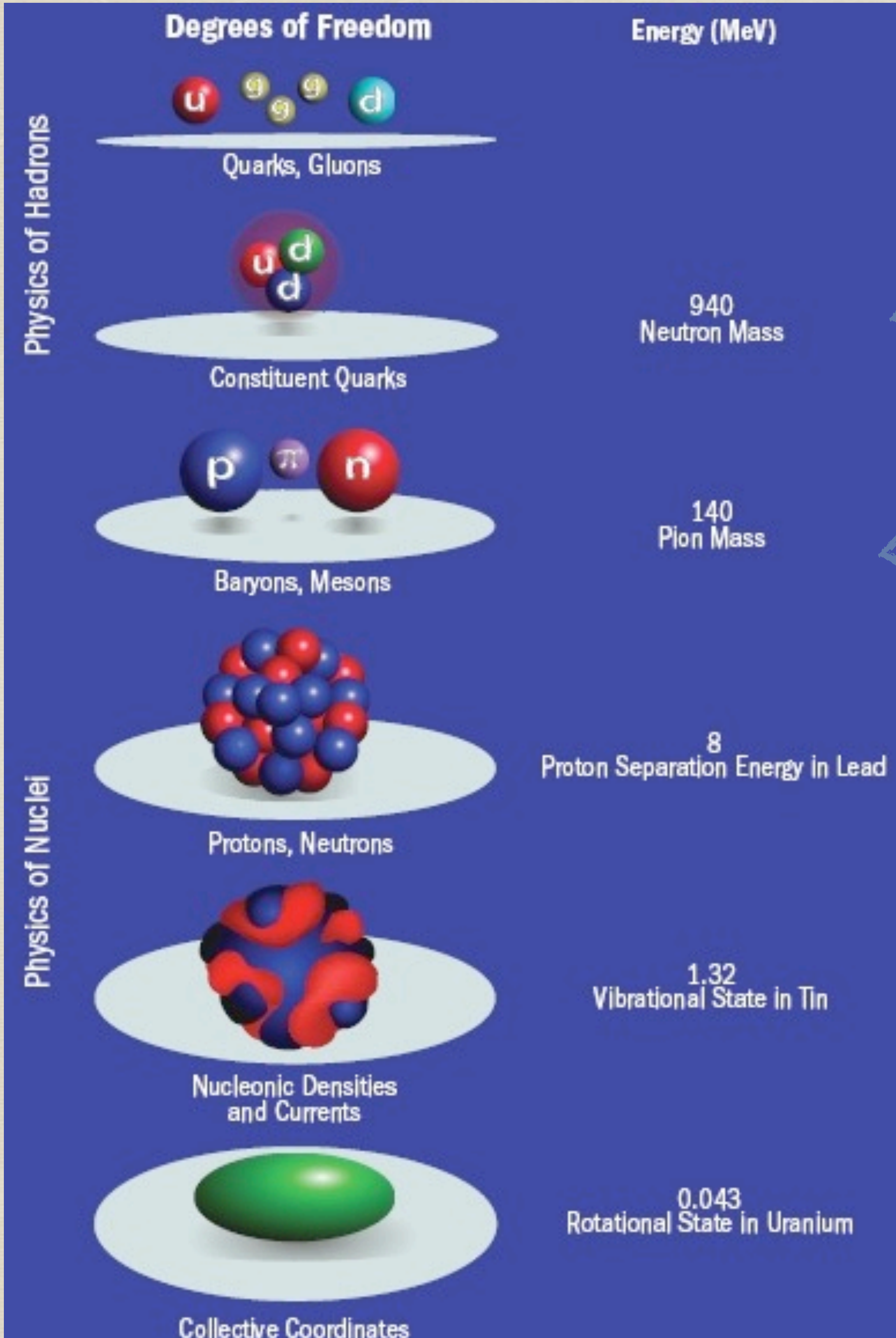
NUCLEAR PHYSICS FROM LATTICE QCD

Kostas Orginos



QCD

Hadron structure and spectrum

Hadronic Interactions

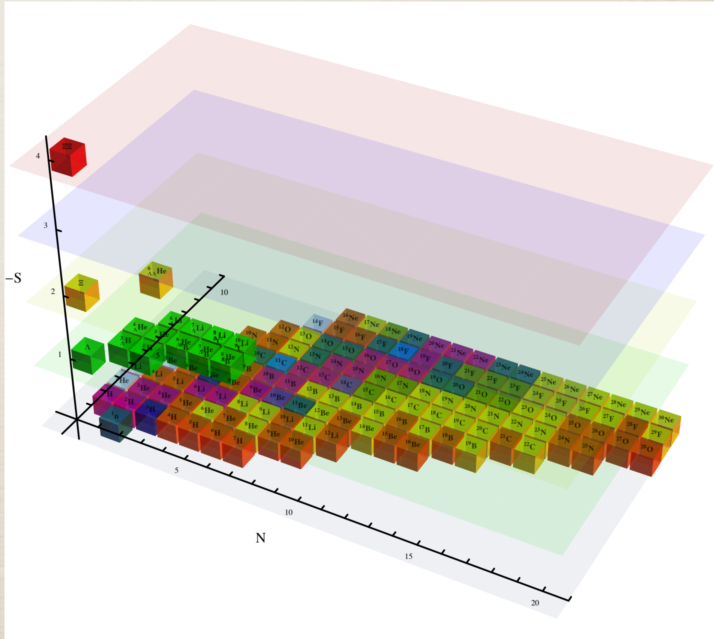
Nuclear physics

Figure by W. Nazarewicz

Hadron Interactions

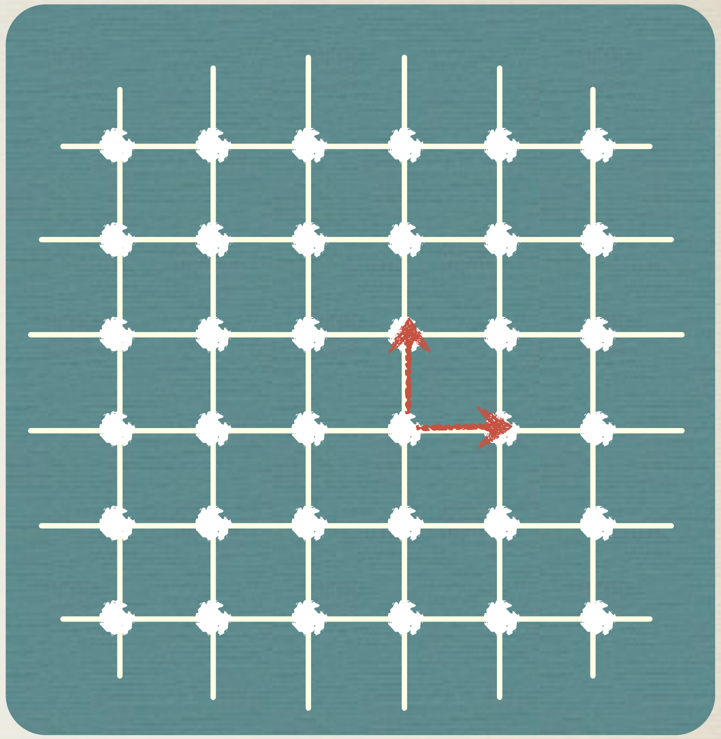
Goals:

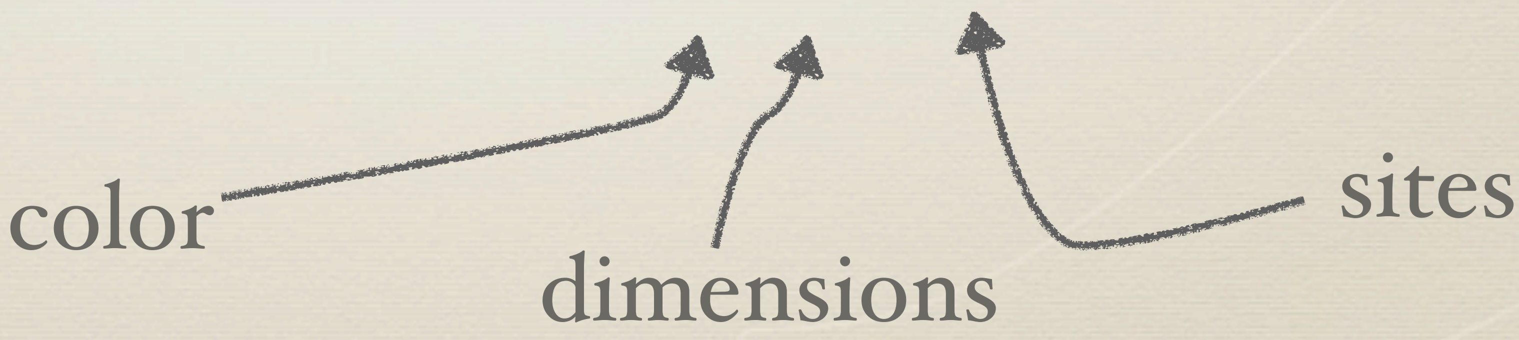
- Challenge: Compute properties of nuclei from QCD
  - Spectrum and structure
- Confirm well known experimental observation for two nucleon systems
- Explore the largely unknown territory of hyper-nuclear physics
- Provide input for the equation of state for nuclear matter in neutron stars
- Provide input for understanding the properties of multi-baryon systems



Scales of the problem

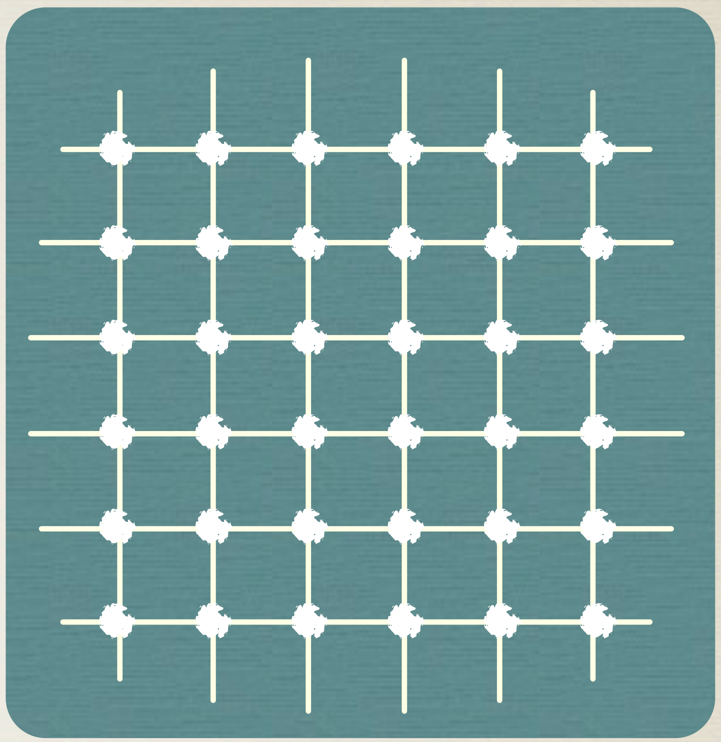
- Hadronic Scale: 1fm ~ 1×10^{-13} cm
- Lattice spacing << 1fm
  - take a=0.1fm
- Lattice size La >> 1fm
  - take La = 3fm
- Lattice 32^4
- Gauge degrees of freedom: $8 \times 4 \times 32^4 = 3.4 \times 10^7$





The pion mass is an additional small scale

$\sim 1/m_\pi \sim 1.4$fm



Single hadron volume corrections

$$\sim e^{-m_\pi L}$$

~ 6 fm boxes are needed

Two hadron bound state volume corrections $\sim e^{-\kappa L}$

Binding momentum κ of the deuteron ~ 45MeV

Nuclear energy level splittings are a few MeV

Box sizes of about 10 fm will be needed

Bound States

Luscher Comm. Math. Phys 104, 177 '86

$$E_b = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} - m_1 - m_2 \qquad p^2 < 0$$

$$E_b \approx \frac{p^2}{2\mu} = -\frac{\kappa^2}{2\mu} \qquad \kappa = |p|$$

κ is the "binding momentum" and μ the reduced mass

Finite volume corrections:

