ADS/CFT imprints on the ALICE fireball?

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Based on : arXiv:1206.1521 S.Banerjee, R. Iyer, AM
arXiv: 1206.3311 AM
arXiv: 1307.xxxx E. Iancu, AM, M.Torres

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Plan of the talk

- Introduction - why non-equilibrium holography?
- Non-equilibrium spectral + statistical functions = Quantum kinetic theory
- Holographic prescriptions for the non-equilibrium spectral and statistical functions - Universal non-equilibrium fluctuation-dissipation relation
- Semi-holographic model of thermalization - holographic imprints on fireball dynamics
- Summary

Introduction - why non-equilibrium holography?

Introduction

- Difficult to obtain real-time description of phenomena in quantum field theory
- No well-organized perturbative expansions exist in non-equilibrium states even at weak coupling
- Uncontrolled approximations necessary to obtain quantum kinetic theory - (quantum Vlasov-Boltzmann equation)
- Even no phenomenological framework like Vlasov-Boltzmann equation for describing strongly coupled thermalization
- Heavy ion collision experiments indicate short thermalization time and possible role of strongly coupled dynamics

The holographic framework

- The only non-perturbative approach for real-time physics
- Difficult infrared problems are resolved by known physics of black hole horizon - predictions have a measure of universality
- Semi-holographic models give a new framework for quantum kinetics of thermalization
- Gives model independent signatures like non-equilibrium generalization of fluctuation-dissipation relation

Non-equilibrium spectral + statistical functions = Quantum kinetic theory

Spectral function : off-shell generalization of local density of states (Wigner transform of commutator (bosons) / anti-commutator (fermions))

$$\mathcal{A}(\omega,\mathbf{k},\mathbf{x},t) = \int d^3r\,dt_r\,e^{i(\omega t_r-\mathbf{k}\cdot\mathbf{r})}\Big\langle\Big[O\Big(\mathbf{x}+\frac{\mathbf{r}}{2},t+\frac{t_r}{2}\Big),O\Big(\mathbf{x}-\frac{\mathbf{r}}{2},t-\frac{t_r}{2}\Big)\Big]\Big\rangle$$

Statistical function : off-shell generalization of particle distribution in phase space (Wigner transform of anti-commutator (bosons) / commutator (fermions))

