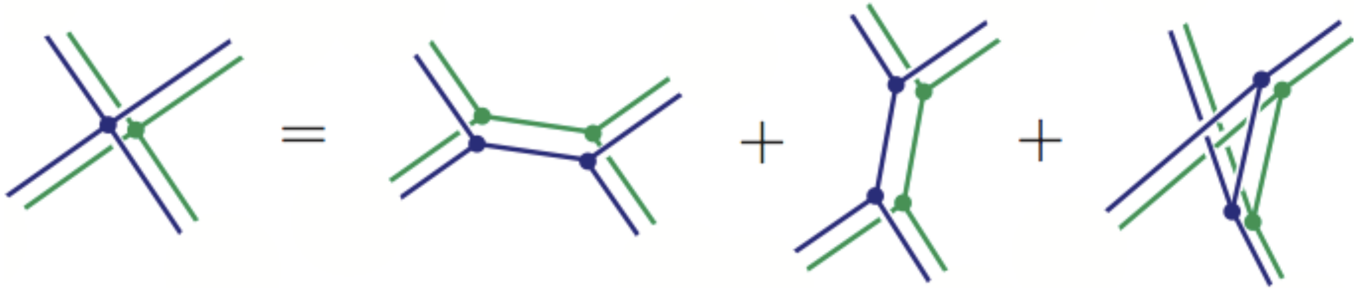
Supergravity from 2 and 3-Algebra Gauge Theory



Henrik Johansson

CERN

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Twelfth Workshop on Non-Perturbative Quantum Chromodynamics,

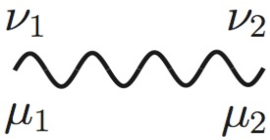
l'Institut d'Astrophysique de Paris

Work with: Zvi Bern, John Joseph Carrasco; Yu-tin Huang, Sangmin Lee

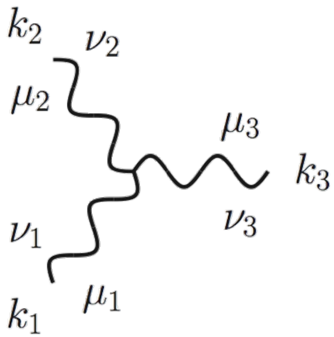
Text-Book: perturbative gravity is complicated !

de Donder gauge:

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



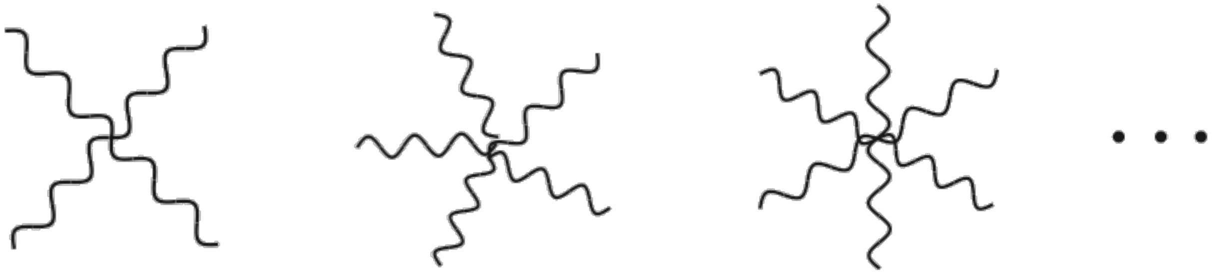
$$= \frac{1}{2}\left[\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} + \eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2} - \frac{2}{D-2}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\right]\frac{i}{p^2+i\epsilon}$$



$$\begin{aligned}= \mathrm{sym}[&-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \tfrac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3})\\
&+P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3})\\
&+P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3})\\
&+2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]\end{aligned}$$

After symmetrization
~ 100 terms !

higher order
vertices...