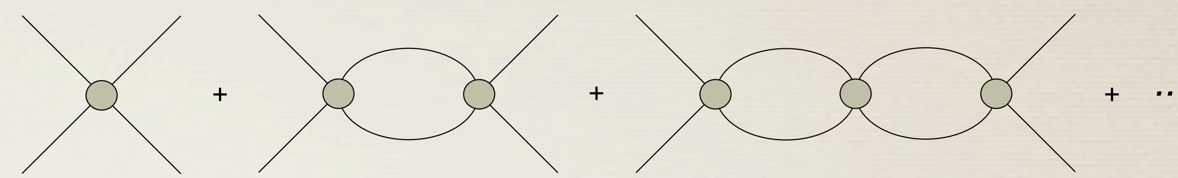
$$\Delta E_b = -3|A|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}\left(e^{-\sqrt{2}\kappa L}\right)$$

cubic group irrep: A_1^+

Scattering on the Lattice

Luscher Comm. Math. Phys 105, 153 '86

Elastic scattering amplitude (s-wave): $A(p) =$



$$A(p) = \frac{4\pi}{m} \frac{1}{p\, cot\delta \;-\; i\, p}$$

At finite volume one can show: $E_n = 2\sqrt{p_n^2 + m^2}$

$$p \cot \delta(p) \;=\; \frac{1}{\pi L}\, \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right)$$

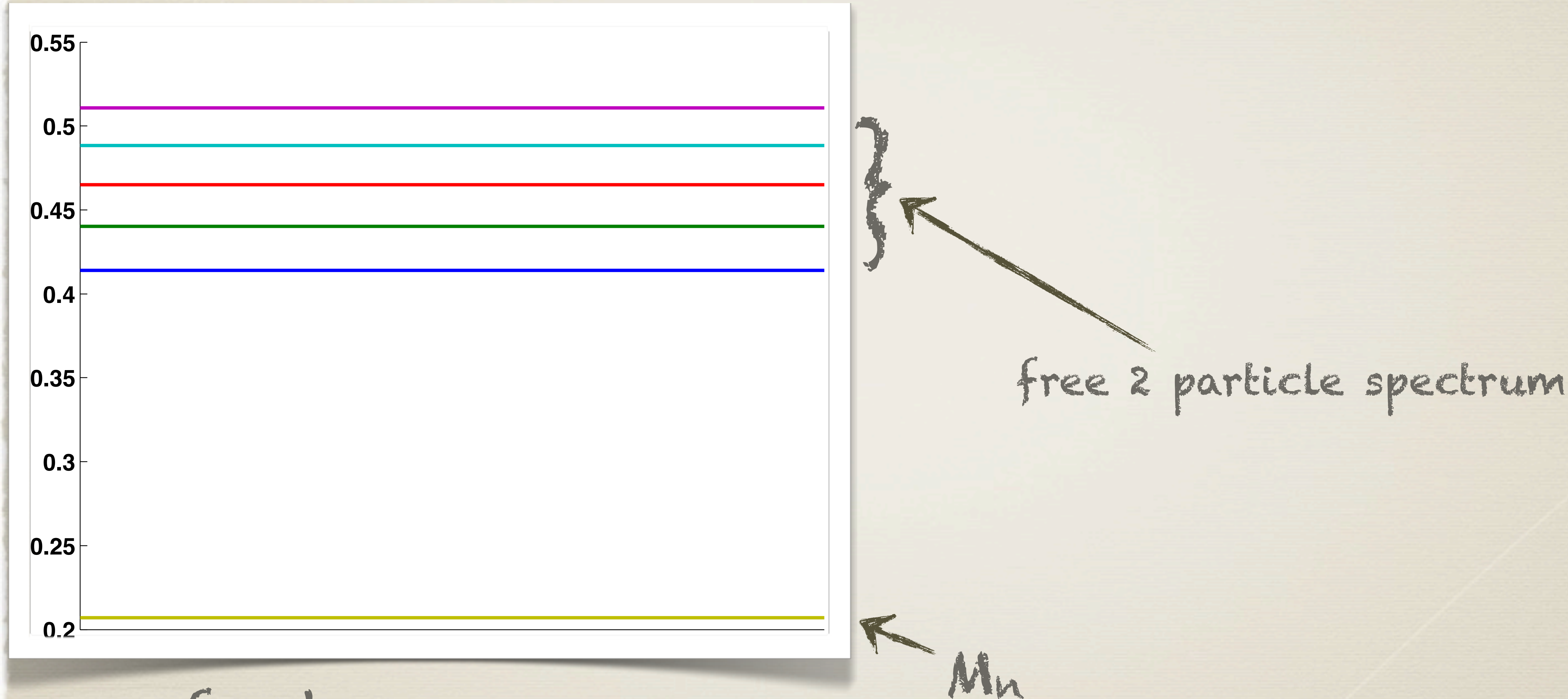
$$\mathbf{S}\,(\,\eta\,) \;\equiv\; \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} \;-\; 4\pi\Lambda$$

Small p: $p \cot \delta(p) = \frac{1}{a} + \frac{1}{2} r p^2 + ...$

a is the scattering length

Two Nucleon spectrum

free nucleons



3fm box

24^3 lattice

anisotropy factor 3.5

$M_\pi = 390$ MeV

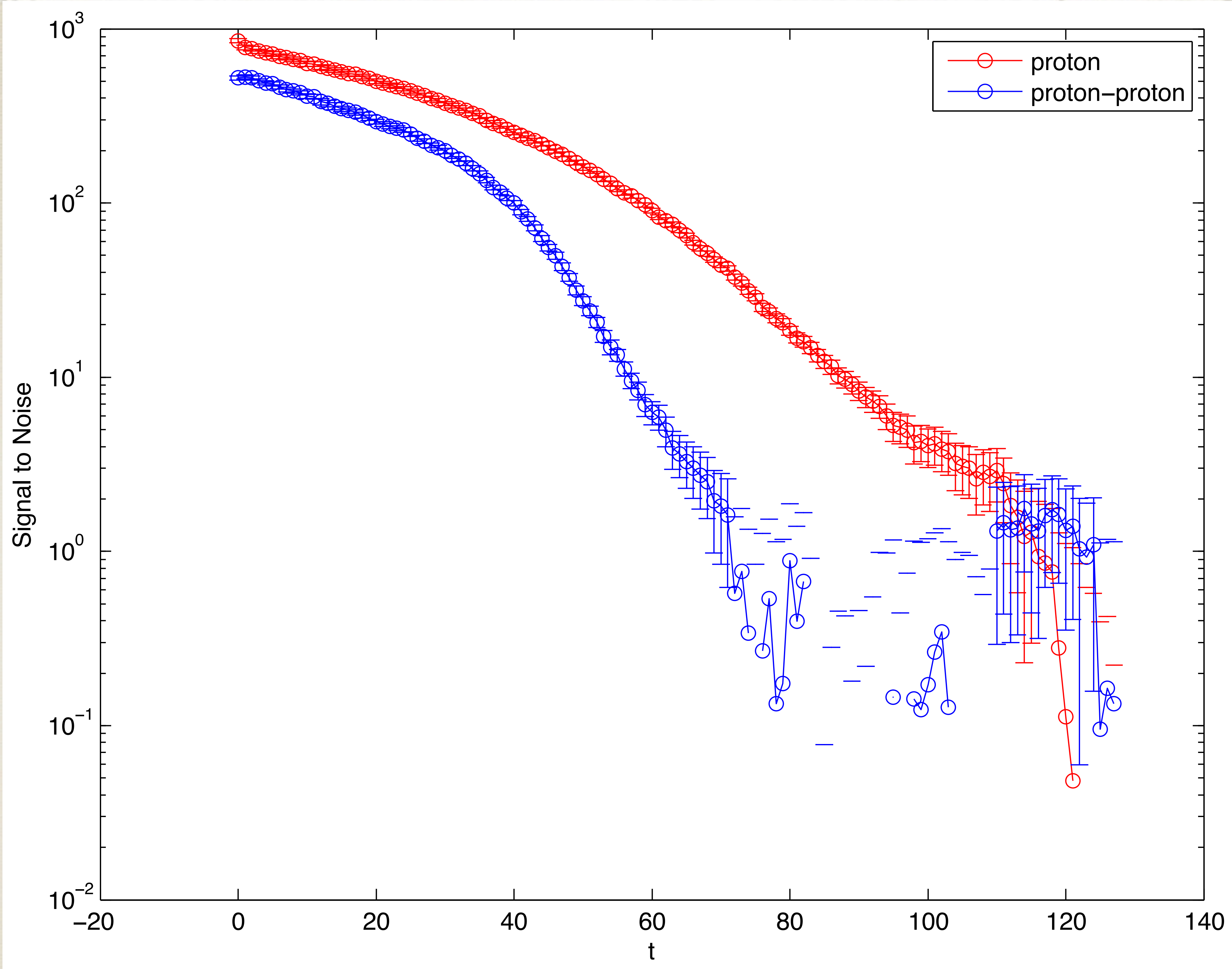
Signal to Noise ratio for correlation functions

$$C(t) = \langle N(t)\bar{N}(0)\rangle \sim Ee^{-M_N t}$$

$$var(C(t)) = \langle N\bar{N}(t)N\bar{N}(0)\rangle \sim Ae^{-2M_N t} + Be^{-3m_\pi t}$$

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} =\sim Ae^{-(M_N - 3/2m_\pi)t}$$

* The signal to noise ratio drops exponentially with time
* The signal to noise ratio drops exponentially with decreasing pion mass
* For two nucleons: $StoN(2N) = StoN(1N)^2$



Signal to Noise

$32^3 \times 256$
M_π=390MeV

anisotropy factor 3.5

NPLQCD data

Challenges for Nuclear physics

* New scales that are much smaller than characteristic QCD scale appear
* The spectrum is complex and more difficult to extract from euclidean correlators
* Construction of multi-quark correlations functions may be computationally expensive
* Monte-Carlo evaluation of correlation functions converges slowly

Challenges for Nuclear physics

* New scales that are much smaller than characteristic QCD scale appear
* The spectrum is complex and more difficult to extract from euclidean correlators
* Construction of multi-quark correlations functions may be computationally expensive
* Monte-Carlo evaluation of correlation functions converges slowly

We really need better algorithms to deal with an exponentially hard problem

Interpolating fields

Most general multi-baryon interpolating field

$$\bar{\mathcal{N}}^h = \sum_{\mathbf{a}} w_h^{a_1, a_2 \cdots a_{n_q}} \bar{q}(a_1)\bar{q}(a_2)\cdots\bar{q}(a_{n_q})$$

The indices a are composite including space, spin, color and flavor that can take N possible values

- The goal is to calculate the tensors w
- The tensors w are completely antisymmetric
- Number of terms in the sum are $$\frac{N!}{(N-n_q)!}$$

Imposing the anti-symmetry:

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

Reduced weights

Totally anti-symmetric tensor

$$\epsilon^{1,2,3,4,\cdots,n_q} = 1$$

* Total number of reduced weights: $\dfrac{N!}{n_q!(N-n_q)!}$

Hadronic Interpolating field

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{M_w} \tilde{W}_h^{(b_1,b_2\cdots b_A)} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_A} \bar{B}(b_{i_1})\bar{B}(b_{i_2})\cdots\bar{B}(b_{i_A})$$

hadronic reduced weights

baryon composite interpolating field

$$\bar{B}(b) = \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(a_1,a_2,a_3),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,i_3} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\bar{q}(a_{i_3})$$

Basak et.al. PhysRevD.72.074501 (2005)

Calculation of weights

* Compute the hadronic weights
* Replace baryons by quark interpolating fields
* Perform Grassmann reductions
* Read off the reduced weights for the quark interpolating fields
* Computations done in: `algebra (C++)`

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{M_w} \tilde{W}_h^{(b_1,b_2\cdots b_A)} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_A} \bar{B}(b_{i_1})\bar{B}(b_{i_2})\cdots\bar{B}(b_{i_A})$$

$$\downarrow$$

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \sum_{\mathbf{i}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