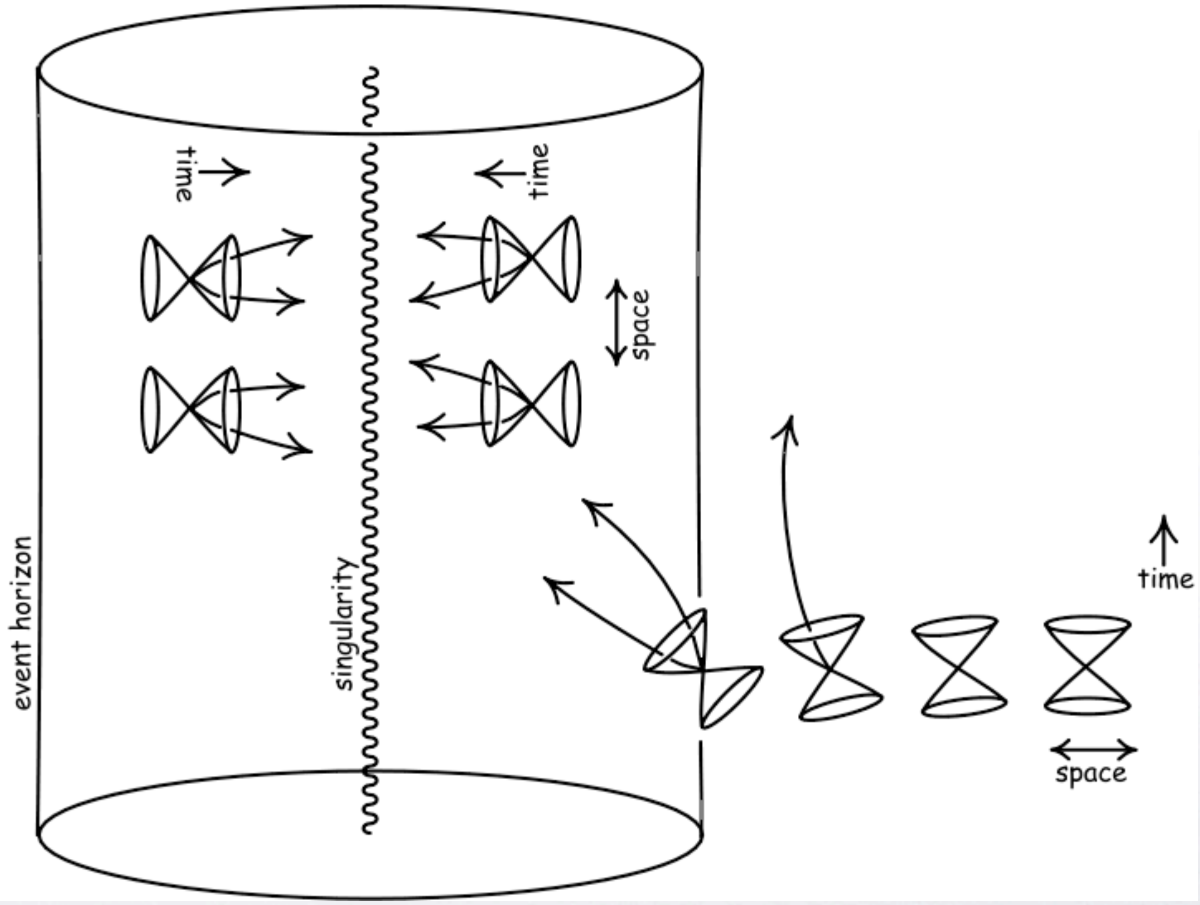
$$G_{\mathcal{K}}(\omega,\mathbf{k},\mathbf{x},t) = -\frac{i}{2}\int d^3r\,dt_r\,e^{i(\omega t_r-\mathbf{k}\cdot\mathbf{r})}\Big\langle\Big\{O\Big(\mathbf{x}+\frac{\mathbf{r}}{2},t+\frac{t_r}{2}\Big),O\Big(\mathbf{x}-\frac{\mathbf{r}}{2},t-\frac{t_r}{2}\Big)\Big\}\Big\rangle$$

The closed and coupled equations (Kadanoff-Baym equations) for evolution of spectral and statistical functions give off-shell quantum kinetic equations. It needs uncontrolled approximations via 2PI effective action technology.

Holographic prescriptions for non-equilibrium spectral and statistical functions

CAUSALITY and BLACK HOLE

- The incoming boundary condition preserves causality
- Quasinormal modes are normalizable and incoming metric pertubations - dual to thermal relaxation modes
- Quasi-normal modes describe non-linear dynamics even far away from equilibrium (fluid/ gravity correspondence, AM and R.Iyer, Mateos et. al, etc.)



Thermal retarded propagator

- Use incoming boundary condition to specify the solution of the scalar field equation uniquely up to a constant
- The thermal retarded propagator can be obtained from causal response (Son, Starinets)

$$\Phi(r, \mathbf{x}, t) = \Phi(r, \omega, \mathbf{k}) e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})}$$

$$\Phi(r, \omega, \mathbf{k}) \approx A^{\text{in}}(\omega, \mathbf{k}) \Big(1 - \frac{r}{r_h}\Big)^{-i\frac{\omega}{4\pi T}}$$

near horizon

$$\Phi(r, \omega, \mathbf{k}) \approx J(\omega, \mathbf{k}) r^{d-\Delta} + O(\omega, \mathbf{k}) r^{\Delta}$$

asymptotically

$$G_R(\omega, \mathbf{k}) = \frac{O(\omega, \mathbf{k})}{J(\omega, \mathbf{k})}$$

Non-equilibrium retarded propagator

Consider the black hole perturbed by a hydrodynamic shear-mode which has the dispersion relation :

$$\omega_{(h)} = -i\frac{\mathbf{k}_{(h)}^2}{4\pi T}, \quad \mathbf{k}_{(h)} \cdot \delta\mathbf{u}(\mathbf{k}_{(h)}) = 0.$$