$\sim 10^3$ terms

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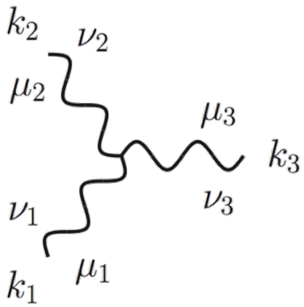
On-shell simplifications

Graviton plane wave: $\varepsilon^{\mu}(p)\varepsilon^{\nu}(p)\, e^{ip\cdot x}$

↑ Yang-Mills polarization

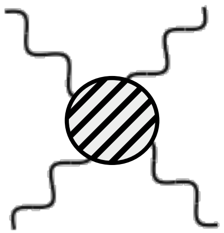
On-shell 3-graviton vertex:



$$= \quad i\kappa\Big(\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3} + \text{cyclic}\Big)\Big(\eta_{\nu_1\nu_2}(k_1-k_2)_{\nu_3} + \text{cyclic}\Big)$$

↑ Yang-Mills vertex

Gravity scattering amplitude:



Yang-Mills amplitude ↓

$$M^{\rm GR}_{\rm tree}(1,2,3,4) = \frac{st}{u} A^{\rm YM}_{\rm tree}(1,2,3,4) \otimes A^{\rm YM}_{\rm tree}(1,2,3,4)$$

$$\xrightarrow{d=3} \; A^{\rm CSm}_{\rm tree}(1,2,3,4) \otimes A^{\rm CSm}_{\rm tree}(1,2,3,4)$$

↑ Chern-Simons-matter theory

Gravity processes = squares of gauge theory ones - entire S-matrix

Bern, Carrasco, HJ [BCJ]

Outline

- Motivation *D*=3 amplitudes
- Duality between Color and Kinematics
  - Kinematical Lie 2-Algebra (Yang-Mills theory)
  - Kinematical Lie 3-Algebra (Chern-Simons-matter theory)
  - Gravity as a Double Copy of YM and CSm theories
- Amplitudes in BLG, ABJM and *D*=2 SUGRA
  - Tree-Amplitude relations
  - Dimensional reduction: *D*=2 ABJM
  - Integrability of *D*=2 SUGRA?
- Conclusions

Why Amplitudes in *D*=3 (or *D*=2)

→Travaglini's talk

- *N*=8 Bagger-Lambert-Gustavsson (BLG) theory
- *N*=6 Aharony-Bergman-Jafferis-Maldacena (ABJM) theory
- Chern-Simons-matter (CSM) theories – enticing gauge theories
- The celebrated AdS_4/CFT_3
- In *D*=2: supergravity integrability   Nicolai, Warner

Comparing CSM ←→ SYM "Same but different"

- Similar phenomena as in *D*=4 SYM
  - Yangian/Dual conformal sym. (ABJM)   Bargheer, Loebbert, Meneghelli; Huang, Lipstein
  - Grassmannian formulation (ABJM)   Lee; Huang, Lee
  - Color-kinematics duality (BLG, ABJM,...)   Bargheer, He, McLoughlin; Huang, HJ.

2-algebra Color-Kinematics Duality

D-dim. Yang-Mills theories are controlled by a kinematic Lie algebra

- Amplitude represented by cubic graphs:

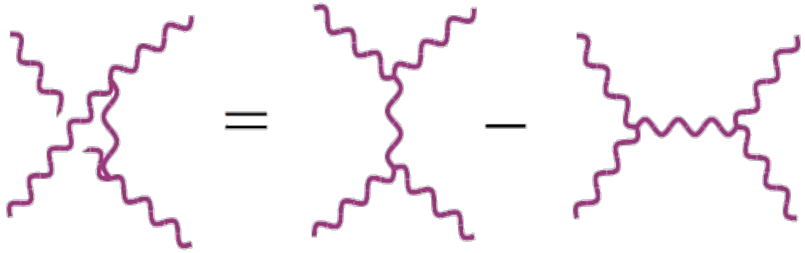
$$\mathcal{A}_m^{(L)} = \sum_{i \in \text{cubic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

numerators

color factors

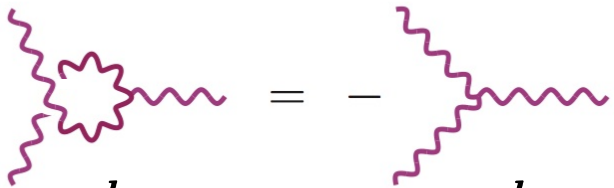
propagators

Color & kinematic numerators satisfy same relations:



$$f^{adc} f^{ceb} = f^{eac} f^{cbd} - f^{abc} f^{cde}$$

Jacobi identity



$$f^{bac} = - f^{abc}$$

antisymmetry

Duality: color ↔ kinematics

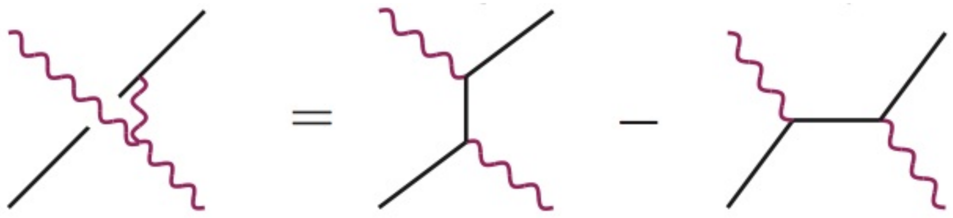
Bern, Carrasco, HJ

Some details of color-kinematics duality

Bern, Carrasco, HJ

can be checked for 4pt on-shell ampl. using Feynman rules

Example with two quarks:



$$\varepsilon_2\cdot(\overline{u}_1 \not{V} u_3)\cdot\varepsilon_4 = \overline{u}_1 \not\varepsilon_4 \not{p}_t \not\varepsilon_2 u_3 - \overline{u}_1 \not\varepsilon_2 \not{p}_s \not\varepsilon_4 u_3$$

$$f^{cba}T^c_{ik} = T^b_{ij}T^a_{jk} - T^a_{ij}T^b_{jk}$$

1. $(A^\mu)^4$ contact interactions absorbed into cubic graphs
   - by hand 1=*s*/*s*
   - or by auxiliary field $B \sim (A^\mu)^2$
2. Beyond 4-pts duality not automatic ➔ Lagrangian reorganization
3. Known to work at tree level: all-*n* example — Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove
4. Enforces (BCJ) relations on partial amplitudes ➔ (*n*-3)! Basis

also in string theory: Bjerrum-Bohr, Damgaard, Vanhove; Stieberger

Gravity is a double copy

- Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i \in \text{cubic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

BCJ

$$\mathcal{M}_m^{(L)} = \sum_{i \in \text{cubic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

- The two numerators can belong to different theories:

n_i $\tilde{n}_i$

($\mathcal{N}$=4) × ($\mathcal{N}$=4) → $\mathcal{N}$=8 sugra

($\mathcal{N}$=4) × ($\mathcal{N}$=2) → $\mathcal{N}$=6 sugra

($\mathcal{N}$=4) × ($\mathcal{N}$=0) → $\mathcal{N}$=4 sugra

($\mathcal{N}$=0) × ($\mathcal{N}$=0) → Einstein gravity + axion+ dilaton

similar to Kawai-Lewellen-Tye but works at loop level

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3-algebra Color-Kinematics Duality

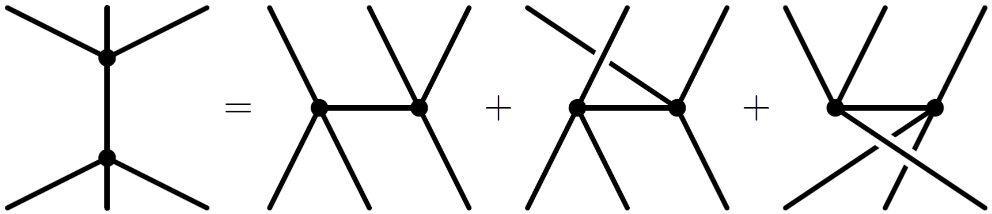
D=3 Chern-Simons matter theories obey color-kinematics duality

Bargheer, He, McLoughlin; Huang, HJ.

3-algebra Fundamental identity (Jacobi identity):

Bagger, Lambert; Gustavsson

$$f^{abc[d}f^{egh]a} = 0$$



$$c_s = c_t + c_u + c_v \Leftrightarrow n_s = n_t + n_u + n_v$$

4 and 6 point checks shows that the double copy of BLG Is N = 16 $E_{8(8)}$ SG of Marcus and Schwarz

BLG ='square root' of *N*=16 SG

$$A_4^{\rm BLG} = \sqrt{M_4^{\mathcal{N}=16}} = \sqrt{\frac{\delta^{16}(Q)}{stu}}$$

D≤3 supergravity is a double copy of CSM

- Gravity amplitudes obtained by replacing color with kinematics — BCJ

$$\mathcal{A}_m^{(L)} = \sum_{i \in \text{quartic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

$$\mathcal{M}_m^{(L)} = \sum_{i \in \text{quartic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

Bargheer, He, McLoughlin; Huang, HJ.