Interpolating fields

NPLQCD arXiv:1206.5219

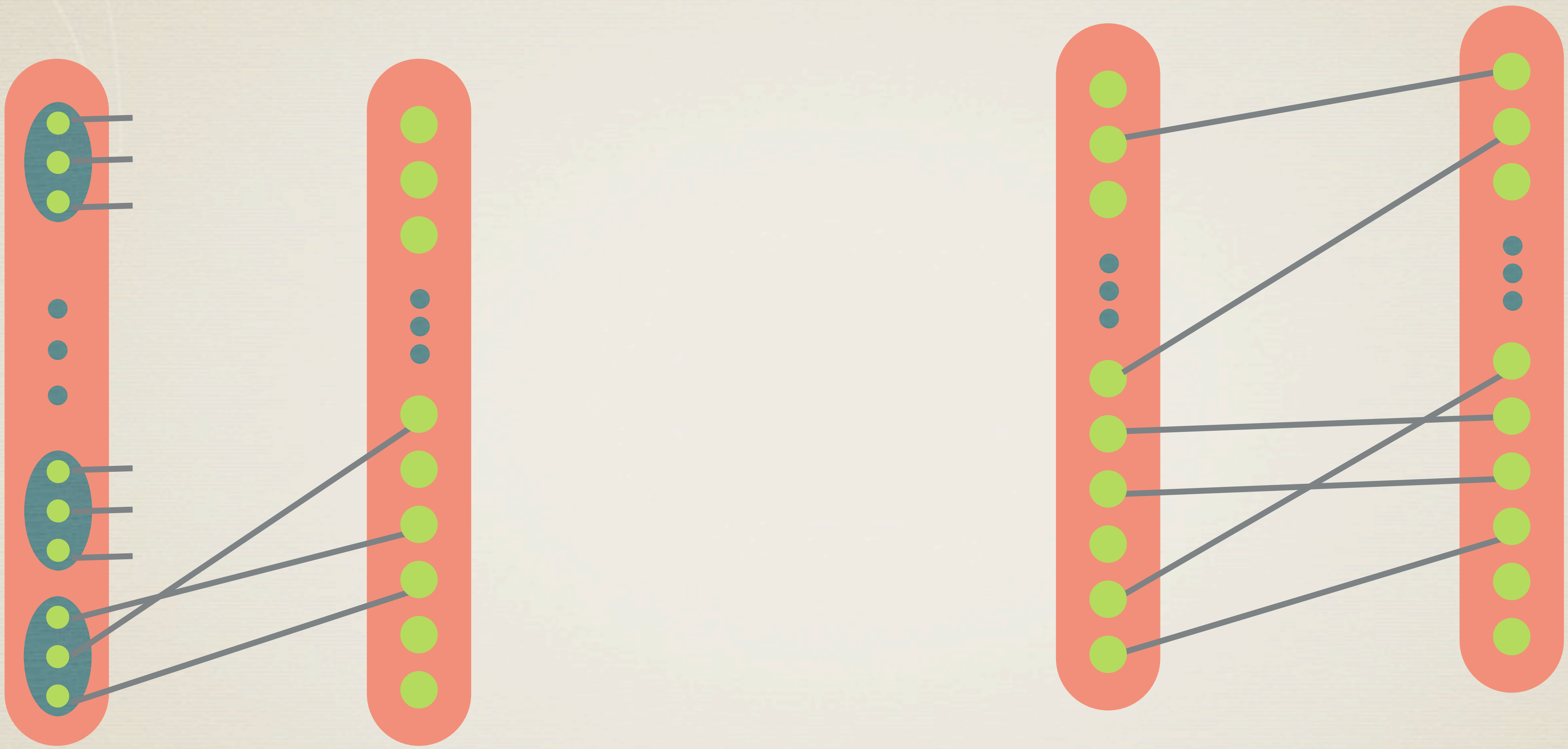
Label	A	s	I	J^π	Local SU(3) irreps	int. field size
N	1	0	1/2	$1/2^+$	$\mathbf{8}$	9
Λ	1	-1	0	$1/2^+$	$\mathbf{8}$	12
Σ	1	-1	1	$1/2^+$	$\mathbf{8}$	9
Ξ	1	-2	1/2	$1/2^+$	$\mathbf{8}$	9
d	2	0	0	1^+	$\overline{\mathbf{10}}$	21
nn	2	0	1	0^+	$\mathbf{27}$	21
$n\Lambda$	2	-1	1/2	0^+	$\mathbf{27}$	96
$n\Lambda$	2	-1	1/2	1^+	$\mathbf{8}_A$, $\overline{\mathbf{10}}$	48, 75
$n\Sigma$	2	-1	3/2	0^+	$\mathbf{27}$	42
$n\Sigma$	2	-1	3/2	1^+	$\mathbf{10}$	27
$n\Xi$	2	-2	0	1^+	$\mathbf{8}_A$	96
$n\Xi$	2	-2	1	1^+	$\mathbf{8}_A$, $\mathbf{10}$, $\overline{\mathbf{10}}$	52,66,75
H	2	-2	0	0^+	$\mathbf{1}$, $\mathbf{27}$	90,132
^{3}H, ^{3}He	3	0	1/2	$1/2^+$	$\overline{\mathbf{35}}$	9
$^3_\Lambda$H$(1/2^+)$	3	-1	0	$1/2^+$	$\overline{\mathbf{35}}$	66
$^3_\Lambda$H$(3/2^+)$	3	-1	0	$3/2^+$	$\overline{\mathbf{10}}$	30
$^3_\Lambda$He, $^3_\Lambda\tilde{\mathrm{H}}$, $nn\Lambda$	3	-1	1	$1/2^+$	$\mathbf{27}$, $\overline{\mathbf{35}}$	30,45
$^3_\Sigma$He	3	-1	1	$3/2^+$	$\mathbf{27}$	21
^{4}He	4	0	0	0^+	$\overline{\mathbf{28}}$	1
$^4_\Lambda$He, $^4_\Lambda$H	4	-1	1/2	0^+	$\overline{\mathbf{28}}$	6
$^4_{\Lambda\Lambda}$He	4	-2	1	0^+	$\mathbf{27}$, $\overline{\mathbf{28}}$	15, 18
$\Lambda\Xi^0 pnn$	5	-3	0	$3/2^+$	$\overline{\mathbf{10}}$ + ...	1

Interpolating fields

NPLQCD arXiv:1206.5219

Label	A	s	I	J^π	Local SU(3) irreps	int. field size
N	1	0	1/2	$1/2^+$	$\mathbf{8}$	9
Λ	1	-1	0	$1/2^+$	$\mathbf{8}$	12
Σ	1	-1	1	$1/2^+$	$\mathbf{8}$	9
Ξ	1	-2	1/2	$1/2^+$	$\mathbf{8}$	9
d	2	0	0	1^+	$\overline{\mathbf{10}}$	21
nn	2	0	1	0^+	$\mathbf{27}$	21
$n\Lambda$	2	-1	1/2	0^+	$\mathbf{27}$	96
$n\Lambda$	2	-1	1/2	1^+	$\mathbf{8}_A$, $\overline{\mathbf{10}}$	48, 75
$n\Sigma$	2	-1	3/2	0^+	$\mathbf{27}$	42
$n\Sigma$	2	-1	3/2	1^+	$\mathbf{10}$	27
$n\Xi$	2	-2	0	1^+	$\mathbf{8}_A$	96
$n\Xi$	2	-2	1	1^+	$\mathbf{8}_A$, $\mathbf{10}$, $\overline{\mathbf{10}}$	52,66,75
H	2	-2	0	0^+	$\mathbf{1}$, $\mathbf{27}$	90,132
^{3}H, ^{3}He	3	0	1/2	$1/2^+$	$\overline{\mathbf{35}}$	9
$^3_\Lambda$H$(1/2^+)$	3	-1	0	$1/2^+$	$\overline{\mathbf{35}}$	66
$^3_\Lambda$H$(3/2^+)$	3	-1	0	$3/2^+$	$\overline{\mathbf{10}}$	30
$^3_\Lambda$He, $^3_\Lambda\tilde{\text{H}}$, $nn\Lambda$	3	-1	1	$1/2^+$	$\mathbf{27}$, $\overline{\mathbf{35}}$	30,45
$^3_\Sigma$He	3	-1	1	$3/2^+$	$\mathbf{27}$	21
^{4}He	4	0	0	0^+	$\overline{\mathbf{28}}$	1
$^4_\Lambda$He, $^4_\Lambda$H	4	-1	1/2	0^+	$\overline{\mathbf{28}}$	6
$^4_{\Lambda\Lambda}$He	4	-2	1	0^+	$\mathbf{27}$, $\overline{\mathbf{28}}$	15, 18
$\Lambda\Xi^0 pnn$	5	-3	0	$3/2^+$	$\overline{\mathbf{10}}$ + ...	1

Contraction methods



* quark to hadronic interpolating fields
* quark to quark interpolating fields

Quarks to Quarks

$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)] = \int \mathcal{D}q\mathcal{D}\bar{q}\; e^{-\mathcal{S}_{QCD}} \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h'^{(a'_1,a'_2\cdots a'_{n_q}),k'}\;\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\times \sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} q(a'_{j_{n_q}})\cdots q(a'_{j_2})q(a'_{j_1}) \times \bar{q}(a_{i_1})\bar{q}(a_{i_2})\cdots\bar{q}(a_{i_{n_q}})$$

$$= \; e^{-\mathcal{S}_{eff}} \;\; \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h'^{(a'_1,a'_2\cdots a'_{n_q}),k'}\;\; \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times$$

$$\times \sum_{\mathbf{j}}\sum_{\mathbf{i}} \epsilon^{j_1,j_2,\cdots,j_{n_q}} \epsilon^{i_1,i_2,\cdots,i_{n_q}} S(a'_{j_1};a_{i_1})S(a'_{j_2};a_{i_2})\cdots S(a'_{j_{n_q}};a_{i_{n_q}})$$

Define the matrix:

$$G(j,i)^{(a'_1,a'_2\cdots a'_{n_q});(a_1,a_2\cdots a_{n_q})} = S(a'_j;a_i)$$

Quarks to Quarks

The matrix:

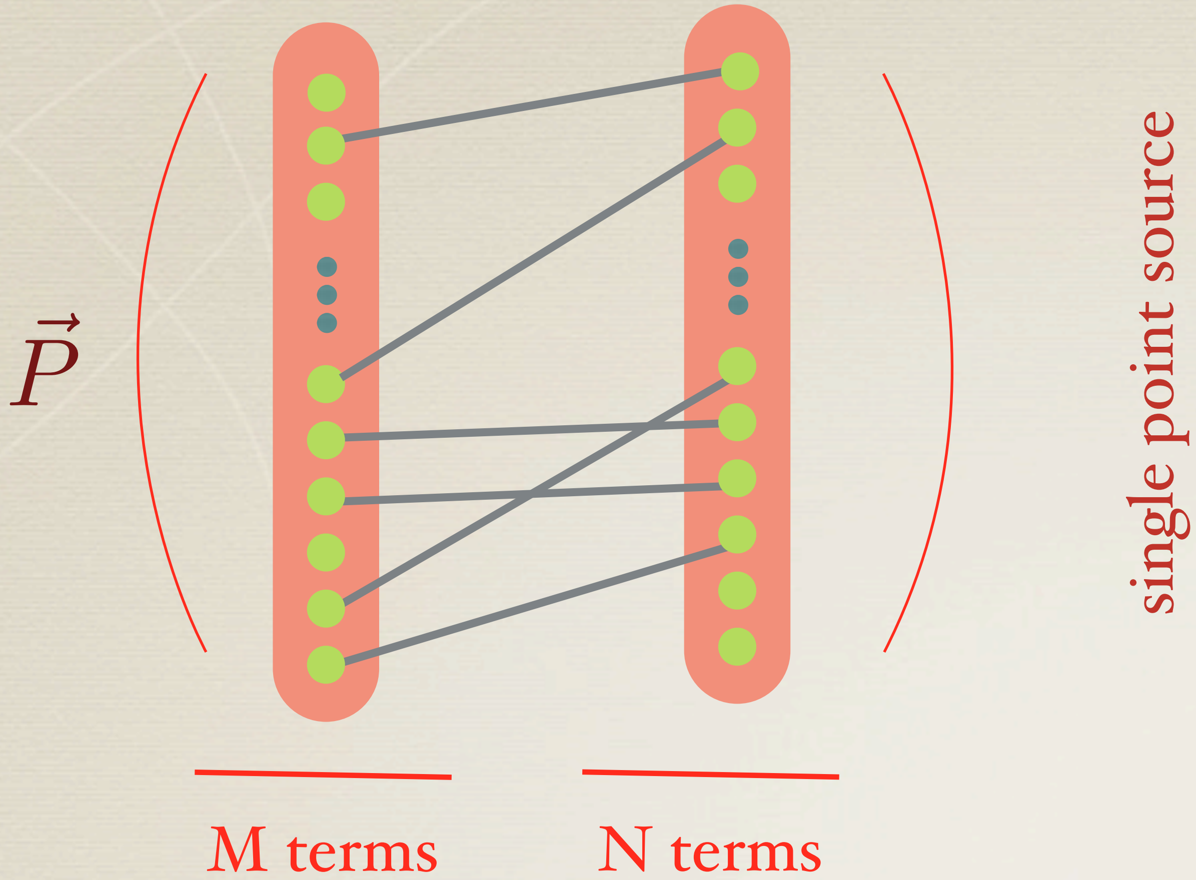
$$G(j,i)^{(a'_1,a'_2\cdots a'_{n_q});(a_1,a_2\cdots a_{n_q})} = S(a'_j;a_i)$$