The explicit metric is:

$$ds^2 = \frac{l^2}{r^2}\frac{dr^2}{f\left(\frac{rr_0}{l^2}\right)} + \frac{l^2}{r^2}\Big(-f\left(\frac{rr_0}{l^2}\right)dt^2 + dx^2 + dy^2 - 2\Big(1 - f\left(\frac{rr_0}{l^2}\right)\Big)\delta u_i(\mathbf{k}_{(\mathrm{h})})e^{i(\mathbf{k}_{(\mathrm{h})}\cdot\mathbf{x} - \omega_{(\mathrm{h})}t)}dtdx^i\Big)$$

$$+\frac{2l^2}{r^2}\Big(-i\frac{l^2}{3r_0}\,k_{(\mathrm{h})i}\,\delta u_j(\mathbf{k}_{(\mathrm{h})})e^{i(\mathbf{k}_{(\mathrm{h})}\cdot\mathbf{x} - \omega_{(\mathrm{h})}t)}\,h\left(\frac{rr_0}{l^2}\right)dx^i dx^j\Big) + O(\epsilon^2, \delta\mathbf{u}^2),$$

General solution of minimally coupled scalar field:

$$\Phi(r,\mathbf{x},t) = \int d\omega \int d^d k A(\omega,\mathbf{k})\Big(\Phi^{(eq)}(r,\omega,\mathbf{k}) + \Phi^{(neq)}(r,\omega,\mathbf{k},\mathbf{k}_{(h)})e^{i\mathbf{k}_{(h)}\cdot\mathbf{x}}e^{-\frac{\mathbf{k}_{(h)}^2}{4\pi T}t}\Big)$$

$$e^{-i\omega t}e^{i\mathbf{k}\cdot x}$$

Near the horizon:

$$\Phi^{(neq)}(r,\omega,\mathbf{k},\mathbf{k}_{(h)}) \approx \alpha(r,\omega,\mathbf{k},\mathbf{k}_{(h)})\Big(1-\frac{r}{r_h}\Big)^{-i\frac{\omega}{4\pi T}-\frac{\mathbf{k}_{(h)}^2}{16\pi^2 T^2}}$$

Incoming homogeneous solution

$$+\beta(r,\omega,\mathbf{k},\mathbf{k}_{(h)})\Big(1-\frac{r}{r_h}\Big)^{i\frac{\omega}{4\pi T}-\frac{\mathbf{k}_{(h)}^2}{16\pi^2 T^2}}$$

Outgoing homogeneous solution

$$+\mathcal{C}(\omega,\mathbf{k},\mathbf{k}_{(h)})\left(\delta\mathbf{u}(\mathbf{k}_{(h)})\cdot\mathbf{k}\right)\left(1-\frac{r}{r_h}\right)^{-i\frac{\omega}{4\pi T}}$$

Particular solution

$\alpha(r,\omega,\mathbf{k},\mathbf{k}_{(h)})$ and $\beta(r,\omega,\mathbf{k},\mathbf{k}_{(h)})$ are arbitrary

$\mathcal{C}(\omega,\mathbf{k},\mathbf{k}_{(h)})$ is fixed by equations of motion

In order to get a regular solution we need :

$$\alpha(\omega, \mathbf{k}, \mathbf{k}_{(h)}) = \beta(\omega, \mathbf{k}, \mathbf{k}_{(h)}) = 0.$$

The above prescription holds when the black brane is perturbed by any arbitrary quasinormal mode (thermal relaxational mode in dual theory). Can also be generalized for the full background solution of (non-linear) Einstein's equations with multiple background quasi-normal modes.

The scalar field solution can be expanded systematically in derivative and amplitude expansions.

We obtain corrections to both the source (non-normalizable mode) and the expectation value (normalizable mode) of the dual operator.

$$\Phi^{(neq)}(r, \omega, \mathbf{k}, \mathbf{k}_{(h)}) \approx J^{(neq)}(\omega, \mathbf{k}, \mathbf{k}_{(h)}) r^{d-\Delta} + O^{(neq)}(\omega, \mathbf{k}, \mathbf{k}_{(h)}) r^{\Delta}$$

These have systematic derivative and amplitude expansions.