- No string understanding (cf. Kawai-Lewellen-Tye)
- Details more subtle than in SYM ⊗ SYM — Huang, HJ, Lee
  - BLG ⊗ BLG works in *D=3* (verified: tree level ≤ 10pts)
  - ABJM ⊗ ABJM works in *D=3* at 4,6pts, but not ≥8pts
  - ABJM ⊗ ABJM works in *D=2* (verified: tree level ≤ 10pts)

BLG, ABJM and D=2 SUGRA

ABJM and BLG theory

ABJM: *N*=6 CSm theory with U(*N*)×U(*N*) gauge group →Travaglini's talk

Matter are the only propagating *d.o.f.*: bi-fundamental representation

Chiral $(N, \overline{N})$ multiplet:

$$\Phi = \phi^4 + \eta^A \psi_A + \frac{1}{2}\epsilon_{ABC}\eta^A\eta^B\phi^C + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\psi_4$$

1 3 3 1

In total 16 states (same spectrum as *N*=4 SYM, but chiral)

BLG: *N*=8 CSm theory with SU(2)×SU(2) = SO(4) gauge group

Matter is non-chiral $N = \overline{N}$

$$\Phi = \phi + \eta^A\psi_A + \frac{1}{2}\eta^A\eta^B\phi_{AB} + \frac{1}{3!}\eta^A\eta^B\eta^C\epsilon_{ABCD}\overline{\psi}^D + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D\epsilon_{ABCD}\overline{\phi}$$

1 4 6 4 1

In total 16 states

ABJM and BLG are three-algebras

Bi-fundamental matter theories are three-algebra theories

Bagger, Lambert;
Bagger, Bruhn

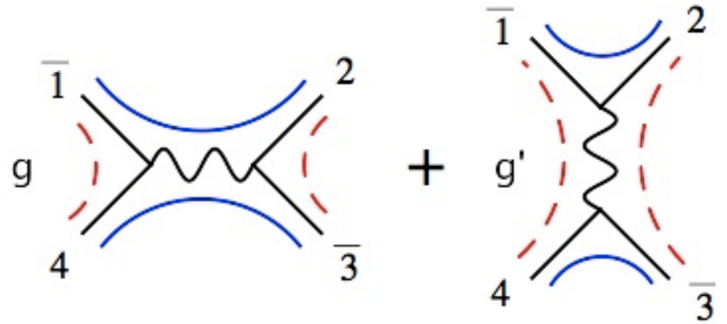
Triple product of *N*×*M* matrices;

$$[M^a, M^b; M^{\bar{c}}] \equiv (M^a M^{\bar{c}} M^b - M^b M^{\bar{c}} M^a)^{\alpha'}{}_{\beta} \equiv f^{ab\bar{c}}{}_d (M^d)^{\alpha'}{}_{\beta}$$

Structure constants satisfy fundamental identity (Jacob identity)

$$f^{ab\bar{f}}{}_g f^{gcd\bar{e}} - f^{acd}{}_g f^{gb\bar{f}\bar{e}} - f^{bcd}{}_g f^{ag\bar{f}\bar{e}} + f^{gcd\bar{f}} f^{ab}{}_g{}^{\bar{e}} = 0$$

Obtained from Feynman diag.



$$f^{ab\bar{c}\bar{d}} = g(T_A)^{a\bar{c}}(T^A)^{b\bar{d}} + g'(T_B)^{a\bar{d}}(T^B)^{b\bar{c}}$$

Interesting choices: $g = -g'$ or $g = g'$

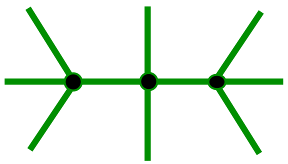
Symmetries of structure constants

- **ABJM theory** $f^{ab\bar{c}\bar{d}} = -f^{ab\bar{d}\bar{c}}$ complex, antisymmetric in pairs
- **BLG theory** f^{abcd} real and totally antisymmetric
- ***N*=5 CSM theory** $f^{ab\bar{c}\bar{d}} = -f^{ab\bar{d}\bar{c}}$ or $f^{ab\bar{c}\bar{d}} = f^{ab\bar{d}\bar{c}}$

real, (anti)symmetric in pairs — Bagger, Bruhn

Consider amplitudes:

$$\mathcal{A}_m = i\left(\frac{2\pi}{k}\right)^{\frac{m-2}{2}} \sum_{i\in\text{quartic}} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}}$$



$$c_i = f^{ab\bar{c}\bar{d}} f^{de\bar{f}\bar{g}} \ldots f^{wx\bar{y}\bar{z}}$$

What are their properties?