The Correlation function:

$$[\mathcal{N}_1^h(t)\bar{\mathcal{N}}_2^h(0)] = \sum_{k'=1}^{N'_w}\sum_{k=1}^{N_w} \tilde{w}_h^{\prime(a'_1,a'_2\cdots a'_{n_q}),k'}\ \tilde{w}_h^{(a_1,a_2\cdots a_{n_q}),k} \times \left| G^{(a'_1,a'_2\cdots a'_{n_q});(a_1,a_2\cdots a_{n_q})} \right|$$

Total momentum projection is implicit in the above

Quarks to Quarks



Naive Cost: $n_u! n_d! n_s! \times NM$

Actual Cost: $n_u^3 n_d^3 n_s^3 \times MN$

- Loop over all source and sink terms
- Compute the determinant for each flavor
- Cost is polynomial in quark number

H-dibaryon

R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977)

- Proposed by R. Jaffe 1977
  - Perturbative color-spin interactions are attractive for (uuddss)
  - Diquark picture of scalar diquarks (ud)(ds)(su)
- Experimental searches of the H have not found it
  - BNL RHIC (+model): Excludes the region [-95, 0] MeV
    A. L. Trattner, PhD Thesis, LBL, UMI-32-54109 (2006).
  - KEK: Resonance near threshold
    C. J. Yoon et al., Phys. Rev. C 75, 022201 (2007).
- Several Lattice QCD calculations have been addressing the existence of a bound H

$S = -2$,
$B = 2$,
$J^P = 0^+$

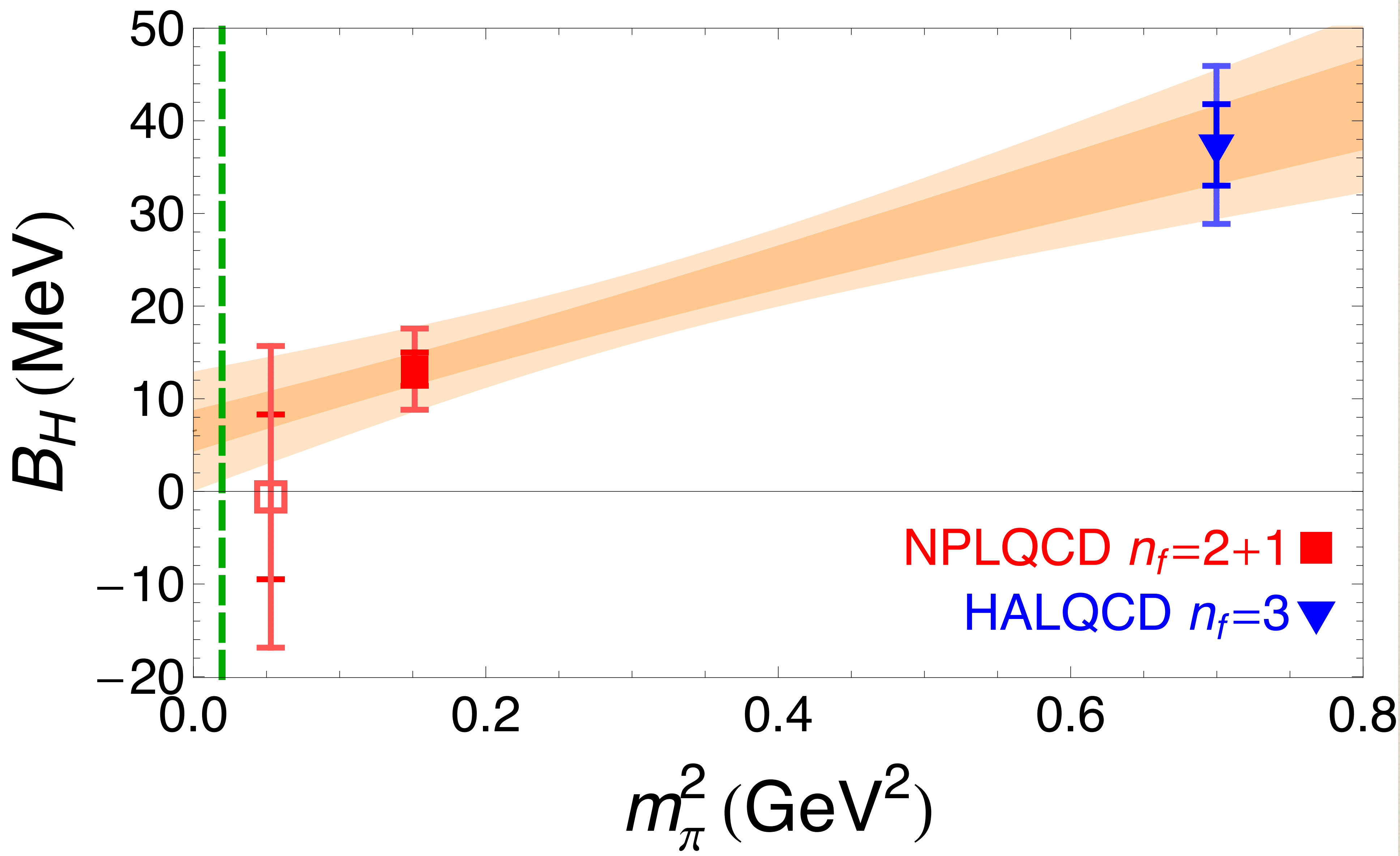
NPLQCD: lattice set up

Hadron Spectrum/JLAB

* Anisotropic 2+1 clover fermion lattices
  * a ~ 0.125fm (anisotropy of ~ 3.5)
  * pion mass ~ 390 MeV
  * Volumes 16^3 x 128, 20^3 x 128, 24^3 x 128, 32^3 x 256 — largest box 4fm
  * Smeared source - 3 sink interpolating fields
* Interpolating fields have the structure of s-wave Λ-Λ system
  * I=0, S=-2, A_1 , positive parity

H-dibaryon: Towards the physical point

[S. Beane et.al. arXiv:1103.2821 Mod. Phys. Lett. A26: 2587, 2011]



H-dibaryon:

Is bound at heavy quark masses.
May be unbound at the physical point

HALQCD:Phys.Rev.Lett.106:162002,2011

ChiPT studies indicate the same trend:
P. Shanahan et.al. arXiv:1106.2851
J. Haidenbauer, Ulf-G. Meisner arXiv:1109.3590

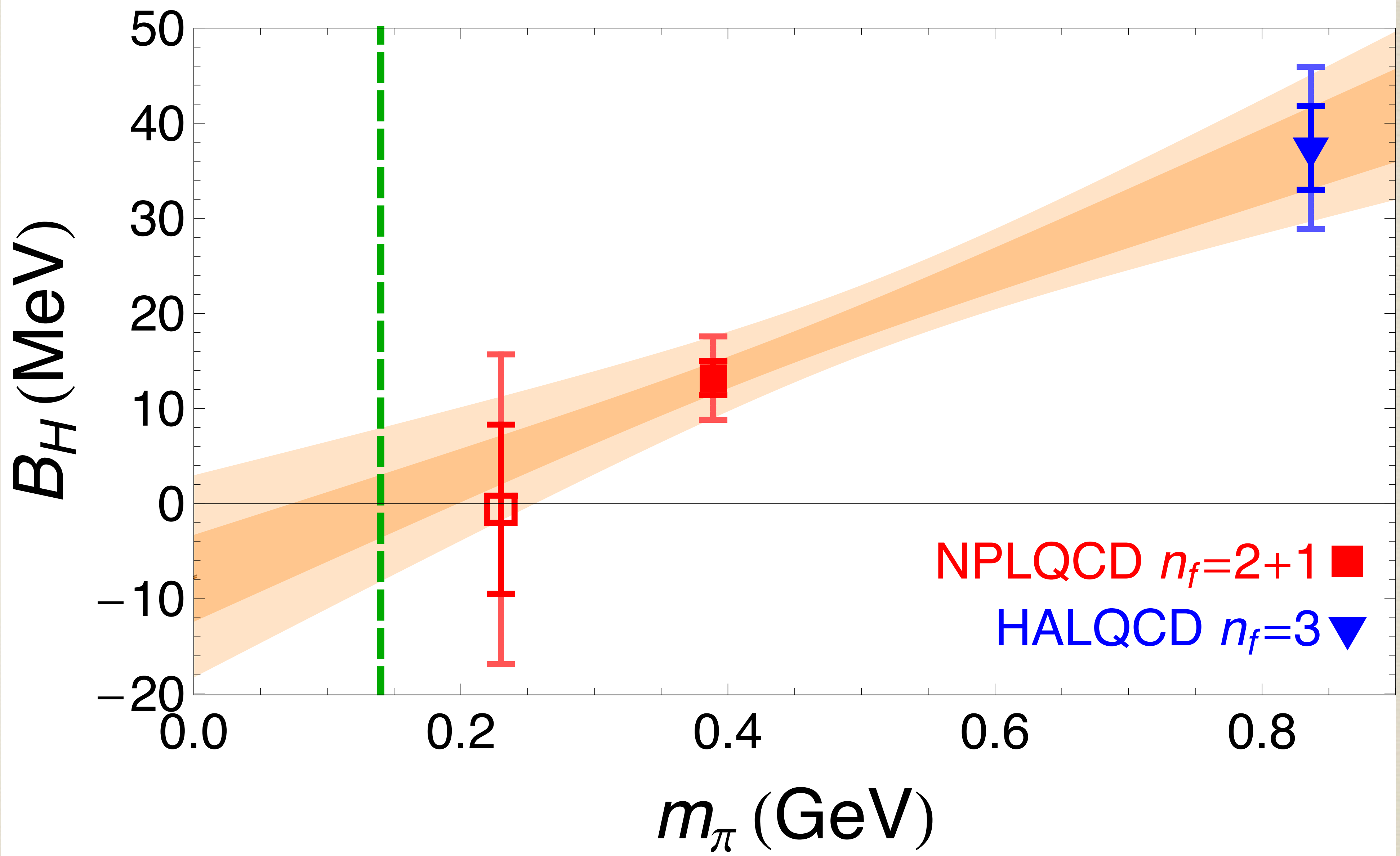
NPLQCD

H-dibaryon: Towards the physical point

[S. Beane et.al. arXiv:1103.2821 Mod. Phys. Lett. A26: 2587, 2011]

H-dibaryon:

Is bound at heavy quark masses.
May be unbound at the physical point



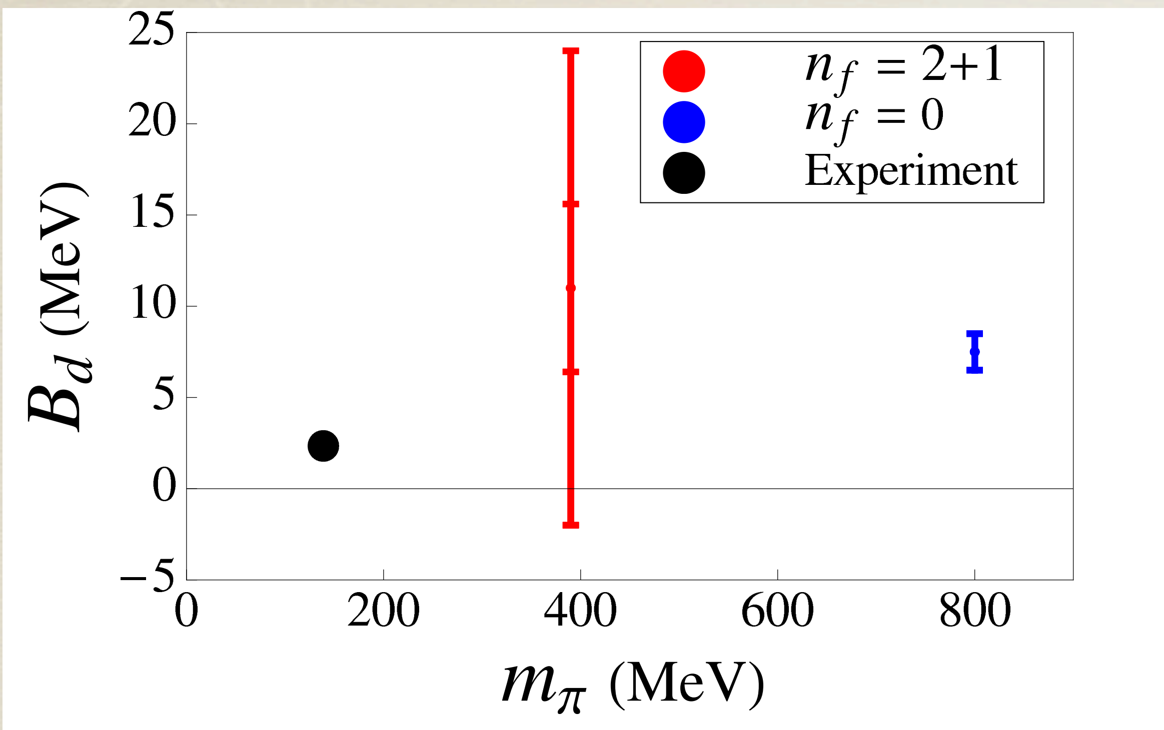
HALQCD:Phys.Rev.Lett.106:162002,2011

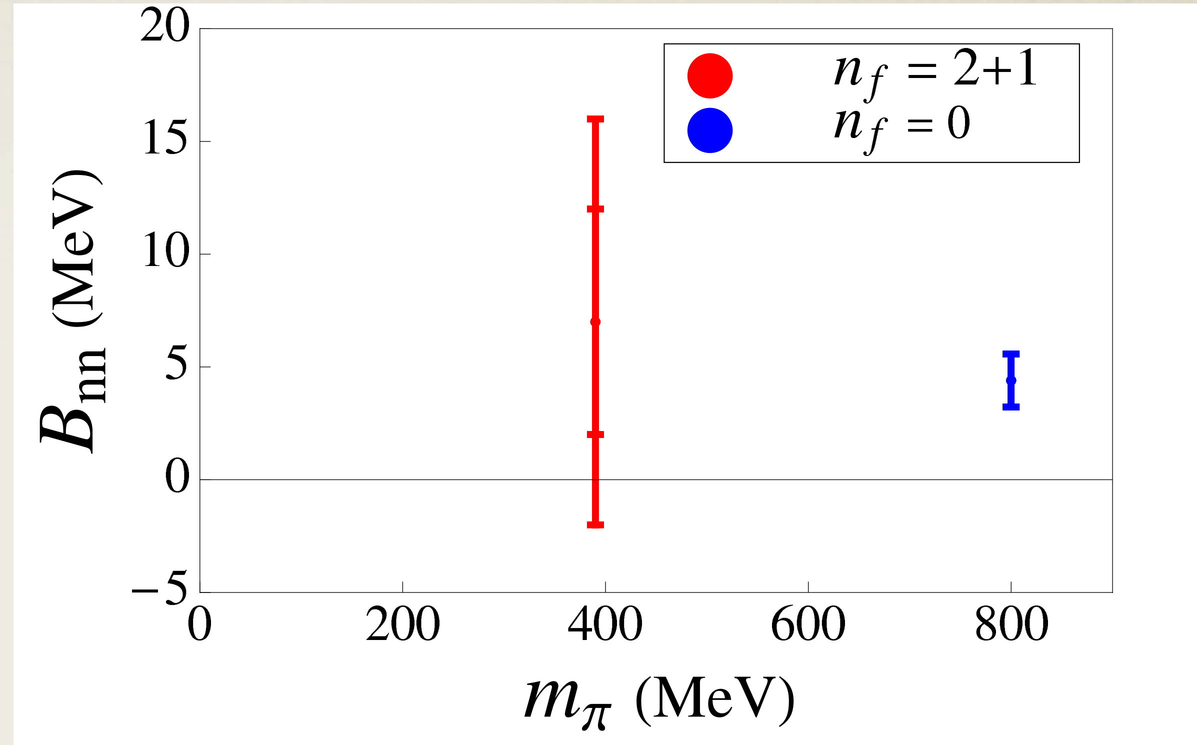
ChiPT studies indicate the same trend:
P. Shanahan et.al. arXiv:1106.2851
J. Haidenbauer, Ulf-G. Meisner arXiv:1109.3590

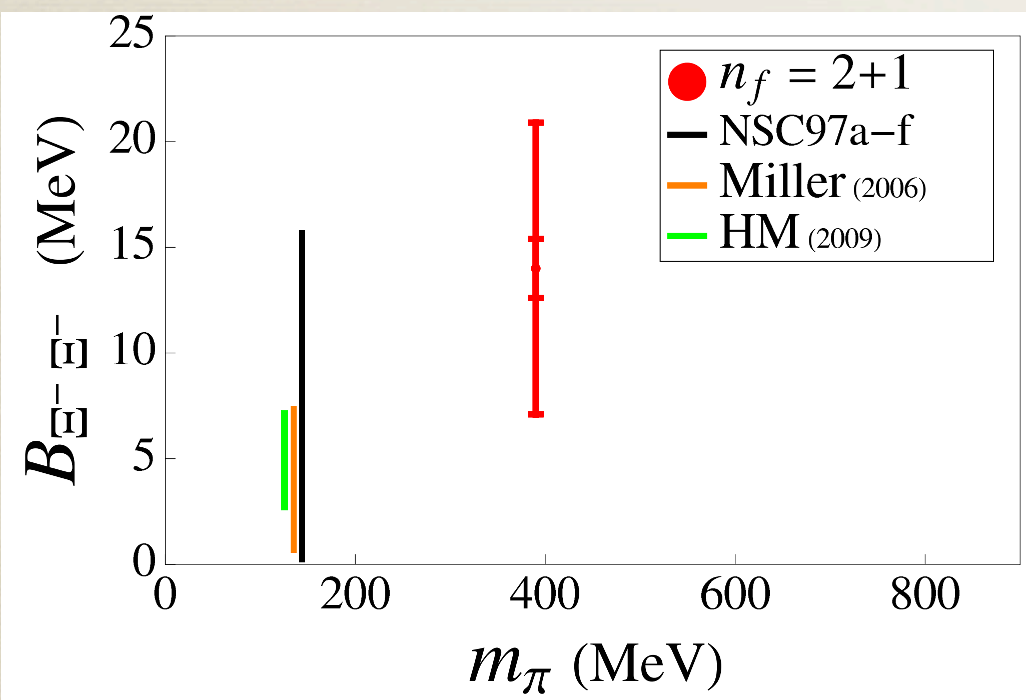
NPLQCD

Two baryon bound states

[S. Beane et.al. arXiv:1108.2889 submitted to Phys.Rev.D]







V. G. J. Stoks and T. A. Rijken
Phys. Rev. C 59, 3009 (1999)
[arXiv:nucl-th/9901028

G. A. Miller,
arXiv:nucl-th/0607006

J. Haidenbauer, Ulf-G. Meisner
Phys.Lett.B684,275-280(2010)
arXiv:0907.1395

n_f=0:
Yamazaki, Kuramashi, Ukawa
Phys.Rev. D84 (2011) 054506
arXiv: 1105.1418

gauge fields 2+1 flavors (JLab)
anisotropic clover m_π~ 390MeV

NPLQCD

Avoiding the noise

Work with heavy quarks

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} =\sim Ae^{-(M_N - 3/2 m_\pi)t}$$

Lattice Setup

* Isotropic Clover Wilson with LW gauge action
  * Stout smeared (1-level)
  * Tadpole improved
* SU(3) symmetric point
  * Defined using m_π/m_Ω
* Lattice spacing 0.145fm
  * Set using Υ spectroscopy
* Large volumes
  * 24^3 x 48 32^3 x 48 48^3 x 64
  * 3.5fm   4.5fm   7.0fm