$$J^{(neq)}(\omega, \mathbf{k}, \mathbf{k}_{(h)}) = J_1(\omega, \mathbf{k}) \delta \mathbf{u}(\mathbf{k}_{(h)}) \cdot \mathbf{k} + J_2(\omega, \mathbf{k}) (\delta u(\mathbf{k}_{(h)})_i k_{(h)j} \, k_i k_j + \ldots$$
$$O^{(neq)}(\omega, \mathbf{k}, \mathbf{k}_{(h)}) = O_1(\omega, \mathbf{k}) \delta \mathbf{u}(\mathbf{k}_{(h)}) \cdot \mathbf{k} + O_2(\omega, \mathbf{k}) (\delta u(\mathbf{k}_{(h)})_i k_{(h)j} \, k_i k_j + \ldots$$

The non-equilibrium retarded propagator is $G_R(x_1, x_2) = \frac{O(x_1)}{J(x_2)}$

Note $A(\omega, \mathbf{k})$ cancels between numerator and denominator

After Wigner transform we get the form below in any quasinormal mode background

$$\omega_{(\mathrm{b})} = \omega_{\mathrm{R(b)}}(\mathbf{k}_{(\mathrm{b})}) - i\omega_{\mathrm{I(b)}}(\mathbf{k}_{(\mathrm{b})})$$

$$G_R(\omega, \mathbf{k}, \mathbf{x}, t) = \int d\omega_0 \int d^d k_0 \Bigg[G_R^{(\mathrm{eq})}(\omega_0, \mathbf{k}_0)\delta(\omega - \omega_0)\delta^2(\mathbf{k} - \mathbf{k}_0)$$

$$- G_R^{(\mathrm{eq})}(\omega_0, \mathbf{k}_0)\frac{1}{2\pi i}\left(\frac{O^{(neq)}(\omega_0, \mathbf{k}_0, \mathbf{k}_{(\mathrm{b})})}{O^{(eq)}(\omega_0, \mathbf{k}_0)} - \frac{J^{(neq)}\left(\omega_0, \mathbf{k}_0, \mathbf{k}_{(\mathrm{b})}\right)}{J^{(eq)}(\omega_0, \mathbf{k}_0)}\right)$$

$$\delta^2\left(\mathbf{k} - \mathbf{k}_0 + \frac{\mathbf{k}_{(\mathrm{b})}}{2}\right)\frac{1}{\left(\omega - \omega_0 + \frac{1}{2}\left(\omega_{\mathrm{R(b)}}(\mathbf{k}_{(b)}) - i\omega_{\mathrm{I(b)}}(\mathbf{k}_{(\mathrm{b})})\right)\right)}$$

$$e^{i\mathbf{k}_{(\mathrm{b})}\cdot\mathbf{x}} e^{-i\left(\omega_{\mathrm{R(b)}}(\mathbf{k}_{(b)}) - i\omega_{\mathrm{I(b)}}(\mathbf{k}_{(\mathrm{b})})\right)t}\Bigg]$$

Contributions from non-linear dynamics of quasinormal modes can be similarly evaluated.

Non-equilibrium contributions are co-moving with the relaxational modes in the non-equilibrium state

The holographic spectral function tells us about

(i) relaxation modes of the system

(ii) their collective non-linear dynamics

Non-equilibrium statistical function

At equilibrium, the statistical and spectral functions are related by the thermal fluctuation-dissipation relation :

$$G_{\mathcal{K}}(\omega, \mathbf{k}) = -i\Big(\frac{1}{2} + n_{BE}(\omega)\Big)\mathcal{A}(\omega, \mathbf{k}) \quad \text{for bosons}$$

$$G_{\mathcal{K}}(\omega, \mathbf{k}) = -i\Big(\frac{1}{2} - n_{FD}(\omega)\Big)\mathcal{A}(\omega, \mathbf{k}) \quad \text{for fermions}$$

The non-equilibrium correction to fluctuation-dissipation relation should depend on hydrodynamic variables, shear-stress, etc.

Therefore, these should be obtained from the boundary conditions at the horizon.

Take the non-equilibrium coefficients of homogeneous modes to be non-zero at each order in derivative/amplitude expansion.

Use them to capture the shifts of individual weights of known non-equilibrium retarded and advanced propagators in non-equilibrium Feynman propagator.

Fix them using field theoretic constraints - symmetry of G_F(x_1, x_2) in x_1 and x_2, and Hermiteanity.