1) Kleiss-Kuijf relations
2) Color-kinematics duality → BCJ relations
3) double copy = supergravity

ABJM amplitude relations

Bargheer, He, McLoughlin; Huang, HJ.

Consider ABJM at 6pts

$$\mathcal{A}_m = \sum_{i\in\text{quartic}} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}} \quad \xrightarrow{\text{Solve Jacobi}} \quad A_{(i)} = \sum_{j=1}^{p} \Theta_{ij} n_j$$

At 6pts (ABJM):

$$\Theta_{ij} = \begin{pmatrix} \frac{1}{s_1} & \frac{1}{s_2}+\frac{1}{s_9} & \frac{1}{s_9} & -\frac{1}{s_9} & 0 \\ \frac{1}{s_8} & -\frac{1}{s_8} & \frac{1}{s_3} & \frac{1}{s_4}+\frac{1}{s_8} & 0 \\ \frac{1}{s_7} & -\frac{1}{s_7} & -\frac{1}{s_6}-\frac{1}{s_7} & \frac{1}{s_6}+\frac{1}{s_7} & \frac{1}{s_5}+\frac{1}{s_6}+\frac{1}{s_7} \\ 0 & -\frac{1}{s_9} & -\frac{1}{s_3}-\frac{1}{s_9} & \frac{1}{s_9} & -\frac{1}{s_5} \\ 0 & -\frac{1}{s_2} & \frac{1}{s_6} & -\frac{1}{s_4}-\frac{1}{s_6} & -\frac{1}{s_6} \end{pmatrix}$$

5×5 matrix has rank 4, but only in *D*=3 and on-shell !

5-term amplitude relation:

$$\mathrm{Ker}(\Theta^T)\cdot A = \sum_{i=1}^{5} C_{ik} A_{(i)} = 0$$

$$\mathrm{Det}(\Theta_{i1}, \Theta_{i2}, \ldots, A_{(i)}, \ldots, \Theta_{ip}) = 0$$

BLG amplitude relations

Huang, HJ, Lee

Consider BLG at 6pts

$$\mathcal{A}_m = \sum_{i\in\text{quartic}} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}} \quad \xrightarrow{\text{Solve Jacobi}} \quad A_{(i)} = \sum_{j=1}^{p} \Theta_{ij} n_j$$

At 6pts (BLG):

$$\Theta_{ij} = \begin{pmatrix} \frac{1}{s_{123}} + \frac{1}{s_{135}} & \frac{1}{s_{126}} + \frac{1}{s_{156}} & \frac{1}{s_{156}} & \frac{1}{s_{135}} - \frac{1}{s_{156}} & \frac{1}{s_{135}} \\ \frac{1}{s_{124}} + \frac{1}{s_{135}} & -\frac{1}{s_{124}} & \frac{1}{s_{134}} & \frac{1}{s_{124}} + \frac{1}{s_{125}} + \frac{1}{s_{135}} & \frac{1}{s_{135}} \\ \frac{1}{s_{135}} + \frac{1}{s_{145}} & -\frac{1}{s_{145}} & -\frac{1}{s_{136}} - \frac{1}{s_{145}} & \frac{1}{s_{135}} + \frac{1}{s_{136}} + \frac{1}{s_{145}} & \frac{1}{s_{135}} + \frac{1}{s_{136}} + \frac{1}{s_{145}} + \frac{1}{s_{146}} \\ -\frac{1}{s_{135}} & -\frac{1}{s_{156}} & -\frac{1}{s_{134}} - \frac{1}{s_{156}} & -\frac{1}{s_{135}} + \frac{1}{s_{156}} & -\frac{1}{s_{135}} - \frac{1}{s_{146}} \\ -\frac{1}{s_{135}} & -\frac{1}{s_{126}} & \frac{1}{s_{136}} & -\frac{1}{s_{125}} - \frac{1}{s_{135}} - \frac{1}{s_{136}} & -\frac{1}{s_{135}} - \frac{1}{s_{136}} \end{pmatrix}$$

5×5 matrix has rank 3, but only in *D*=3 and on-shell !