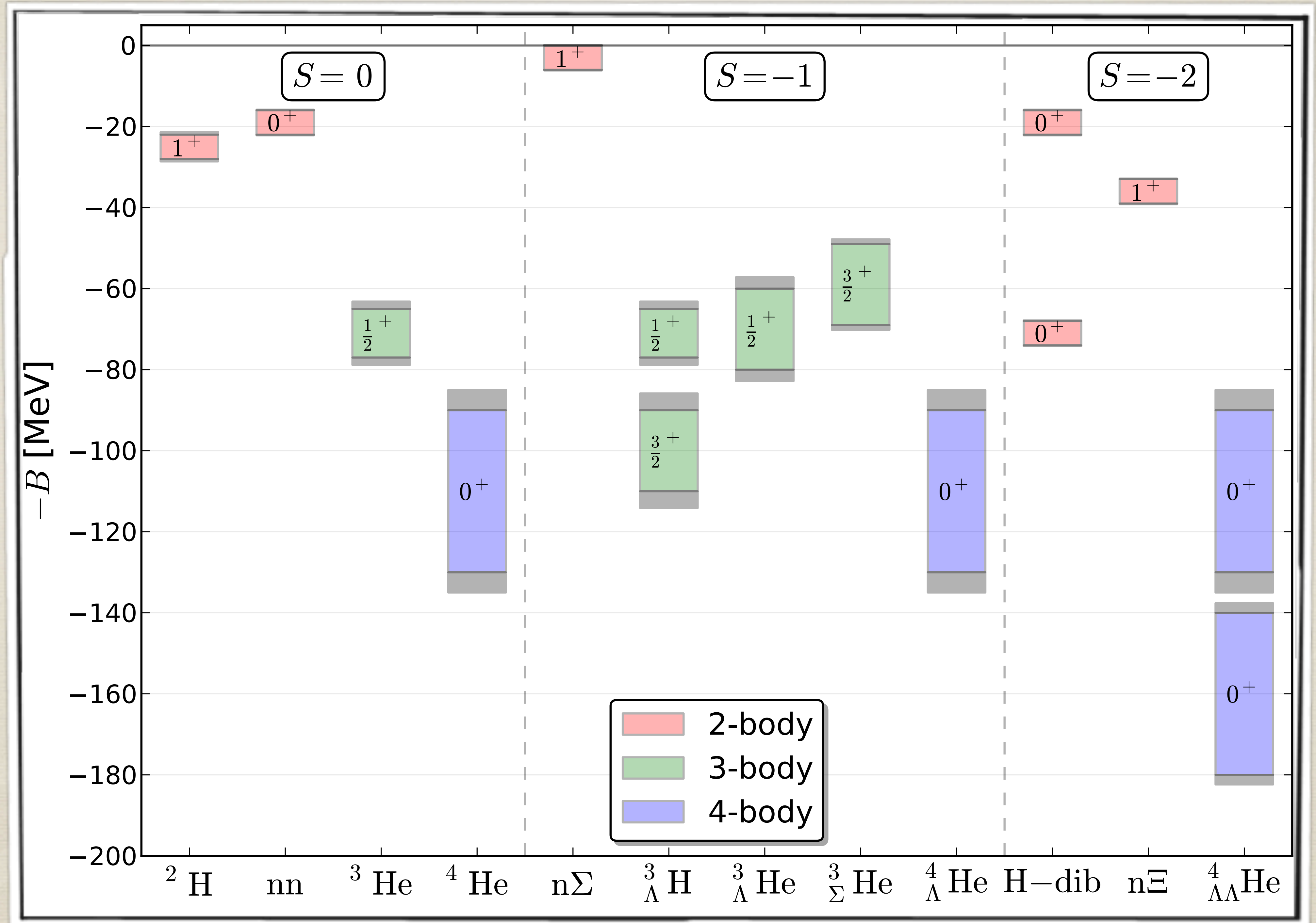
NPLQCD arXiv:1206.5219

6000 configurations,
200 correlation functions per configuration

computer time: XSEDE/NERSC

Nuclear spectrum

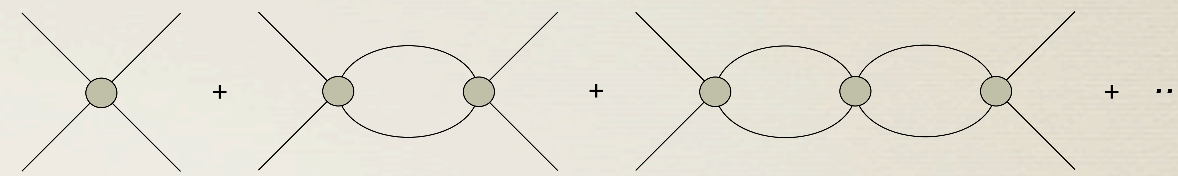
NPLQCD



Nucleon Phase shifts

Luscher Comm. Math. Phys 105, 153 ’86

Elastic scattering amplitude (s-wave): $A(p) =$



$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \; - \; i\,p}$$

At finite volume one can show: $E_n = 2\sqrt{p_n^2 + m^2}$

$$p \cot \delta(p) \;=\; \frac{1}{\pi L}\, \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} \; - \; 4\pi\Lambda$$

Small p: $p \cot \delta(p) = \frac{1}{a} + \frac{1}{2} r p^2 + ...$

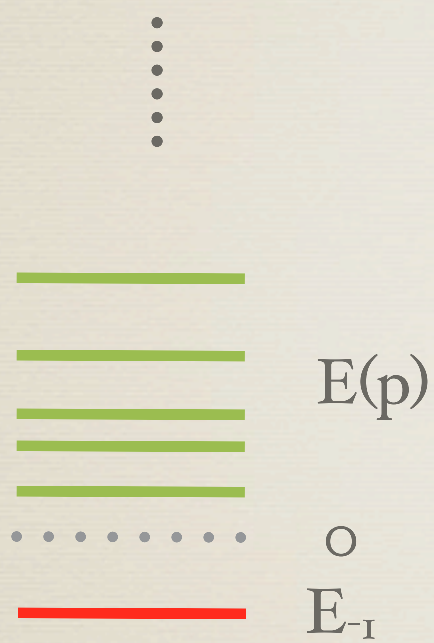
a is the scattering length

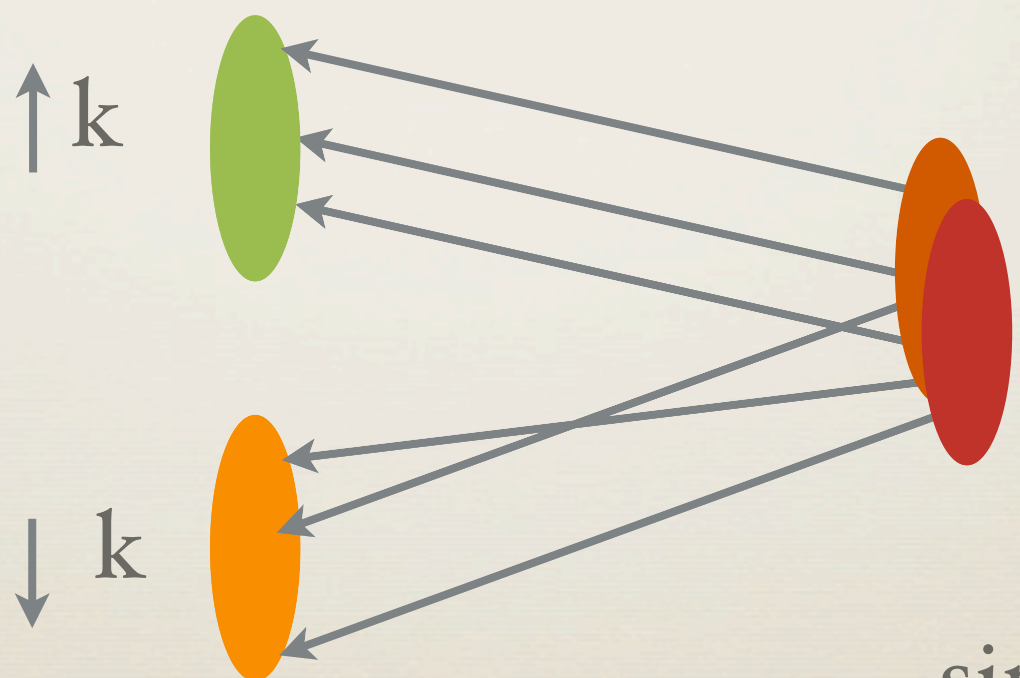
Two Body spectrum in a box

$$E(p) = 2\sqrt{p^2 + m^2} - 2m$$

$$p\, cot\delta(p) = S(\frac{p^2 L^2}{4\pi^2})$$

$$p\, cot\delta(p) = \frac{1}{a} + r^2 p^2 + \cdots$$



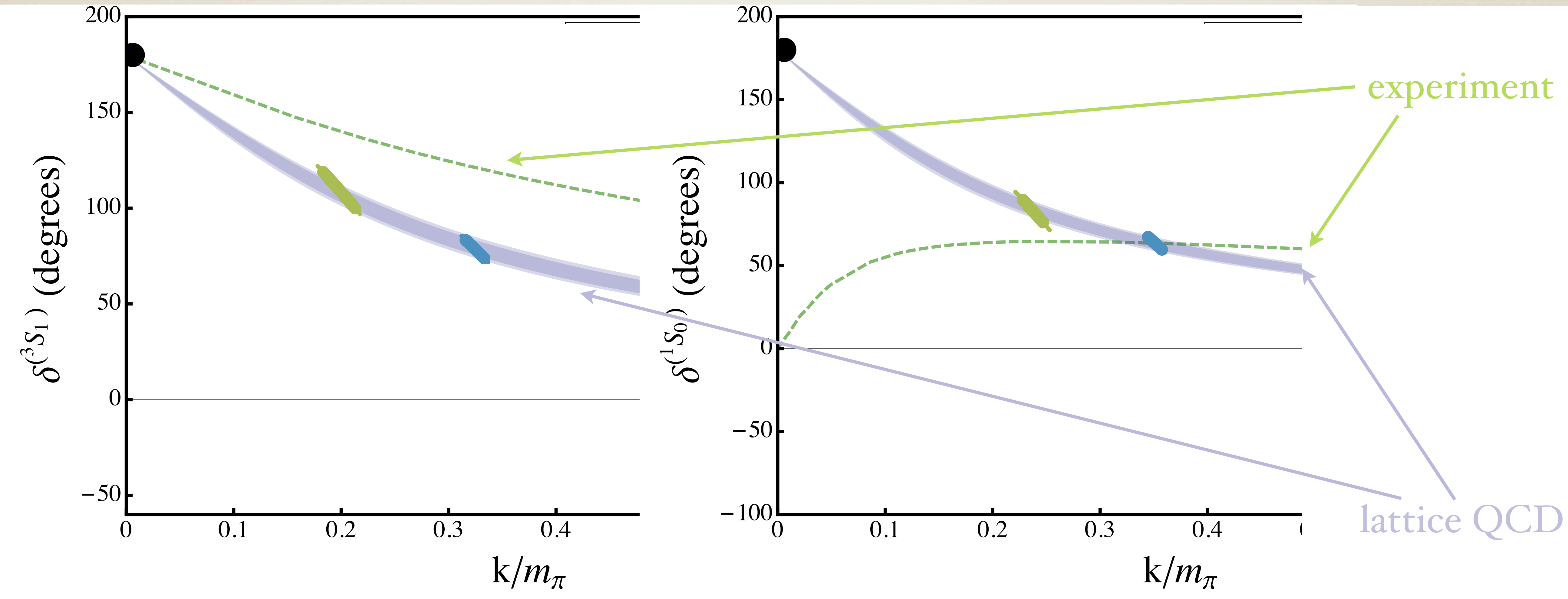


single point source

"back to back momentum"