It follows for all non-equilibrium states in absence of sources :

$$G_F(\omega, \mathbf{k}, \mathbf{x}, t) = \Big(n_{\mathrm{BE}}(\omega) + 1\Big) G_R(\omega, \mathbf{k}, \mathbf{x}, t) - n_{\mathrm{BE}}(\omega) G_A(\omega, \mathbf{k}, \mathbf{x}, t)$$

$$G_K(\omega, \mathbf{k}, \mathbf{x}, t) = -i\Big(\frac{1}{2} + n_{\mathrm{BE}}(\omega)\Big) \mathcal{A}(\omega, \mathbf{k}, \mathbf{x}, t)$$

The Bose-Einstein distribution is fixed by the final equilibrium temperature.

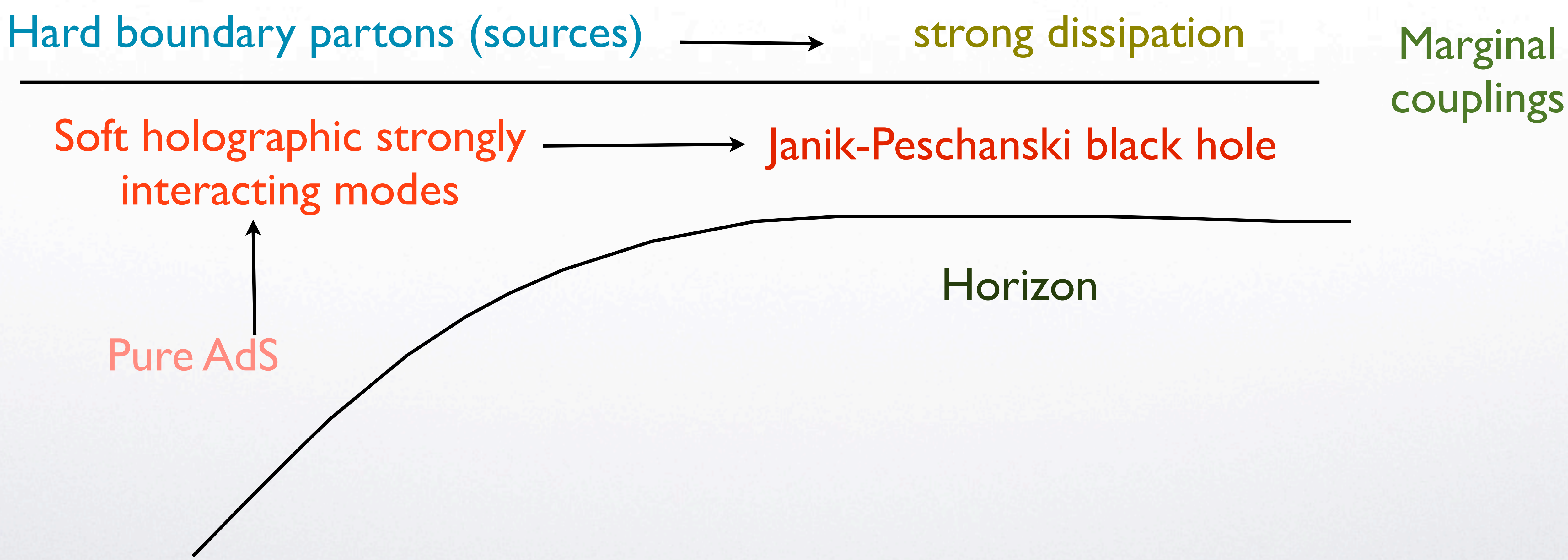
Note there is no violation of causality - as in absence of sources the final temperature is determined by the initial energy.

This is a highly non-local relation both in space and time - cannot be reproduced by any quasi-particle picture.

We have now implemented these prescriptions for the fermion on the computer (AM, S. Banerjee, G. Policastro; to appear soon).

Semi-holographic thermalization

General semi-holographic models by including all relevant and marginal couplings



According to saturation physics :

Most of the initial partons released after heavy ion collisions are hard and have typical momentum of Q_s = 2-3 GeV (at ALICE).

The gluons have large occupation numbers of O(1/g^2) - therefore can be described by classical Yang-Mill's equations.