4-term amplitude relation:

$$\text{Ker}(\Theta^T) \cdot A = \sum_{i=1}^{4} C_{ik} A_{(i)} = 0$$

$$\text{Det}(\Theta_{i1}, \Theta_{i2}, \ldots, A_{(i)}, \ldots, \Theta_{ip}) = 0$$

BLG and ABJM amplitude relations

Huang, HJ, Lee

BLG: 4-term amplitude relation:

$$0 = \sum_{i=2}^{5} S_i A_{(i)}$$

$$S_2 = s_{124}(s_{156}(s_{145}s_{146} - s_{135}s_{136}) + s_{126}(s_{146}(s_{135} + s_{156}) - s_{136}(s_{145} + s_{156}))),$$
$$S_3 = s_{145}(s_{156}(s_{136}(s_{124} + s_{126} + s_{135}) + s_{146}(s_{136} - s_{126})) - s_{126}s_{146}(s_{124} + s_{135} + s_{136})),$$
$$S_4 = s_{156}(s_{136}s_{145}(s_{124} + s_{126} + s_{135}) + s_{146}(s_{136}(s_{126} + s_{135}) + s_{145}(s_{135} + s_{136}) + s_{124}(s_{126} + s_{145}))),$$
$$S_5 = -s_{126}(s_{145}s_{146}(s_{124} + s_{135} + s_{156}) + s_{136}(s_{135}(s_{145} + s_{146}) + s_{124}(s_{145} + s_{156}) + s_{146}(s_{145} + s_{156}))),$$

(plus one additional relation)

ABJM-type theory (in *D*=2): 4-term amplitude relations:

$$0 = (A_{(1)}s_{123} - A_{(2)}s_{124})(s_{126}s_{146} - s_{136}s_{156}) + (A_{(4)}s_{146} - A_{(5)}s_{136})(s_{126}s_{156} + s_{123}s_{126} + s_{123}s_{156})$$

$$0 = (A_{(2)}s_{124} - A_{(3)}s_{145})(s_{126}s_{146} - s_{136}s_{156}) + (A_{(4)}s_{156} - A_{(5)}s_{126})(s_{145}s_{146} + s_{136}s_{145} + s_{136}s_{146})$$

ABJM and BLG data collection

ABJM theory counts:

Huang, HJ, Lee

external legs	4	6	8	10	$m = 2k+2$
quartic diagrams	1	9	216	9900	$\frac{(3k)!(k+1)!}{(2k+1)!(2!)^k}$
planar amplitudes	1	6	72	1440	$\frac{1}{2}(k+1)!k!$
diagrams in A^{planar}	1	3	12	55	$\frac{(3k)!}{k!(2k+1)!}$
distinct fundamental id's	0	9	432	29700	$\frac{(k-1)(3k)!(k+1)!}{(2k+1)!(2!)^k}$
KK-indep. ampls.	1	5	57	1144	*
BCJ-indep. ampls. $D=2$	1	3	38	987	*

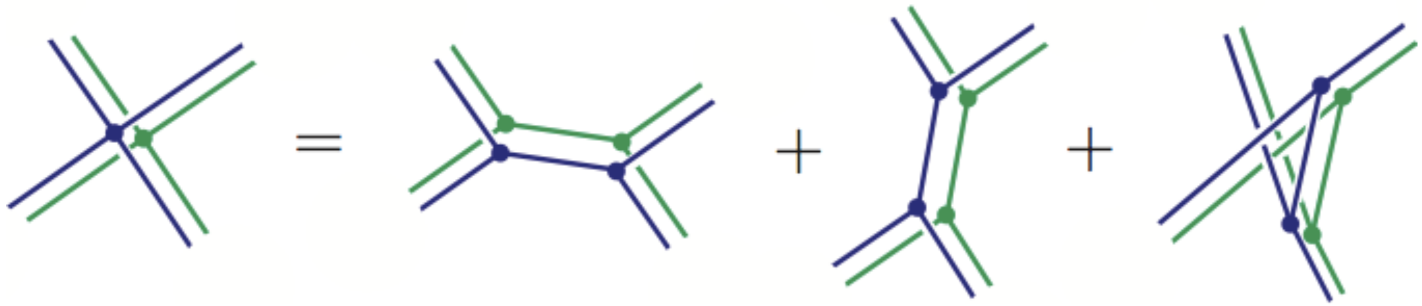
BLG theory counts:

external legs	4	6	8	10	$m = 2k+2$
quartic diagrams	1	10	280	15400	$\frac{(3k)!}{k!(3!)^k}$
distinct fundamental id's	0	15	840	69300	$\frac{3}{2}(k-1)\frac{(3k)!}{k!(3!)^k}$
KK-indep. ampls.	1	5	56	1077	*
BCJ-indep. ampls. $D=3$	1	3	38	1029	*

Note: no simple combinatorial patterns for KK and BCJ counts.