nucleon-nucleon phase shifts

NPLQCD



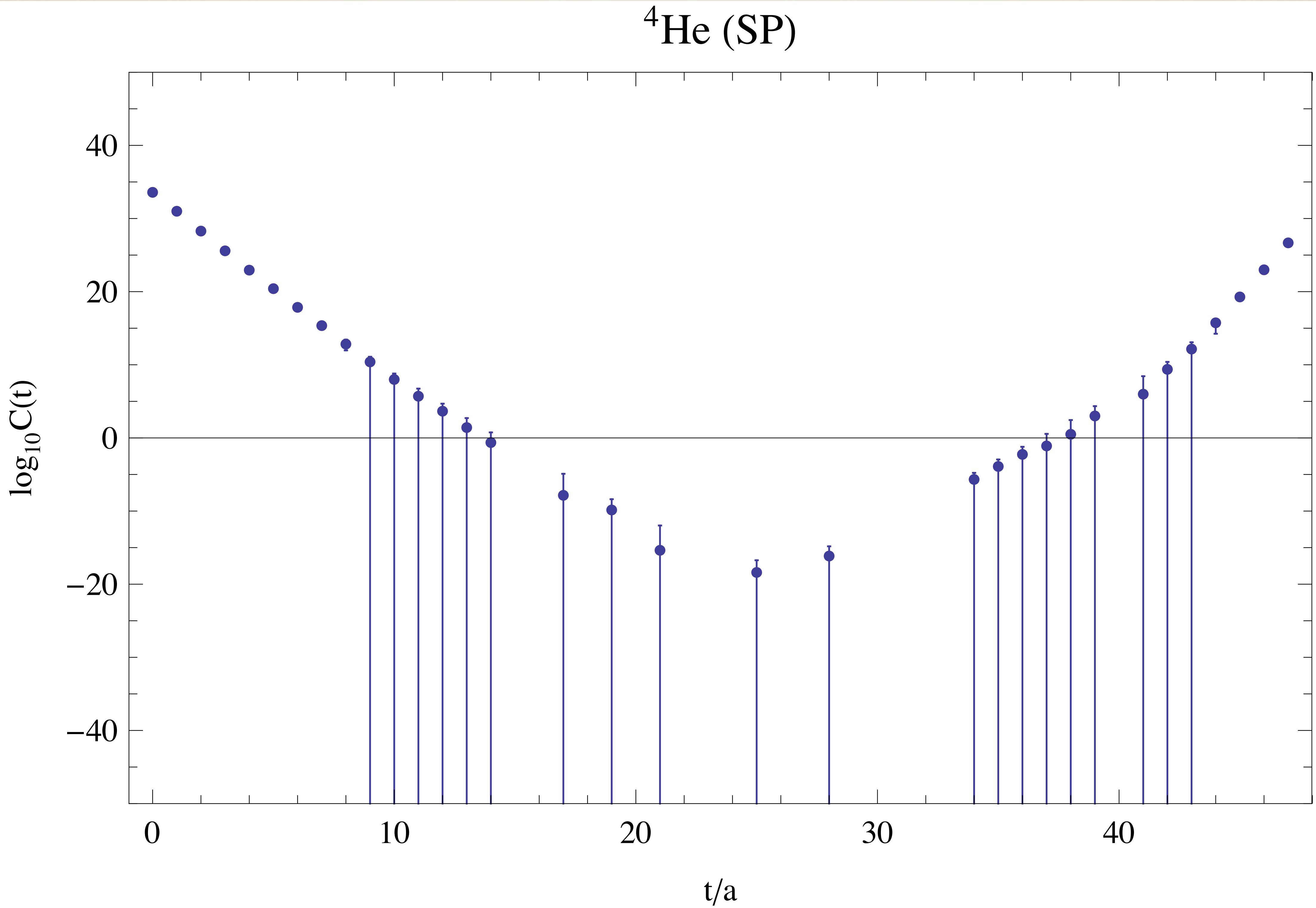
spin triplet

spin singlet

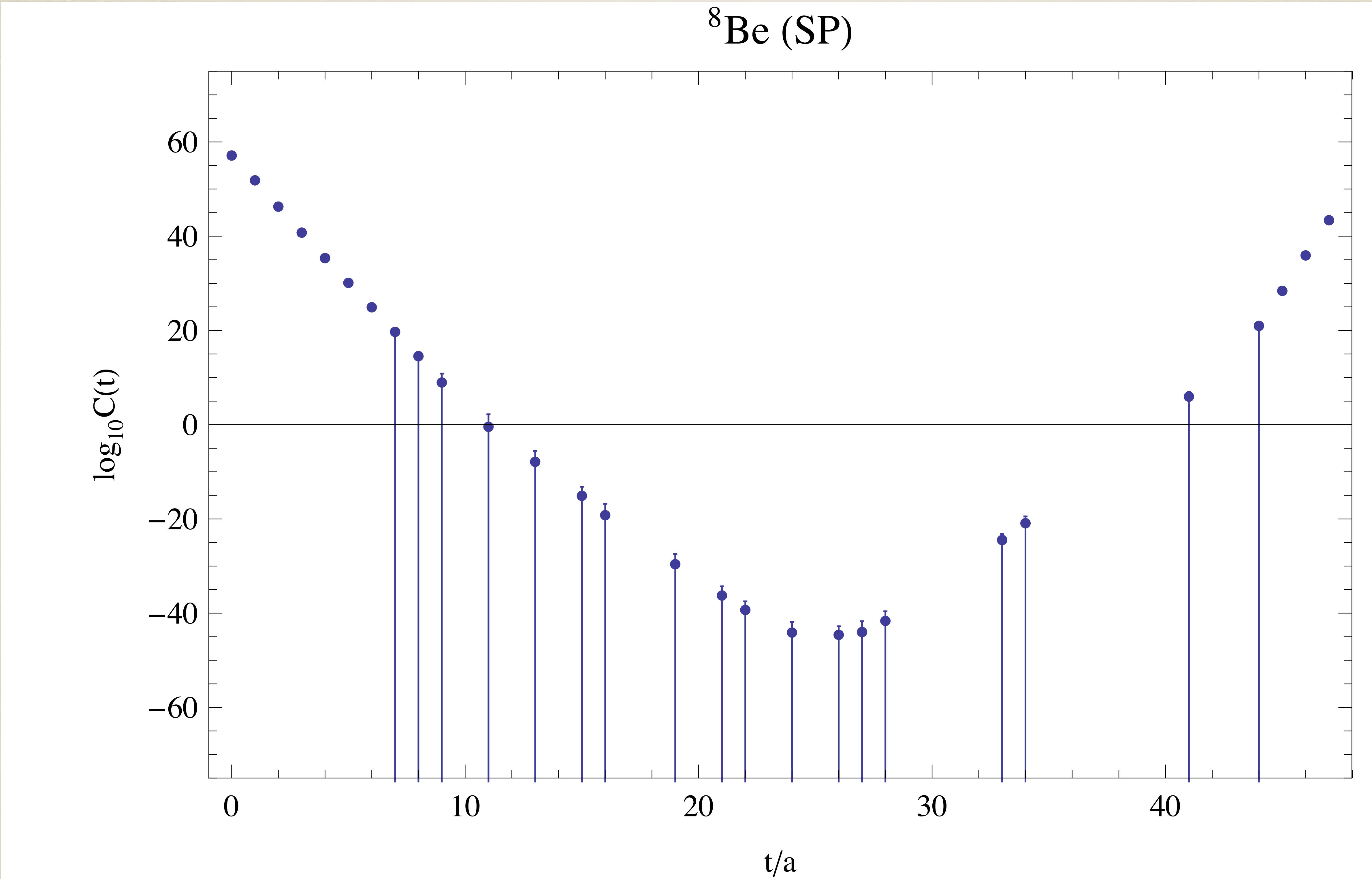
M_π=800MeV

degenerate up down and strange quarks

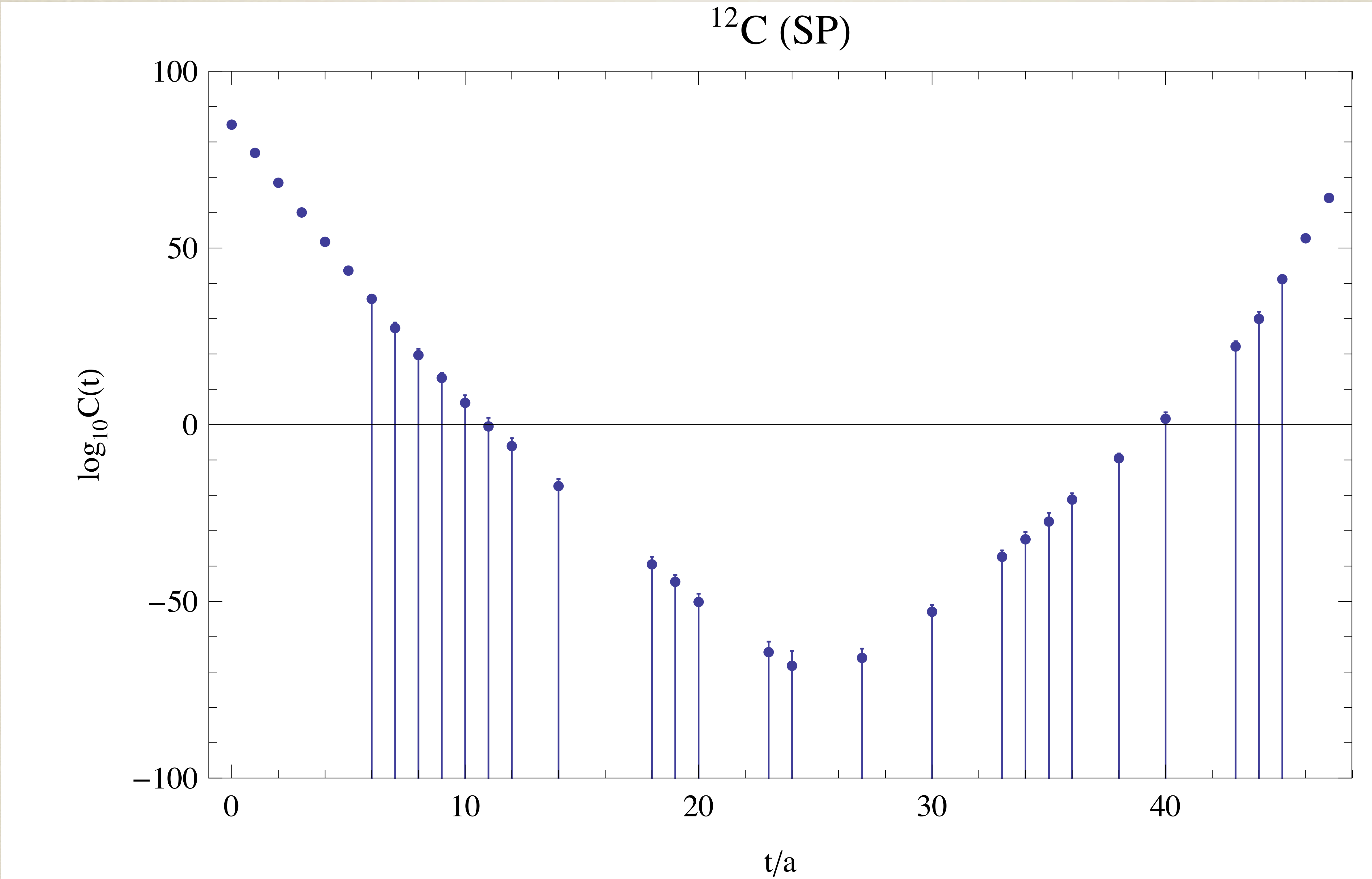
Correlators for large nucleii



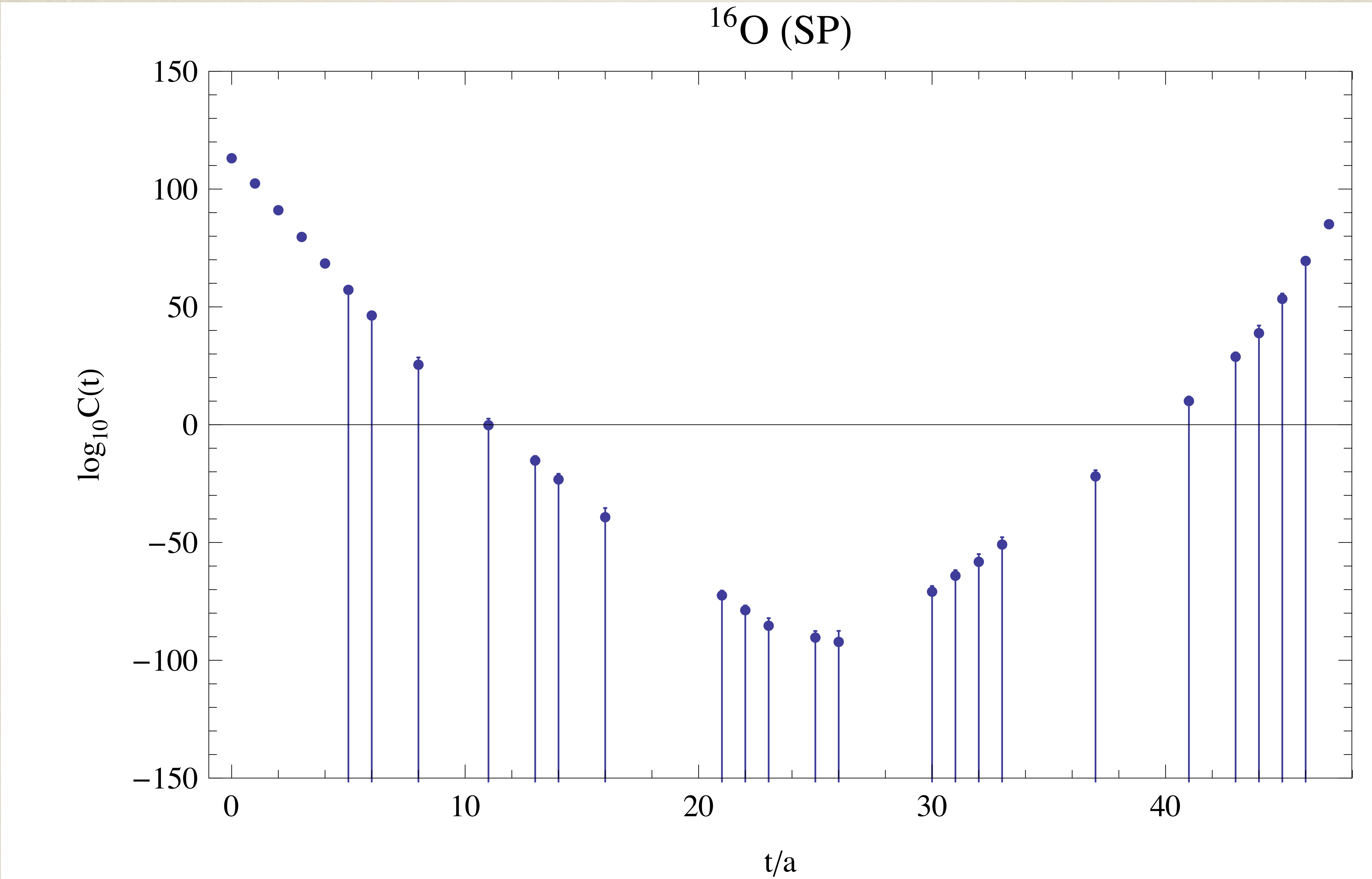
Correlators for large nucleii



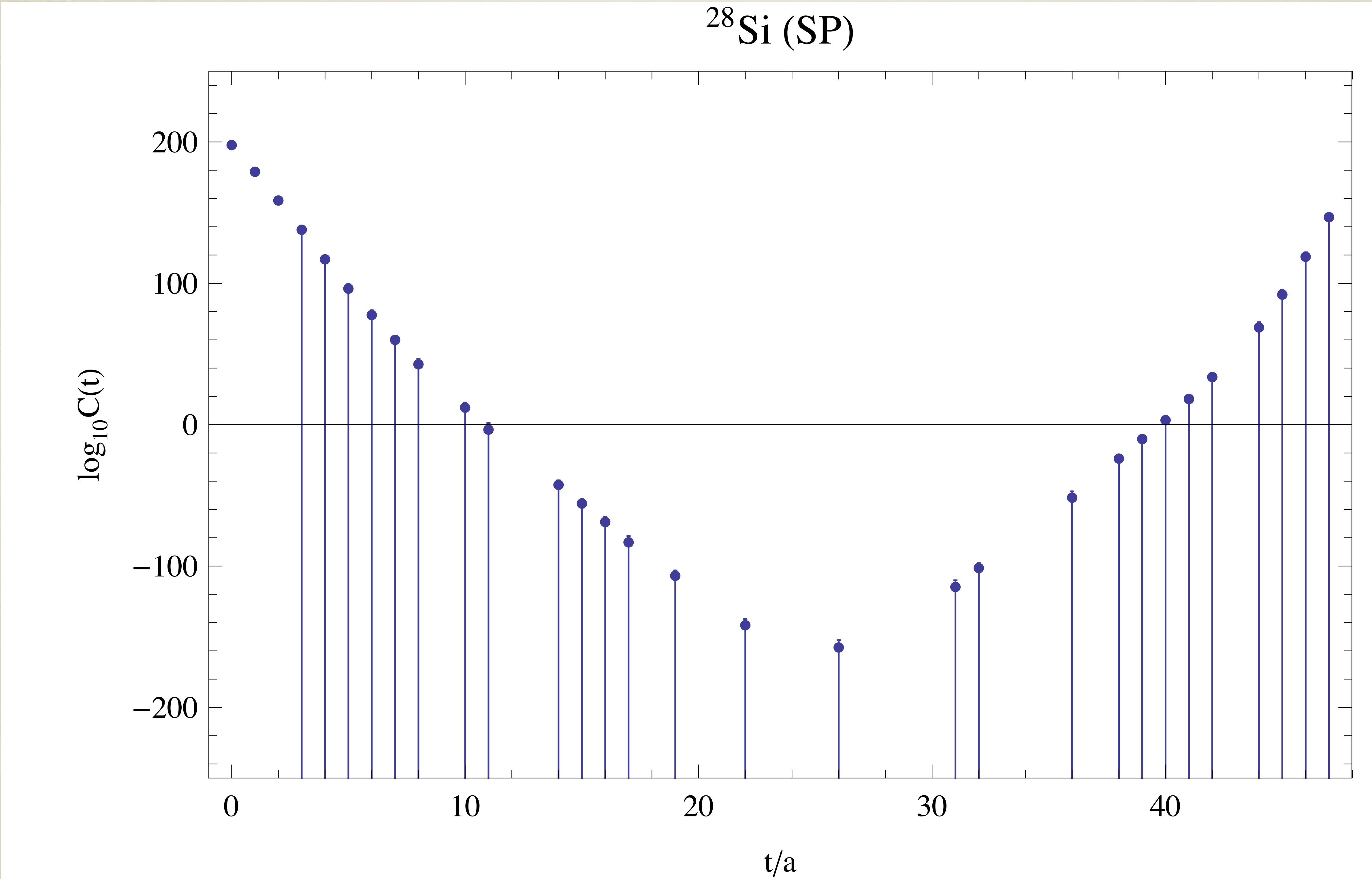
Correlators for large nucleii



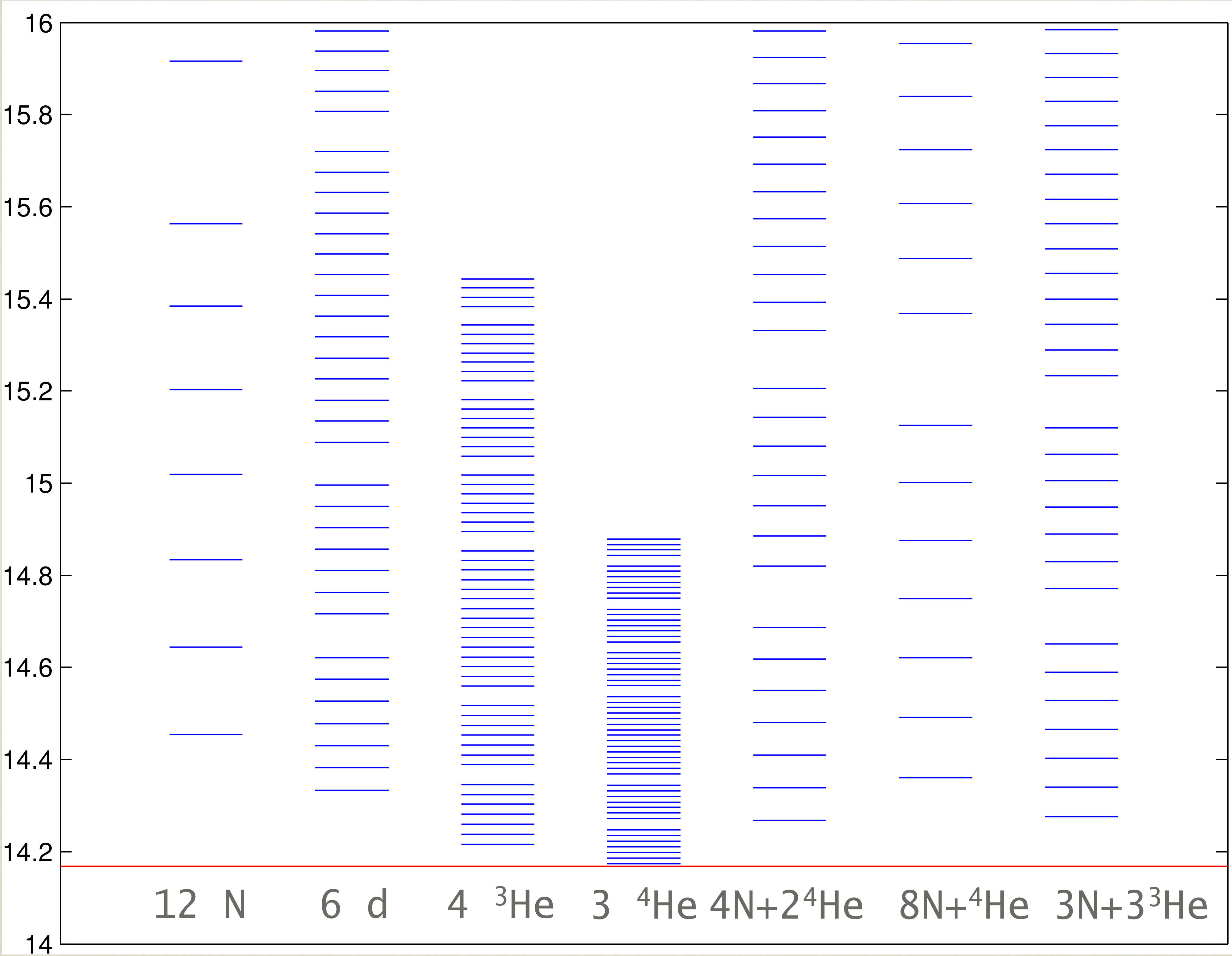
Correlators for large nucleii



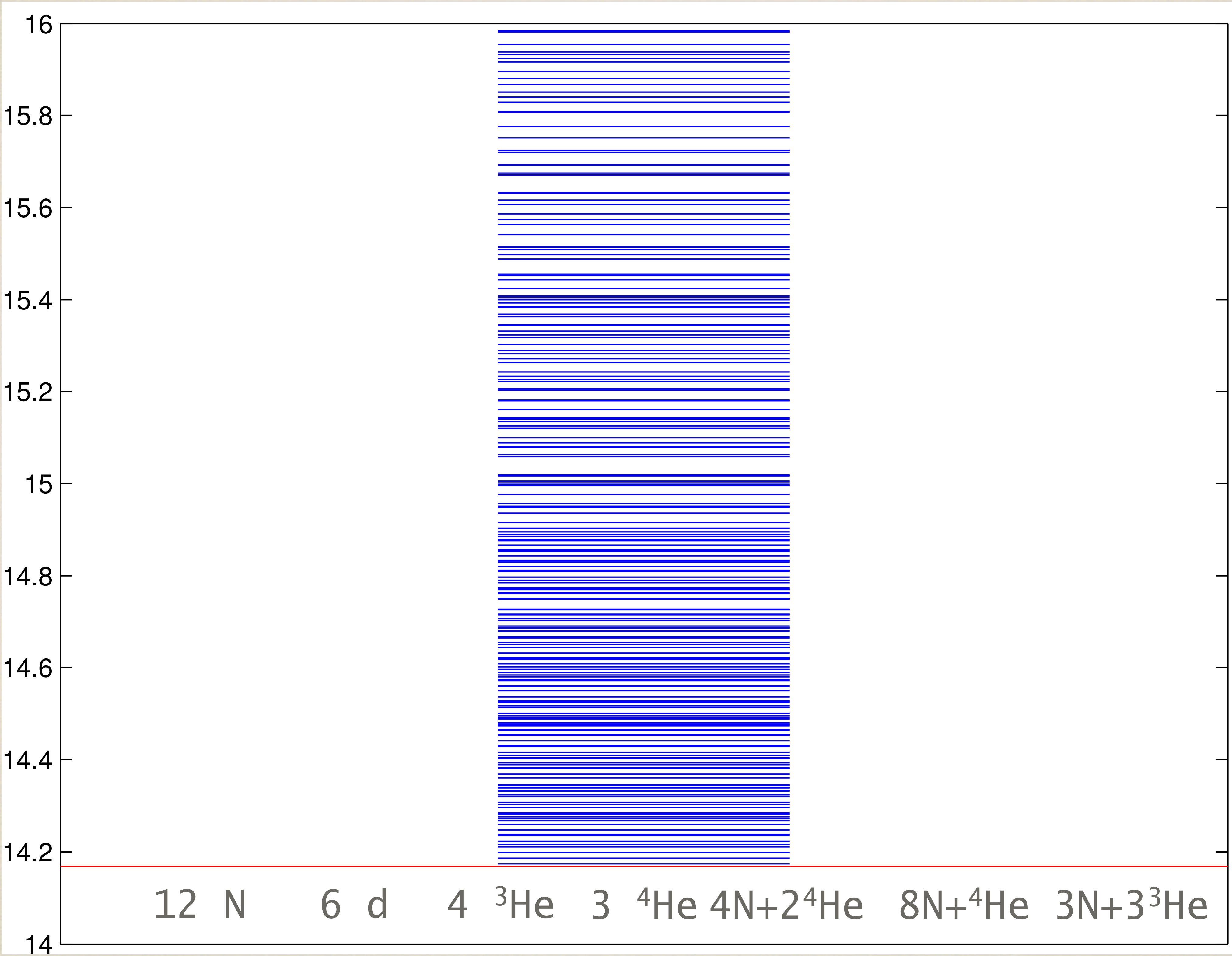
Correlators for large nucleii



Expected Carbon spectrum in the 32^3 box



Expected Carbon spectrum in the 32^3 box



Conclusions

* We have a systematic way of constructing all possible interpolating fields
* Using heavy quarks noise is reduced
* Special care needs to be given to the selection of interpolating fields
    * Minimize number of terms in the interpolating field and optimize the signal
* NPLQCD: Presented results for the spectrum of nuclei with A<5 and S>-3
    * ... and nucleon-nucleon phase shifts

NPLQCD arXiv: 1206.5219,1301.5790

* We have an algorithm for quark contractions in polynomial time for A>5
* Future work must focus on the improved sampling methods to reduce statistical errors at light quark masses