The (light-cone) color sources are in the nucleus and they have event-by-event fluctuations which can be modeled by a Gaussian distribution.

$$A^{\pm} = 0, \quad A^i = \Theta(x^-)\Theta(-x^+)\alpha_1^i(\mathbf{x}_\perp) + \Theta(-x^-)\Theta(x^+)\alpha_2^i(\mathbf{x}_\perp)$$

$$-D_i \alpha_{1,2}^i = \rho_{1,2}(\mathbf{x}_\perp)$$

$$\langle \rho_a^k(\mathbf{x}_\perp)\rho_b^l(\mathbf{y}_\perp)\rangle \approx \delta_{ab}\delta_{kl}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)\frac{Q_s^2(\mathbf{x}_\perp)}{N}$$

The hard partons radiate soft ones.

Let us assume that the soft partons are described by an effective holographic strongly coupled large N CFT.

Also the coupling between hard and soft modes is marginal with a small dimensionless coupling.

$$S = \frac{N^2}{\lambda}\int d^4x\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + +N^2\int d^4x\mathrm{Tr}(j_\mu A^\mu)$$
$$+ N^2\alpha Q_s^{-4}\int d^4x\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\Phi^{\mathrm{CFT}} + N^2\beta Q_s^{-3}\int d^4x\mathrm{Tr}(\overline{\Psi}\Psi)\Phi^{\mathrm{CFT}}$$
$$+ N^2 S_{\mathrm{CFT}}[\Phi^{\mathrm{CFT}}]$$

on- shell action of a 5D gravitational theory

$\Phi^{\rm CFT}$ is the source for the bulk dilaton. This produces soft modes.
The dissipation of boundary hard partons comes mainly via dynamics of bulk graviton.

Integrating out the bulk dilation, we get the new effective action for the glasma :

$$S = \frac{N^2}{\lambda}\int d^4x\,{\rm Tr}(F_{\mu\nu}F^{\mu\nu}) + N^2\int d^4x{\rm Tr}(j_\mu A^\mu)$$
$$+ N^2\sum_{n=2}^{\infty}\alpha^n\frac{Q_s^{-4n}}{n!}\int d^4x_1\int d^4x_2....\int d^4x_n{\rm Tr}(F_{\mu_1\nu_1}F^{\mu_1\nu_1})(x_1){\rm Tr}(F_{\mu_2\nu_2}F^{\mu_2\nu_2})(x_2)$$
$$...{\rm Tr}(F_{\mu_n\nu_n}F^{\mu_n\nu_n})(x_n)G^{(n)}(x_1,x_2,...x_n) \qquad .$$

G(x_1, x_2, ... x_n) is the causal non-equilibrium holographic correlator evaluated in a non-equilibrium geometry

The geometry is self-consistently determined via 5-D Einstein's equations with the boundary condition :

$$R_{MN} - \frac{1}{2} R g_{MN} = \Lambda g_{MN} + T_{MN}[\Phi],$$
$$\nabla^M \nabla_M \Phi = 0,$$
$$\Phi(r = 0, x) = \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu})(x)$$

For small \alpha, we can keep only the two-point non-equilibrium retarded function.

The boundary partons indeed dominate the dynamics at initial time as the bulk is pure AdS (i.e. there are no soft modes). So thermalization time T^{-1} (1/\alpha)^x.

The extra radial dimension can be though of as capturing the dynamics at various rapidity scales - locality in bulk translates to locality in interaction in rapidity space.

After a while, the boundary partons dissipate, and the dynamics is purely that of the quasi-normal modes of the black hole which include hydrodynamics. These describe collective flow - determine viscosity, etc.

Here, we can use the holographic prescriptions for non-equilibrium retarded correlator and the statistical functions for the dilaton.

The non-equilibrium dilaton retarded correlator can be measured by:
(i) collective flow observables
(ii) response of collective flow to initial quantum fluctuations

The non-equilibrium dilaton statistical function can be measured by the pion production.

The universal non-equilibrium fluctuation-dissipation relation is thus an acid test of the semi-holographic models.

Summary

- There are universal holographic prescriptions for evaluating non-equilibrium retarded correlator and non-equilibrium statistical function in states undergoing thermal relaxation.

- One can give a generic semi-holographic model for thermalization - one can build a detailed theory for calculating observables of collective flow, response to event-by-event fluctuations in initial conditions, pion interferometry, etc.

- The falsifiable signature of such models is the universal non-equilibrium fluctuation-dissipation relation which holds on an event-by-event basis.

- Future: What about jets?

THANK YOU