Same D=3 Supergravity Either Way

In D=3, supergravity from two different double copies:



CSM ⊗ CSM = SYM ⊗ SYM (kinematic parts) Huang, H.J.

- The extra propagators in SYM ⊗ SYM compensates for dimension mismatch
- SYM has even and odd matrix elements, CSM only even!
- R-symmetry constrains ensure that double copy kills odd SYM contributions

For N=16 SUGRA: all states are SO(16) spinors -> no odd S-matrix elements

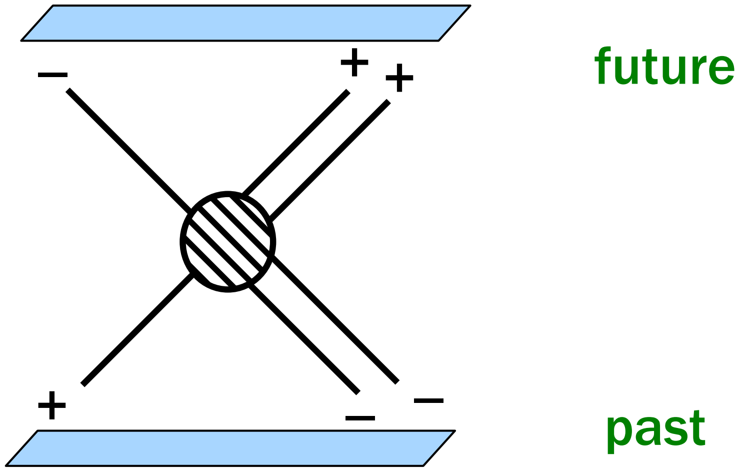
Marcus and Schwarz

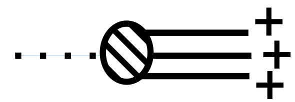
D=2 Supergravity and Integrability

- Easy access to *D*=2 supergravity S-matrix   Huang, HJ, Lee

- Problem: a *D*=2 massless S-matrix has severe IR divergences

- Possible to restrict to amplitudes without soft or collinear div's





Note: vanishing soft channel

Can check integrability of *D*=2 supergravity S-matrix (in restricted momenta)

Nicolai, Warner

D=2 ABJM and supergravity ampls

D=2 ABJM amplitudes:

Huang, HJ, Lee

4pts:

$$A^{\rm ABJM}_{D=2}(\bar{1},2,\bar{3},4) = i\frac{\delta^{(3)}(\sum_{\rm even}\lambda_i\eta_i)\delta^{(3)}(\sum_{\rm odd}\bar{\lambda}_i\eta_i)}{\bar{\lambda}_1\lambda_2\bar{\lambda}_3\lambda_4}$$

6pts:

$$A^{\rm ABJM}_{D=2}(\bar{1}2\bar{3}4\bar{5}6) = i\frac{\delta^{(3)}(\sum_{\rm even}\lambda_i\eta_i)\delta^{(3)}(\sum_{\rm odd}\bar{\lambda}_i\eta_i)}{\bar{\lambda}_1\lambda_2\bar{\lambda}_3\lambda_4\bar{\lambda}_5\lambda_6}\sum_{s=\pm}\delta^{(3)}\left(s\frac{\bar{\lambda}_3\eta_1-\bar{\lambda}_1\eta_3}{\bar{\lambda}_5}+i\frac{\lambda_6\eta_4-\lambda_4\eta_6}{\lambda_2}\right)$$

D=2 supergravity:

4pts: $M_{D=2}(\bar{1},2,\bar{3},4) = [A^{\rm ABJM}_{D=2}(1,\bar{2},3,\bar{4})]^2$ (finite and non-zero)

6pts:

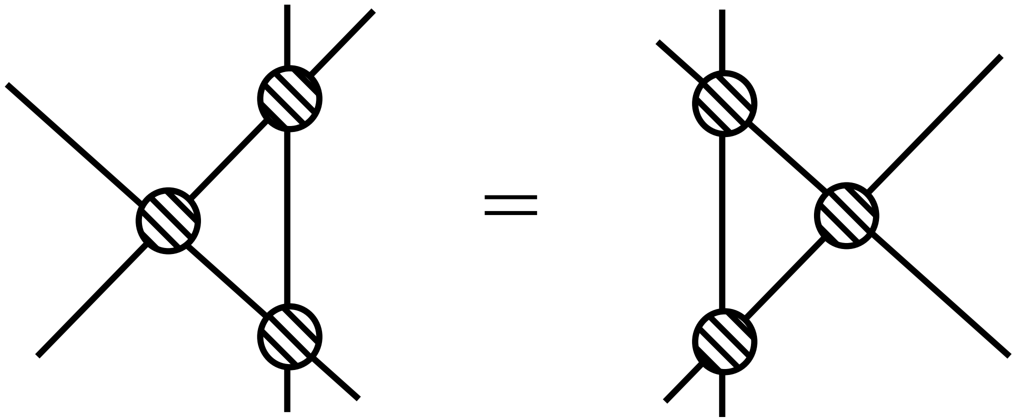
$$\mathcal{M}_6(\bar{1}2\bar{3}4\bar{5}6) = \frac{s_{12}s_{34}s_{56}}{(s_{23}-s_{14})(s_{36}-s_{12})(s_{34}-s_{16})}\Big((s_{34}-s_{16})A_{(1)}\tilde{A}_{(1)} + (s_{36}-s_{12})A_{(2)}\tilde{A}_{(2)} + (s_{23}-s_{14})A_{(3)}\tilde{A}_{(3)}\Big)$$

6pt amplitude vanishes → consistent with integrability

Yang-Baxter Eqn

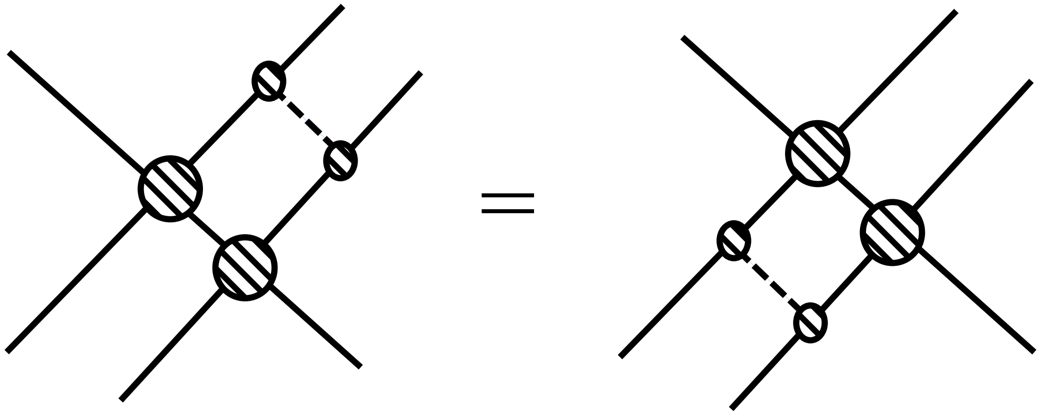
We would like to check the Yang-Baxter Eqn:

Huang, HJ, Lee



Holds in *D*=3 ABJM
and *D*=3 sugra

Problem: one line is massive → take massless limit



Holds in *D*=2 ABJM
and *D*=2 sugra,
but both sides diverge!

→ more checks are needed, as well as better understanding of IR div.

Conclusions

- Yang-Mills theories are controlled by a kinematic Lie 2-algebra
- Chern-Simons-matter theories controlled by a kinematic Lie 3-algebra
- The explicit kinematic algebra is still missing for all but the simplest case of self-dual Yang-Mills.
- With duality manifest: Gravity becomes double copy.
  double copy of CSM theory = double copy of $D \leq 3$ SYM
- BCJ relations/double copy present in $D=3$ for BLG theories
- BCJ relations/double copy present in $D=2$ for ABJM theories
- Simple access to $D=2$ supergravity S-matrix → checks of integrability
- C-K duality is a key tool for nonplanar gauge and gravity calculations.
  - Loop amplitudes in BLG (ABJM)…
  - $N=8$ supergravity UV behavior at 5 (and 7) loops…
  - $N=4$ supergravity UV behavior at 3,4 loops …